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SCHOOL OF ENGINEERING RESEARCH

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CONTENTS.

PAGE

The School of Engineering Research, by Maitland C. Boswell, M.A., Ph.D. 5

The Investigation of Girderless Concrete Floors, by Peter Gillespie, M.Sc., C.E., M.E.I.C., and T. D. Myhea, A.M.W.S.E. 9

An Improvised Vibrometer, by G. R. Anderson, M.A. 43

Preliminary Report on an Investigation into the Concentration of Graphite from some Ontario Ores, by H. E. T. Haultain, C.E., M.I.M.M., F. C. Dyer, B.A.Sc., and J. T. King, B.A.Sc. 47

New and Rapid apparatus for Electro Chemical Analyses, by J. T. King, B.A.Sc. 69

A Research on a 4-Inch Single Stage Turbine Pump, by J. H. Parkin, B.A.Sc., M.E. 83

The Strength of Cast Iron in Bending as Affected by Variations in Cross Section, by J. H. Billings, B.A.Sc., S.M.; Assoc. Mem. Am. Soc. M.E. 109

Heat Transfer Tests of Building Materials, by L. M. Arkley, B.A.Sc., M.E.I.C. 115

The Fusion of Sodium Hydroxide with Some Inorganic Salts, by Maitland C. Boswell, M.A., Ph.D., and J. V. Dickson, B.A.Sc. 131

The Action of Sodium Hydroxide on Carbon Monoxide, Sodium Fonnate and Sodium Oxalate, by Maitland C. Boswell, M.A., Ph.D., and J. V. Dickson, B.A.Sc. 139

The Fusion of Sodium Hydroxide with Several Phenols and Sulphonic Acids, by Maitland C. Boswell, M.A., Ph.D., and J. V. Dickson, B.A.Sc. 147

The Adsorbtion of Arsenious Acid by Ferric Hydroxide, by Maitland C. Boswell, M.A., Ph.D., and J. V. Dickson, B.A.Sc. . . 157

The Calculation of Transmission Line Networks, by T. R. Rosebrugh, M.A. 169

The Overload Limitations of High-Voltage Single-Turn-Primary Current Transformers, by W. B. Buchanan, B.A.Sc. 191

A Process for Electrolytically Refining Nickel, by G. A. Guess, M.A. 241

THE SCHOOL OF ENGINEERING RESEARCH.

BY MAITLAND C. BOSWELL, M.A., PH.D.
Secretary of The School of Engineering Research.

A School of Engineering Research within the Faculty of Applied Science and Engineering of the University of Toronto was established in the spring of 1917 at the suggestion of Dr. W. H. Ellis. The inauguration and setting in operation of this research movement by Dr. Ellis, who has recently retired from the positions of Head of the department of Chemistry and Dean of the Faculty, is a fitting climax to a life long endeavour in the cause of Applied Science in Canada.

The School is under the direct supervision of a Committee of Management composed of fifteen members of the Faculty Council. To this committee is entrusted the selection of researches from among those proposed by members of the staff, and the disposition of funds for conducting them.

The School was organized chiefly for two purposes. The primary aim is the training of men in research methods, thus meeting in a gradually increasing degree the growing demand for research men by the industries and the government research laboratories about to be established in Canada. The second object is the performance of useful research within the School. The gradual realization of these aims will, the committee believes, not only meet most effectively the insistent demand now being made by almost all the organized sections of the community, that the University shall train men for research work and apply science more extensively to the solution of problems relating to the raw materials and industries of the country, but also accomplish simultaneously in the best possible manner the highest development of the various departments of Applied Science and Engineering within the University. It would seem that any organization of research as applied to industry in Canada is very deficient, which does not adequately recognize the necessity for developing primarily the whole of the Research activities, both pure and applied, in the leading universities of the country.

In the selection of problems the Committee of Management is guided by the best practice of the old and well known university research laboratories, and recognizes the necessity for encouraging two kinds of investigation, viz., the specific problem relating to a specific industry or raw material and having a specific end in view, and the general problem which seeks for some underlying principle or generalization.

It is obvious that such an organization must very quickly exert a marked influence upon the undergraduate body. Contact with a larger number of men performing research will more quickly develop the desirable point of view of the investigator, who reads the current literature of his science and the original sources of information, rather than text-books which very often have a tone of finality, and thus tend to leave the impression of science as a finished product, rather than as an organism undergoing constant change and growth. A new incentive will thus be induced in the student mind, to develop his knowledge and laboratory technique, so that he too may play a part in either extending the bounds of knowledge, or assisting in the development of its applications to industry. The importance of this reaction of the School of Engineering Research upon the undergraduate cannot be overestimated and would alone justify the establishment of such a school.

The extent of the actual results of investigation to be obtained, depends upon a number of factors, most of which at the outset cannot possibly be estimated. A study of the history of research departments of universities the world over, invariably shows, however, that progress is at first very slow, but that the published researches show year by year a decided advance both in quality and quantity. This must be carefully borne in mind both by those conducting the investigations, and by those who are watching the extent of the progress from a distance. Any appraisal of the results for the first year or two upon any basis such as obtains in a business enterprise, would be both very unfair and very unwise.

The degree of success attending the initial efforts of the school has been somewhat greater than could reasonably be expected in the year of its organization, and with an expenditure of only four thousand dollars. The further expansion of this research activity within the Faculty of Applied Science and Engineering, to a degree at all commensurate with the needs of the University and the country, can be realized only by the appointment of more research assistants, the purchase of more apparatus, and above all by the partial relief from lecturing and teaching duties of those members of the staff who are performing or conducting research.

This research movement within the Faculty of Applied Science of the University of Toronto has had a very modest beginning, but the influence which it will exert upon the university and the country must eventually be marked and might be very great. One naturally thinks of the great university research laboratories of the world, justly famous for their mighty contributions to the development of our civilization. Is it too ambitious a goal, to aspire in Canada and in the University of Toronto to an equal participation in this great work? I much mistake the spirit of this country if we are content with any position of inferiority in this regard. Once the people and government really come to realize the importance and magnitude of this task of organizing research, and set about it with a comprehensive plan based on full and correct information, and with a far sighted vision of the future, the final result cannot be in doubt. An adequate support for this great work of scientific research in Canada will be forthcoming only when the government comes to the realization that research places an instrument of incalculable power in the hands of any nation which efficiently, but also at the same time sanely, organizes and develops it, and that in so doing a nation in reality forms an alliance with the forces of nature, an alliance which in power and stability vastly exceeds any political or economic alliance which it is possible for a nation to make.

INVESTIGATION OF GIRDERLESS CONCRETE FLOORS.

Reprinted from Journal of the Engineering Institute of Canada.

BY PETER GILLESPIE, M.SC., C.E., M.E.I.C., T. D. MYLREA, A.M.W.S.E.

INTRODUCTORY.

Until the advent of the reinforced concrete ship, the flat slab type of floor was very properly regarded as the most recent development in reinforced concrete, and for buildings which can be divided into rectangular bays nearly square, it is by far the most popular. The principal reasons for this popularity: (1) ease in construction of forms, (2) economy in steel and concrete, (3) ease in placing steel, (4) saving in story heights, (5) absence of deep shadows on ceiling and (6) adaptability to automatic sprinkler lay-out owing to the absence of beams and girders—are too well known to need more than passing mention. Some conception of this popularity may be had from a study of the records of the Building Departments of some of the larger American cities. In 1916, for example, the total value of buildings of the flat slab type erected in the city of Cleveland was about $1,300,000; in the city of Pittsburg approximately $1,200,000; while in the city of Philadelphia it was over $6,300,000. Information furnished by building commissioners shows that it represents about 7% of the total, and from 40 to 90% of the concrete building operations carried on in recent years in the larger cities of the United States.

The chief difficulty in the design of the flat slab, and the one which delayed its more rapid adoption, is that the stresses therein are not readily subject to analysis. From the year 1905, when it was first proposed by the man who later became its most enthusiastic sponsor, until about 1909 it was bitterly opposed by nearly all engineers because of this difficulty. But by this time it had established itself commercially and as a type of construction whose strength had been proven by many load tests. From 1910 on, many articles based on theoretical considerations, appeared in the technical press suggesting methods of analysis. Many of these, however, have since been modified to accord with principles deduced from extensometer tests.

Coincident with the publication of methods of analysis came a flood of so-called "systems" developed in the effort to avoid patent infringements. These "systems" fall into three general classes, viz.: (1) the 4-way system, in which bands of rods run directly and diagonally between columns, the steel being all or partially raised to the upper part of the slab in the region of the column heads; (2) the 2-way system, in which diagonal bands are omitted, extra steel being placed parallel to the direct bands to compensate therefor; and (3) combinations of the two preceding.

The city of Chicago was the first whose building department was seriously confronted with the issuing of permits for this type of building, and solved the difficulty by appointing commissions of consulting engineers to report on all proposed systems. Approval by a commission was necessary before building permits could be obtained. Subsequently the city authorities required extensometer tests to be conducted until data, judged to be sufficient for the compilation of a city ordinance governing flat slab floors, had been accumulated. Such an ordinance—the first of its kind—was compiled and published in 1914 and remains, with but slight modification in force to-day.

The testing of flat slab buildings under load by means of the extensometer was carried on in the United States through the combined efforts of several universities, construction companies and consulting engineers. Much of the test data was kept as private information by the construction companies concerned. Some of it appeared later in the form of University Bulletins.

Early in 1916 the building department of the city of Toronto was asked for a permit to erect a large warehouse of the flat slab type. As Toronto had no building by-law of its own governing flat slabs, an examination was made of those in force in the principal American cities, but it was discovered that there was a marked lack of agreement among them. The City Architect's Department thereupon decided to grant permits for buildings designed according to the Chicago building ordinance previously mentioned, on condition that the buildings when completed be subjected to extensometer tests. Accordingly five buildings, designed for different live loads, and having various panel lengths and schemes of reinforcement, were subjected to such tests by the City Architect's Department working in conjunction with the Department of Applied Mechanics, University of Toronto.

Of these, the T. Eaton factory, the last one tested, has been chosen as the subject of this paper, because in the conduct of the test, there was available the experience and skill in observation acquired from its predecessors.

The form of the paper is somewhat out of the ordinary, in that all observed data are presented. Great difficulty was encountered by the writers when attempts were made to check the published results of others who had conducted similar building tests. In some cases full data as to the structure and a mere summary of the results of the tests were given. In others, rather more complete results of the tests were given but with an entire omission of the details of reinforcing and other constructional features. In consequence it was impossible to check the conclusions reported from the data available. For this reason it was deemed advisable to give full information as to the structure as well as all the observed data, even though the results have been worked out for one live load only. It is the purpose of the writers to compare the bending moments which were found to exist in the actual slab under test, with those prescribed in the better known regulations governing flat slab construction, namely the "Report of the Joint Committee", "The Recommendations of the American Concrete Institute", and "The Ordinances of the city of Chicago".

THE BUILDING.

The building (see Fig. 1) is twelve stories high and has a frontage of about 228 feet on Alice Street and 166 feet on Downey's Lane. It was designed by the William Steele Co., of Philadelphia, and erected under their supervision by the Raymond Construction Co., of Toronto. To both these concerns, as well as to the T. Eaton Co., and the City Architect's Department, acknowledgment is made for permission to put the results of the test into print.

With certain exceptions the floor slabs were similar and were designed for a live load of 125 pounds per square foot. The reinforcement differed from that in any of the other buildings tested, in that the rods resisting positive moment were independent of those resisting negative moment. In the bottom of the slab, short rods, less than one panel in length, were used as a 4-way reinforcement, while over each column cap a mat composed of rods running in two directions only, parallel to the panel sides resisted the negative moments. Another feature was the use of structural steel columns, enclosed in a concrete envelope. These steel cores in the case of the wall columns, extended to the 8th floor and in the case of the interior columns to the 10th floor. By this means the size of column in the lower stories was much reduced. Fig. 2 shows clearly the scheme of the slab and column construction, while details are shown on Plates I and II.

The reinforcing steel was of the grade known as munition steel and Table 1 gives a summary of tests showing its physical and chemical properties. The structural steel used in the columns was of a much softer quality, as may be seen by comparison with Table 2. The concrete used throughout was a 1-2-4 mix, with a limestone coarse aggregate of $\frac{3}{4}''$ size. At the time the tested floor was poured, control cylinders were taken and tested at the age of 60 days at the University of Toronto.

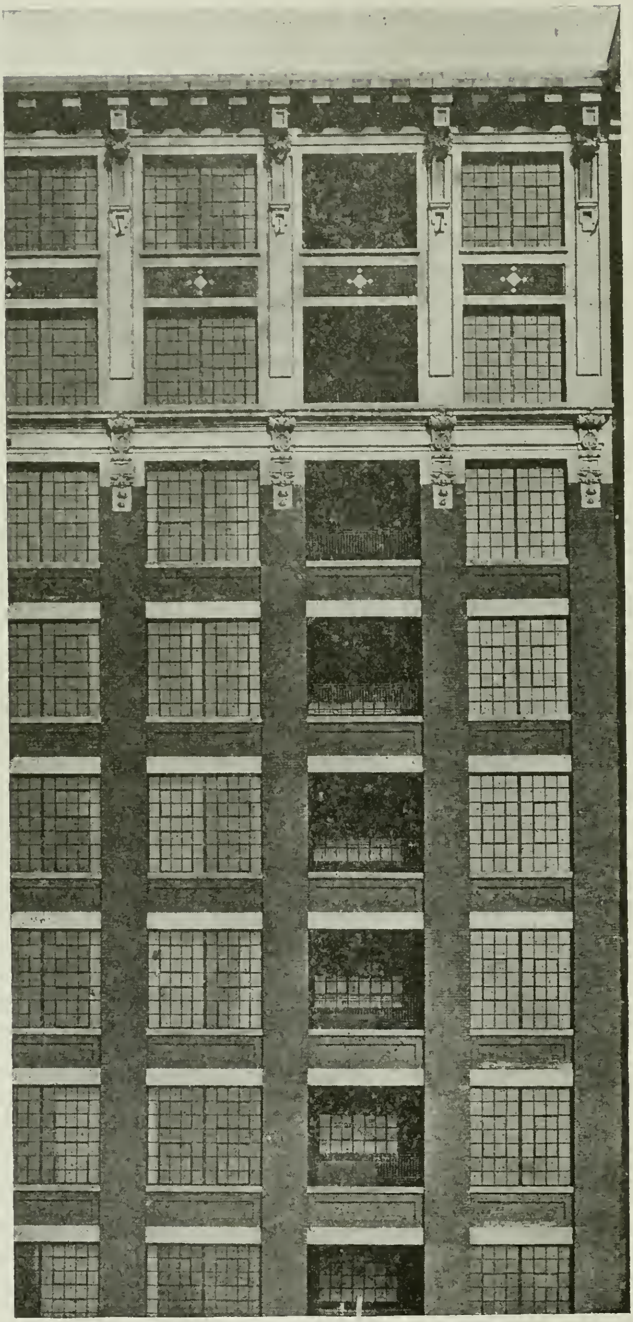


FIG. 1.

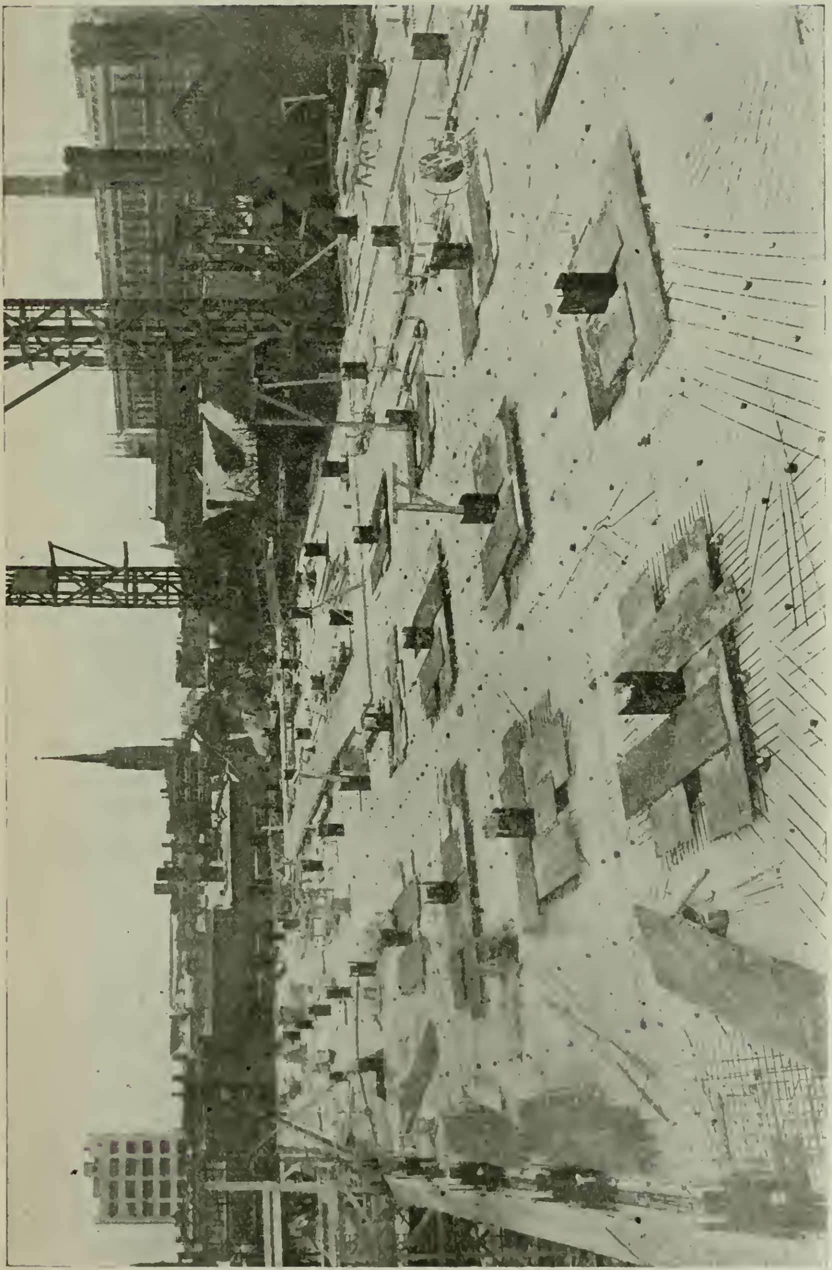


FIG. 2. View showing Slab Reinforcement and Steel Cores of Columns, The T. Eaton Company's New Factory, Toronto.

These cylinders showed an average compressive strength of 2,950 pounds per square inch, and an average modulus of elasticity of 2,350,000 lbs. per square inch which latter value was used in converting concrete deformations into stresses.

THE TEST.

The test was made on four panels on the 8th floor. Observations were also made on one wall column and one interior column in the story below, since it was desired to determine the stresses in the structural steel column cores as well as in the floor slabs. The test load was applied by means of cement in bags, and Fig. 3 shows the maximum load in place.

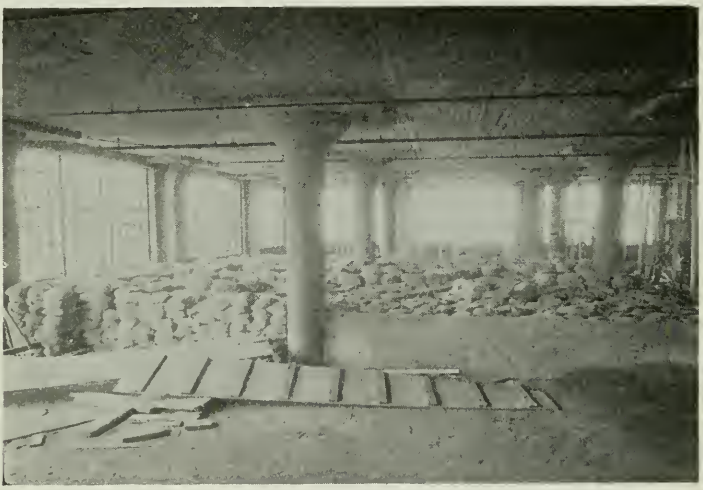


FIG. 3.

TABLE I.
MUNITION STEEL.

Physical Properties					*Chemical Properties*			
Size of rod ins.	Elastic limit lbs. per sq. in.	Ult. strength lbs. per sq. inch.	Elong. in 8 ins. %	Reduction in area %	C.	Mn.	P.	S.
	60610	82420	25.	55.	.30	.56	.015	.042
	49060	83610	30.	44.	.45	.56	.02	.044
1″	43940	83590	29.	55.	.41	.64	.02	.037
	45020	72400	29.	38.	.31	.51	.02	.045
	47320	74800	25.	41.	.31	.53	.02	.04
	44990	72210	25.	38.	.33	.42	.02	.041

TABLE 2.

STRUCTURAL STEEL COLUMN CORES.

Physical Properties				*Chemical Properties*			
Elastic limit lbs. per sq. in.	Ultimate strength lbs. per sq. inch	Elongation in 8 ins. %	Reduction in area %	C.	Mn.	P.	S.
38640	58780	30.	56.	.20	.64	.018	.039
34740	60380	34.	49.	.20	.56	.014	.028
41670	62040	31.	53.	.22	.47	.017	.029
37470	57420	26.	49.	.20	.52	.024	.044
39520	63640	30.	50.	.21	.47	.024	.048

By means of a Berry strain gauge deformation readings were taken on the concrete and reinforcing steel on 243 gauge lines, 89 of which were upon the upper surface of the slab, 121 on the lower surface, 13 on the wall column and 20 on the interior column. The dial of the strain gauge was graduated to read in thousandths of an inch, and with an arm ratio of five to one a change in reading of one division on the dial represented on actual deformation of one five-thousandth of an inch. In the tabulated deformations on Plate III one unit represents one division on the dial of the strain gauge, or a deformation of one five-thousandth of an inch in an eight-inch gauge length. In order that the reinforcing steel might be readily located at the points where gauge holes were to be drilled, small tapered wooden plugs were attached to the reinforcement at these points previous to pouring the concrete, and were readily removed after it had set. Where readings were to be taken

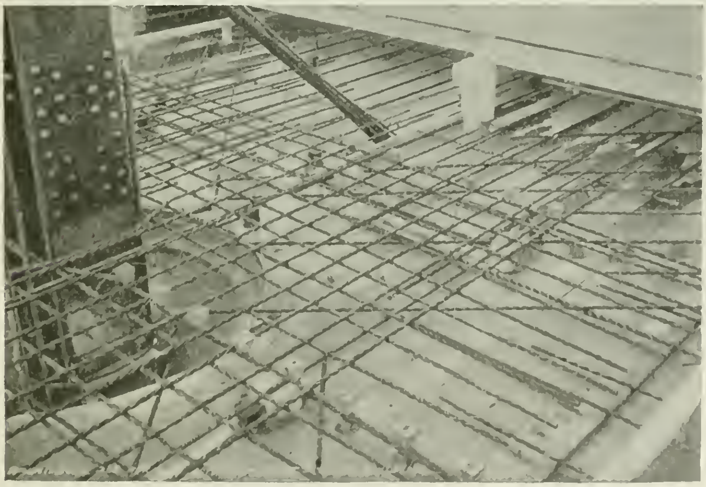


FIG. 4.

on the concrete, small metal plugs were attached to the forms. Removal of the forms exposed the metal surface for the drilling of gauge holes. Figs. 4 and 5 show some of these blocks and plugs in place ready for the pouring of the concrete. The general appearance of these gauge lines when ready for the test may be seen in Figs. 6, 7 and 8 and Plates I and II give their exact location.

There were no construction joints in the floor panels under test, but in the interior column such a joint occurred just below the flared head, and one in the wall column immediately below the bracket as shown on Plate II.

On the 8th floor one series of gauge lines Nos. 201 to 207 was located on the negative reinforcing rod nearest the lintel at column 27. Two other gauge lines Nos. 208 and 209 were located on the outside rod of this band; No. 208 being across the centre line between columns 26 and 27, and No. 209 directly opposite the edge of the dropped head. At column 21 another series of gauge lines Nos. 215 to 221, and 229 and 240 are similarly located, and at right angles to this series were gauge lines Nos. 222 to 228 and 236 to 239 upon the outer and centre rods respectively of the mat. Two rows of gauge lines Nos. 243 to 249 and 261 to 267 were located symmetrically with the centre line of column 20 upon the middle rod of the mat for this column, and gauge lines Nos. 254, 255 and 256 were upon the outside rod of this mat. At column 19 upon the middle and outer rods of the mat were located gauge lines Nos. 270 to 276 and 278 to 284 respectively. Gauge lines Nos. 207, 215, 228, 229, 239, 243, 267 and 270 were less than 8″ from the ends of rods.

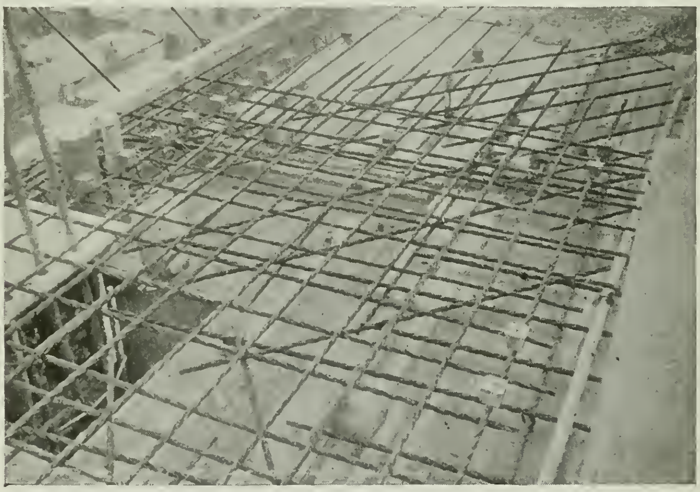


FIG. 5.

For investigation of positive moments at the intersection of diagonal lines in the wall panel and interior panel, two other series of gauge lines Nos. 211 to 214 and 285 to 288 were located, and midway between the columns over the direct bands were located gauge lines Nos. 241 and 268, and 242 and 269 for positive and negative deformations in the concrete. Gauge line No. 210 was located as near the lintel as possible, so that the bending moment in that member might be investigated.

Since there were no diagonal bands to resist negative moment it was necessary to insert steel plugs in the floor surface in order to investigate the deformation along diagonal lines. Such deformations were measured over the edges of the column capitals, and in the neighbourhood of the drop head corners at both wall and interior columns.

The gauge lines on the ceiling were so arranged that as nearly as possible they would pair with corresponding gauge lines on the floor above. In general the ceiling gauge lines were on the steel reinforcement, but on the drop heads, and at No. 20 on the lintel, as well as at several other places where check deformations were measured, they were located on the surface of the concrete. It was hoped that the line of inflection surrounding the column heads might be determined, and gauge lines on both floor and ceiling were located with a view to determining its position.

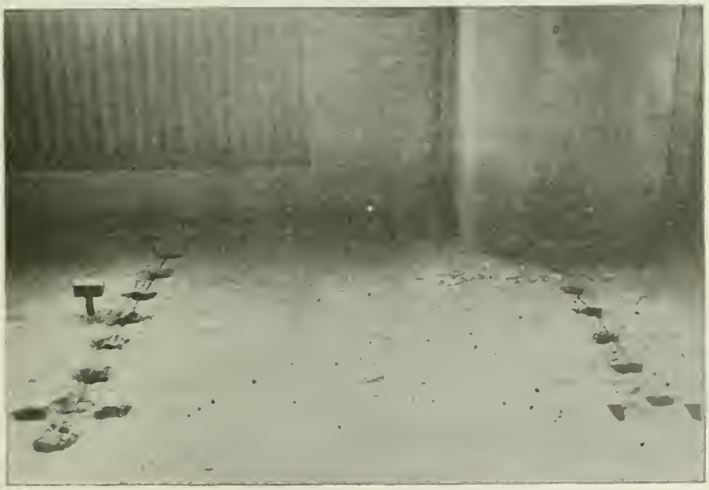


FIG. 6.

The gauge lines on the columns formed an upper and a lower series; the upper one on both wall and interior columns being across the construction joint. On the wall column two gauge lines were located one above the other on each of the four corner reinforcing rods. Two were on the concrete face, and three on the face of the bracket. On the interior column two gauge lines were located one above the other on each corner of the structural steel core and on each of the reinforcing rods. Check readings were taken on the concrete on opposite faces, gauge lines C 14 and C 15 being on the side nearest the windows.

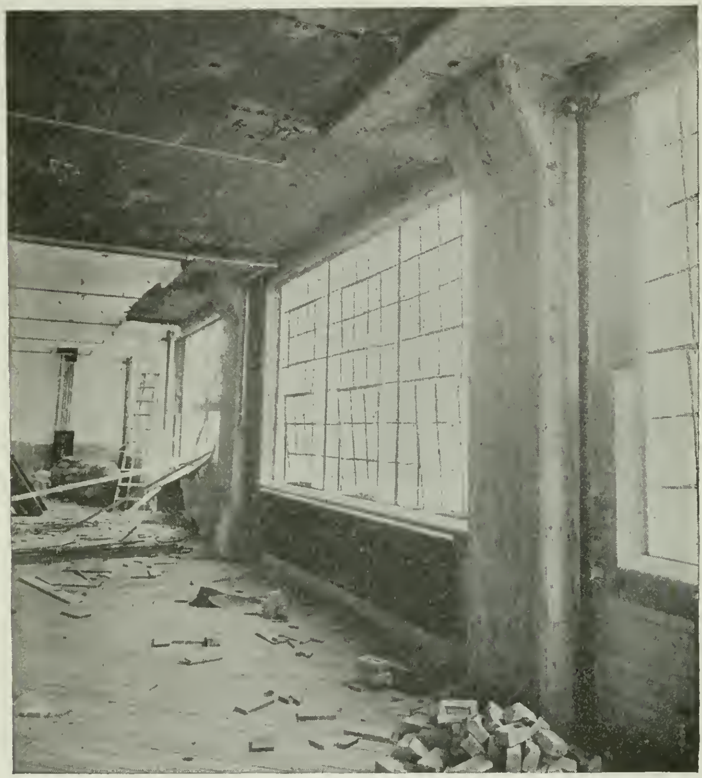


FIG. 7.

Before applying any load, all gauge lines were read and then a load of 125 lbs. per square foot was put on all 4 panels, leaving uncovered only enough room for the observer. To compensate for this sufficient additional material was piled around the edges of the unloaded spaces. When a new set of readings had been taken, an additional 125 lbs. per square foot was added to all 4 panels. This gave a live load of 250 lbs. per square foot or twice that for which the floor was designed. Another set of readings was then taken, and after three days without any alteration in the test load, a further set was taken to note any change in deformations due to the lapse of time. After this the two interior panels were completely unloaded, leaving 250 lbs. per square foot on the two wall panels. The deformations were again observed and the two wall panels unloaded. After the complete removal of the load, a final set of readings was taken to observe the extent of the permanent set.

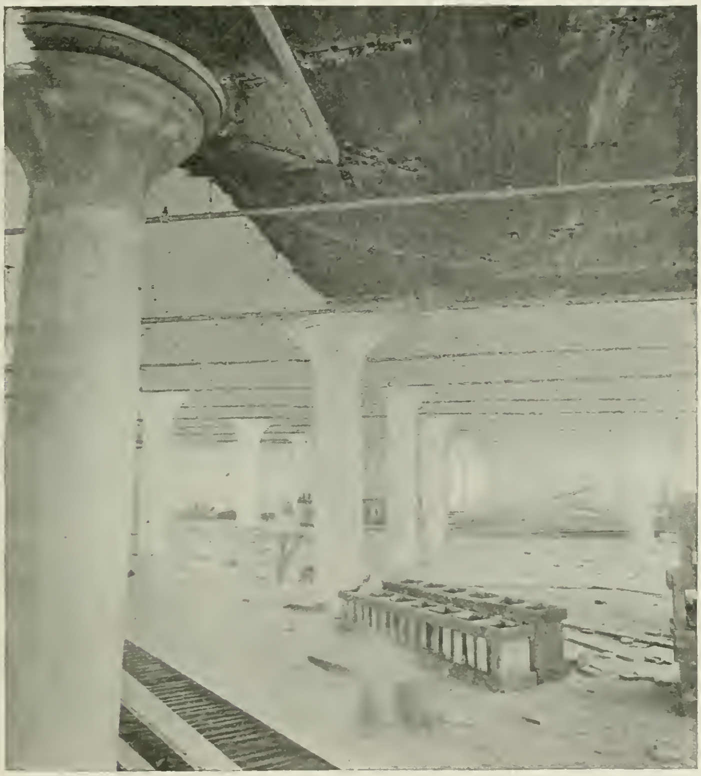


FIG. 8.

Plate III gives the observed deformations under the various conditions of loading, and on Plates I and II these deformations have been converted into stresses in the usual way, assuming the modulus of elasticity for concrete to be the same in tension as in compression.

DEFLECTIONS.

In this test no attempt was made to determine deflection. In some of the previous tests attempts had been made but the difficulties encountered far outweighed the information obtained. In the earlier tests where deflections were measured a substantial scaffold was erected to within a few inches of the ceiling and an Ames dial mounted on a suitable base was used as a deflectometer. Another scaffold, in no way connected to the first one, was erected as an observers' platform. It was often found that the workmen drove spikes into the framing upon which to hang coats with the result that reliable readings were almost impossible to obtain. Fences or barricades merely served to attract attention.

Plate IV gives deflection contours of the slab surface of the Robert Simpson Building under the design load, the vertical interval being .01 of an inch. This diagram is presented for the reason that the recorded deflections for this test were considered the most reliable. From such a diagram it might be possible to check computed moments by means of the slope deflection method. No attempt has been made to make such a check, however, since this deflection diagram pertains to another building.

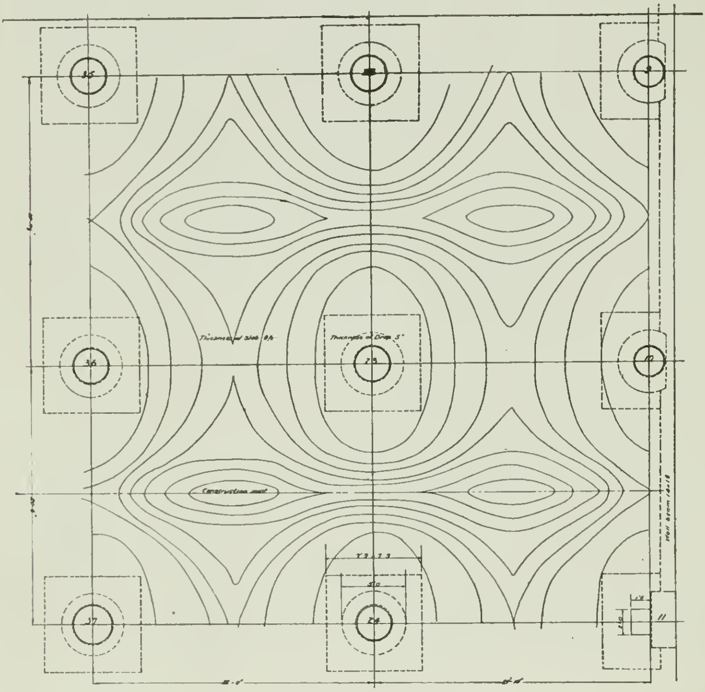
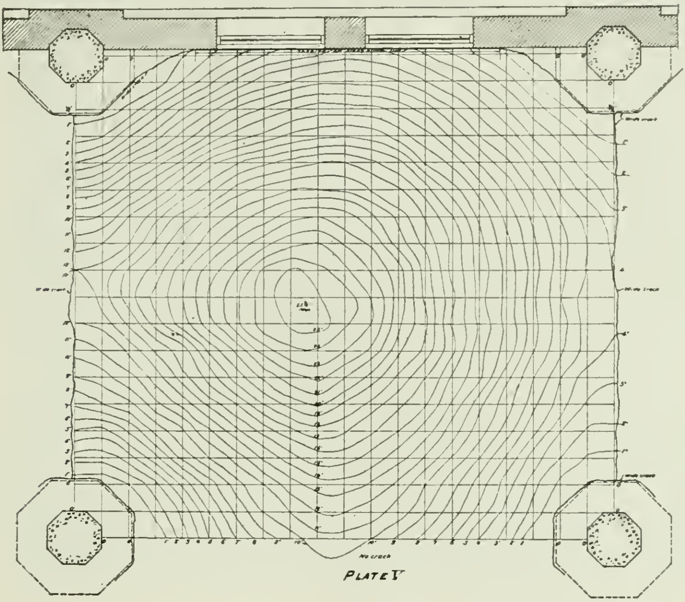


PLATE IV

One peculiarity of the diagram seems to merit attention. It will be noticed that across a line joining wall column No. 10 with the interior columns Nos. 23 and 36 the curvature appears to be more abrupt than that across a line parallel to the wall joining interior columns. This would seem to be the opposite of what would be expected, for with a comparatively rigid lintel along one edge, one could readily conceive of the wall panels troughing parallel to the wall. A striking confirmation of this peculiarity was afforded by the excessive sagging of the floor slabs in the concrete warehouse of the Quaker Oats Co., Peterborough, Ont., which was destroyed by fire December 11th, 1916. Before the building was razed deflection readings were taken on one typical sagging wall panel, the results of which are plotted on Plate V. The vertical interval in this case is one inch instead of .01 inch.



CRACKS.

Under a live load of 125 lbs. per square foot no cracks were visible, but with the increase to 250 lbs. per square foot they became apparent. Those which were visible to the naked eye both on the upper surface of the slab and on the ceiling have been carefully mapped on Plates I and II. The phenomenon of troughing perpendicular to the wall is again indicated by the formation of a very clearly defined crack in the upper surface of the slab, extending from column No. 19 to column No. 21 in an almost unbroken line. Fig. 9 is a photograph tracing the course of this crack between two columns and Fig. 10 is a view of it at close range. It will be noted that the crack extends into the slab to the reinforcement, thus indicating the necessity of providing reinforcing metal across the direct bands.

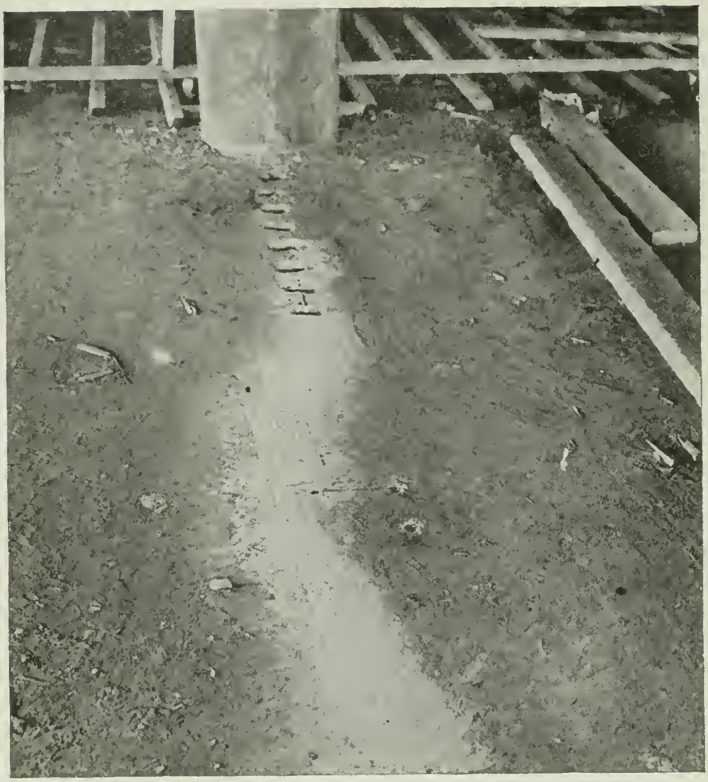


FIG. 9.

In the tensile stresses tabulated on Plates I and II this cracking of the concrete must be taken into account particularly where the gauge line is on the concrete rather than on the reinforcement. An illustration will make this clear. Take as an example gauge line No. 250 on the upper surface of the slab near column No. 20, where no diagonal reinforcement existed. It will be noticed that a crack in the concrete occurred directly between the two gauge points under a 250 lb. load, and that the indicated tensile stress therein is 1789 lbs. per sq. inch, increasing to 2580 lbs. per sq. inch after the load had been in place three days, and further increasing to 3,320 lbs. per sq. inch when the load was removed from the two interior panels. These stresses are clearly impossible, for a live load of 125 lbs. per sq. foot produced a tension of only 258 lbs. per sq. inch in the concrete, which was probably very nearly its ultimate value at this place. When the crack occurred the tension for some little distance on both sides of it was relieved resulting in the drawing back of the edges and the indication of excessive deformations. Had a gauge line on diagonal reinforcement been possible, the indications would doubtless have been different.

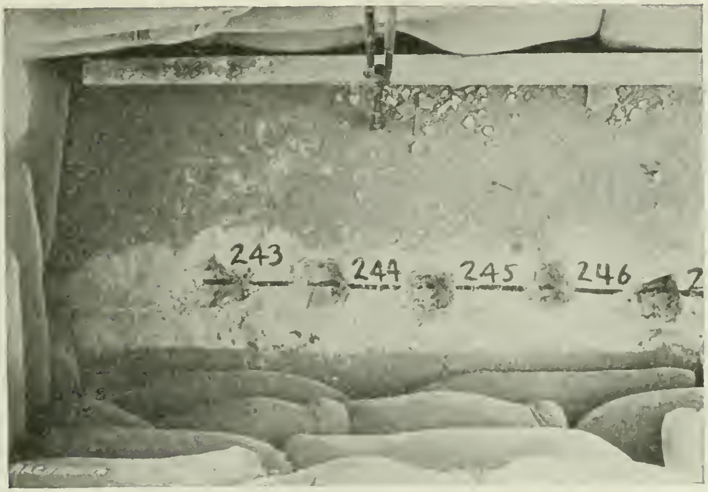


FIG. 10.

THE LINE OF INFLECTION.

The position of the line of inflection was readily determined by following radially from the columns several series of consecutive gauge lines and noting where the tabulated stresses changed from negative to positive or *vice versa*. On the floor it was found to be at an average distance of about 5′ 6″ from the central column, while on the ceiling it varied from 6′ 0″ in the interior panel to 6′ 6″ in the wall panel. A general average would be about .28L which is larger than the .2L commonly assumed. The reason for this probably lies in the fact that the drop panels were larger than usual, being about .38L as against .33L found in common practice. It will be noted that in the wall panel the centre-to-centre distance between columns is smaller than elsewhere. The effect of the large drop panels is thus increased, causing the line of inflection to move toward the centre of the span.

On the ceiling the distance from column centre to line of inflection appears to be greater than on the floor. A horizontal thrust of some magnitude due to arch action existed in the slab, and was probably the cause of the difference. Assume for example, that at a certain section of a slab, a negative bending moment exists producing tension on the upper surface and compression on the lower surface. Any central thrust acting toward the support would decrease the tension in the upper surface of the slab thus bringing the line of inflection closer to the support. At the same time it would increase the compression at the lower surface, which would have the effect of moving the line of inflection there, farther from the support. It will be seen later that the line of action of the thrust was practically central in the region of contraflexure.

TENSION IN CONCRETE.

The assumption that in a reinforced concrete beam the concrete takes no tension, is known to be erroneous. The same holds true for the flat slab. Under moderate loads a very large amount of the tensile resistance is undoubtedly furnished by the concrete. As the loading increases and the tensile deformations become greater than the extensibility of the concrete, rupture occurs at the surface. With the continued addition of load the rupture approaches nearer and nearer to the neutral surface, and owing to its shortened lever arm, the resultant of the tensile stress in the unbroken concrete supplies a resisting moment of diminishing magnitude. For this reason it is justifiable in design to neglect the part that the tensile resistance of the concrete may play. A casual inspection of the tabulated deformations on Plate III and stresses on Plates I and II, will make this point clear, for it will be noted that the deformations and stresses as recorded in column No. 2 are out of proportion to those appearing in column No. 1. The sudden increase with the change of load is probably due to the fact that rupture of the concrete occurred, throwing upon the reinforcement a larger proportion of the tension.

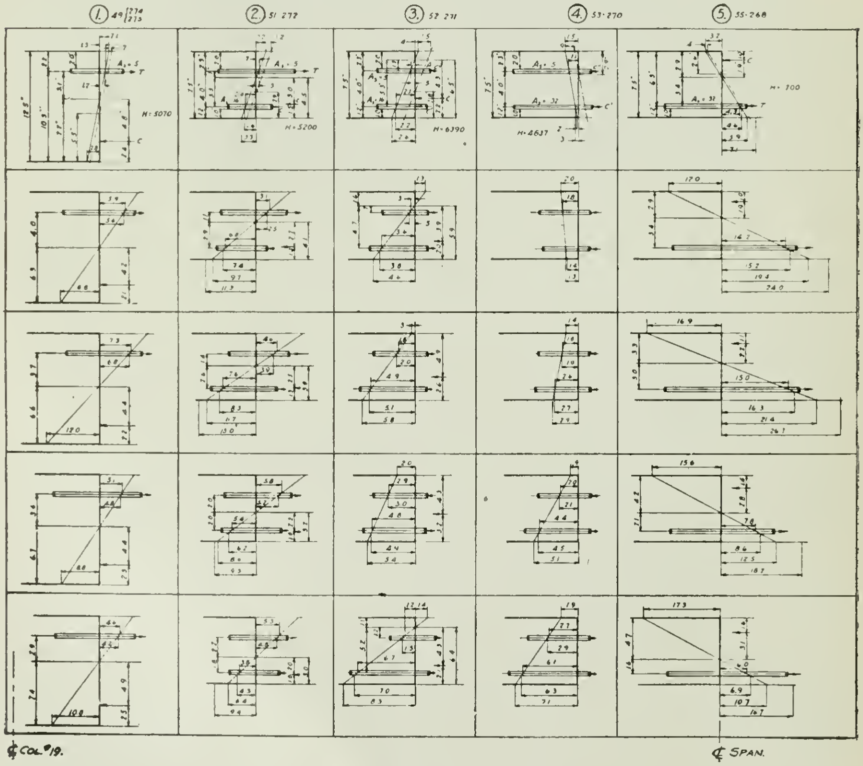


PLATE VI.

The stress at which concrete ruptures is a variable quantity even in the same slab. In order to take account of it in the computations it was necessary to fix upon some arbitrary value. After an examination of the results of computations based on various assumed values of strength and a consideration of the tests on the control cylinders, this was finally placed at 200 pounds per square inch. It was assumed that unobserved stresses due to dead load are proportional to those produced by live load. If, for example, the dead load is 94 lbs. per sq. ft., the live load 125 lbs. per sq. ft. and the observed stress due to the latter in a specific instance is 115 lbs. per sq. inch, the assumed unobserved stress due to dead load only, is $\frac{94}{125} \times 115$ or 85 lbs. per sq. inch. The total existing stress is therefore the sum of these or 200 lbs. per sq. inch, the ultimate tensile value assumed as above. This condition would be produced by an actual tensile deformation of 3.4 divisions on the dial or by an indicated tensile deformation of 1.93 divisions.

Plates VI and VII have been prepared showing to scale the deformations at 17 pairs of gauge lines on the upper and lower surfaces of the

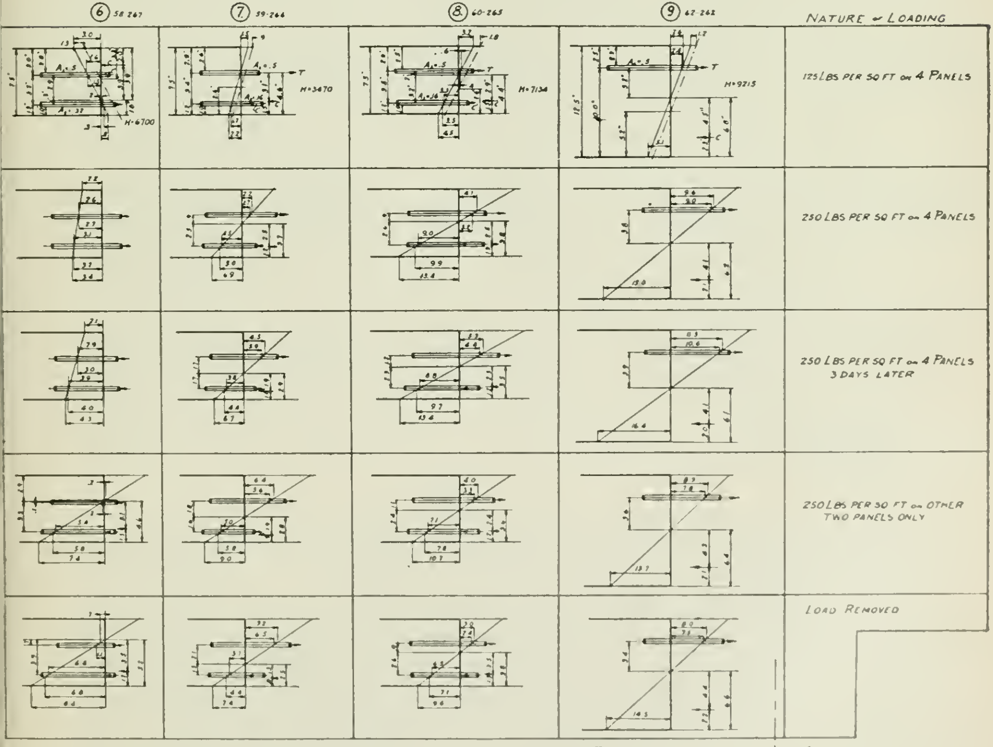

⑥ 58 267
⑦ 59-266
⑧ 60-265
⑨ 62-262
NATURE OF LOADING
M=6700
M=3470
M=7134
M=9215
125 LBS PER SQ FT ON 4 PANELS
250 LBS PER SQ FT ON 4 PANELS
250 LBS PER SQ FT ON 4 PANELS 3 DAYS LATER
250 LBS PER SQ FT ON OTHER TWO PANELS ONLY
LOAD REMOVED
℄ COL. #20.

PLATE VI

slab between columns Nos. 19 and 21. The thickness of the slab, the amount of reinforcement present and its position are shown, as is also the loading which produced these deformations. A line joining the ordinates representing the respective deformations on the upper and lower surfaces, intersects the vertical plane of the section at the neutral axis in accordance with usual assumptions. On these diagrams interesting conditions are revealed graphically. In any vertical series for example, the shifting of the neutral axis at the section due to changing load is shown. Again under constant load, the movement of the neutral axis and the variation in stress from section to section can be observed by following any horizontal series.

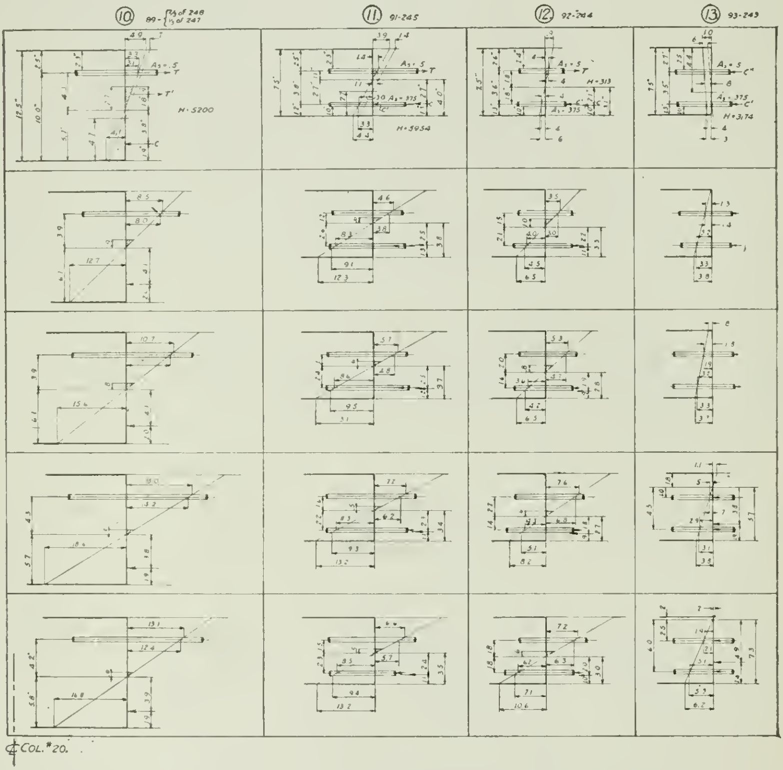


PLATE VII.

In many places particularly under the heavy loads, it will be noticed that the compressive deformations indicated appear to be excessive. This is observed at pair No. 9 after the full load had been left in place for three days and indicates that the linear law had probably been exceeded. This assumption seems to be verified by pair No. 2 where under a live load of 250 lbs. per sq. ft. the indicated deformation in the concrete is 11.3 divisions while the computed deformation as figured from the corresponding steel deformation is but 9.7. In the reduction of the data however this has been ignored and the linear law assumed to be in operation. An analogous condition obtains at pair No. 5. These facts should be kept in mind in considering the question of thrusts along lines connecting columns.

As indicated above, at any point on the tensile side of a neutral surface where the scaled deformation exceeds 1.93 divisions concrete is assumed to have ruptured.

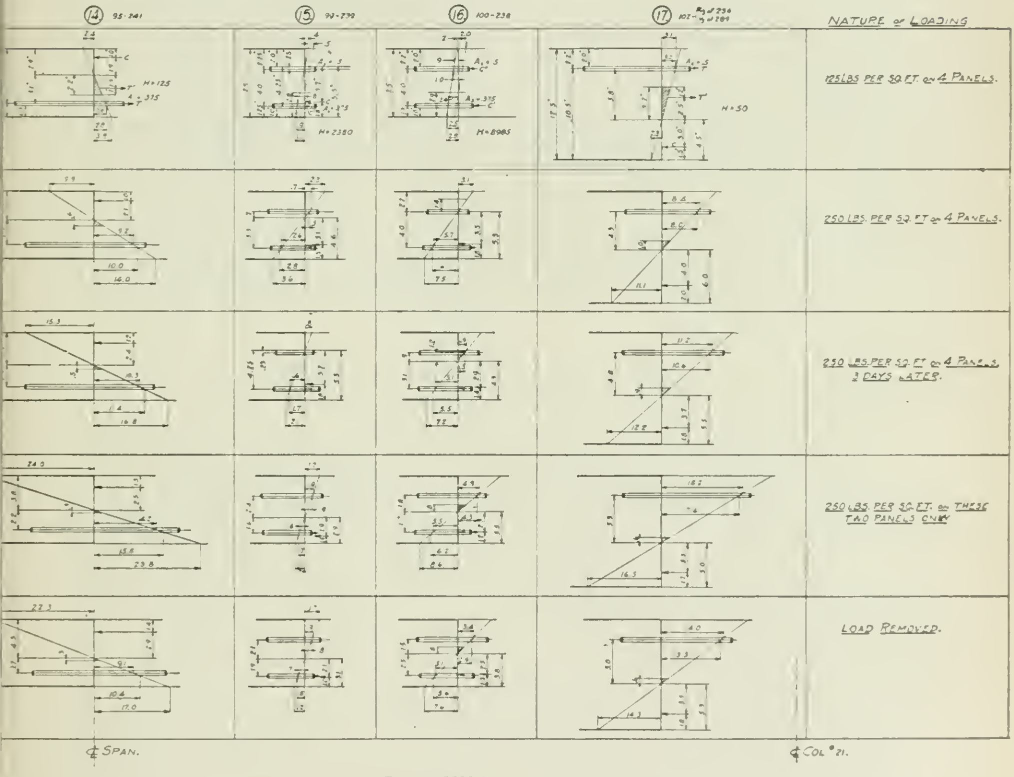

⑭ 95-241
⑮ 97-239
⑯ 100-238
⑰
NATURE OF LOADING
H=125
H=2380
H=8985
H=50
125 LBS PER SQ. FT. ON 4 PANELS.
250 LBS. PER SQ. FT. ON 4 PANELS.
250 LBS. PER SQ. FT. ON 4 PANELS, 3 DAYS LATER.
250 LBS. PER SQ. FT. ON THESE TWO PANELS ONLY
LOAD REMOVED.
℄ SPAN.
℄ COL. #21.

PLATE VII.

COMBINED BENDING MOMENTS AND THRUSTS.

Preliminary to a further examination of Plates VI, VII and IX a brief consideration will be given to the phenomena of thrust combined with bending. Assume a beam of solid section and of homogeneous material of cross-section 6″ × 6″ subjected to such an external action that stresses on two opposite faces of 300 lbs per sq. inch in compression and 100 lbs. per sq. inch in tension exist, these stresses having been presumably observed with the aid of an extensometer. In accordance with usual assumptions, the average stress existing at the cross-section (and the actual stress at the centre of gravity) is a compression of 100 lbs. per sq. inch and the consequent resultant thrust is 3,600 pounds. Now the observed phenomena of stress in this instance may be due to: (1) a central thrust of 3,600 lbs. plus an external bending moment of 7,200 in-lbs. or (2) a thrust of 3,600 lbs. acting with an eccentricity of 2 ins. toward the face which is known to be in compression.

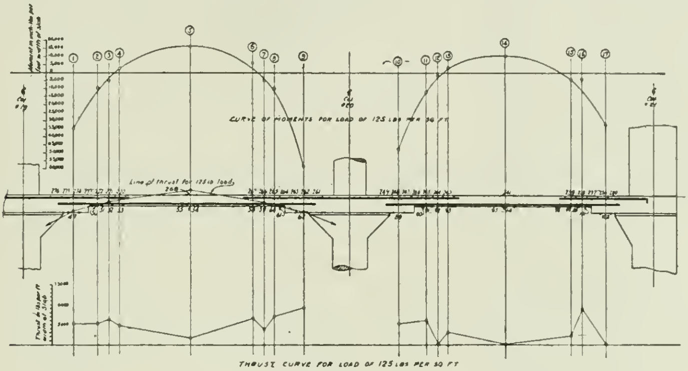
A little reflection will show that the external bending moment is a maximum when the eccentricity is zero, and is zero when the eccentricity is at its maximum of 2 inches, the thrust being constant. Indeed the two conditions above indicated are but the limits in an infinite series, the following being some of the intermediate steps: (3) a thrust of 3,600 lbs. whose eccentricity is 1 inch combined with a true external bending moment of 3,600 inch-pounds and (4) a thrust 3,600 lbs. whose eccentricity is $\frac{1}{2}$ inch combined with a true external bending moment of 5,400 inch-pounds. The member, it will be seen, may act as a true column with eccentric load or one of many combinations of true column and true beam; which of these the observer has perhaps no means of knowing. Of one thing, however, he may be sure and that is the magnitude of the thrust.

THRUSTS.

In this test, much evidence as to the presence of thrust has been found, as an examination of Plates VI, VII and IX will show. In all instances where tension exists in the concrete at a section it is impossible to be certain as to the amount of the thrust. In such cases the values given are to be regarded as indications of what probably occurs. Where observation showed compression to exist in both faces of the slab, there is little doubt as to the extent of the thrust, except such as is due to the inevitable inaccuracies of the instrument which was employed.

The deformations indicated on Plates VI, VII and IX were converted into stresses in the usual way and the total tension and total compression for one foot width of slab found for each pair of gauge lines under the 125 lb. load. In determining the amount of tension taken by the concrete it was considered advisable to ignore all concrete nearer to the surface of the slab than the outer fibre of the tensile reinforcement because this concrete in all cases had been much cut up by gauge holes, and as before mentioned, only that concrete between the neutral axis and the point where its extension would indicate a stress of 200 lbs. per sq. inch due to dead load and live load was considered. When the total tension at any one section was compared with the total compression at the same section, it was found that in every case the total compression exceeded the total tension, and the amount of this excess, for the 125 lb. load has been designated by the letter "H"on these diagrams.

With the span as an axis for ordinates, these thrusts have been plotted vertically in Plate VIII and the thrust curve drawn. The features of the curve are that the thrust apparently increases toward the column and diminishes toward midspan, and that the thrusts are generally less in the wall panel than in the interior one. This latter phenomena is really not difficult to understand since the capacity of wall columns to resist thrust is obviously less than that of interior columns. Why the thrust should augment toward the column is, however, not so clear. Had the slab consisted of a series of radiating spoke-like elements, each independent of its neighbour except at the common hub-like drop panel, the increased radial compression toward the centre would be perhaps capable of explanation. But the independence of radiating elements in the slab cannot, by any reasonable effort of the imagination, be said to exist.


①
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⑨
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⑪⑫⑬
⑭
⑮⑯
⑰
CURVE OF MOMENTS FOR LOAD OF 125 LBS PER SQ FT
Line of thrust for 125 lb load
268
THRUST CURVE FOR LOAD OF 125 LBS PER SQ FT

PLATE VIII.

On the same drawing is shown an approximate line of thrust for the slab considered as an arch. It was constructed on the assumption that the slab behaved entirely as a continuous column without the existence of beam action as such. This is equivalent to assuming the existence of maximum eccentricity and that all bending is due thereto. With the thrust at each section known, its point of application was determined. These points were plotted in position on the cross-section of the slab and connected as shown.

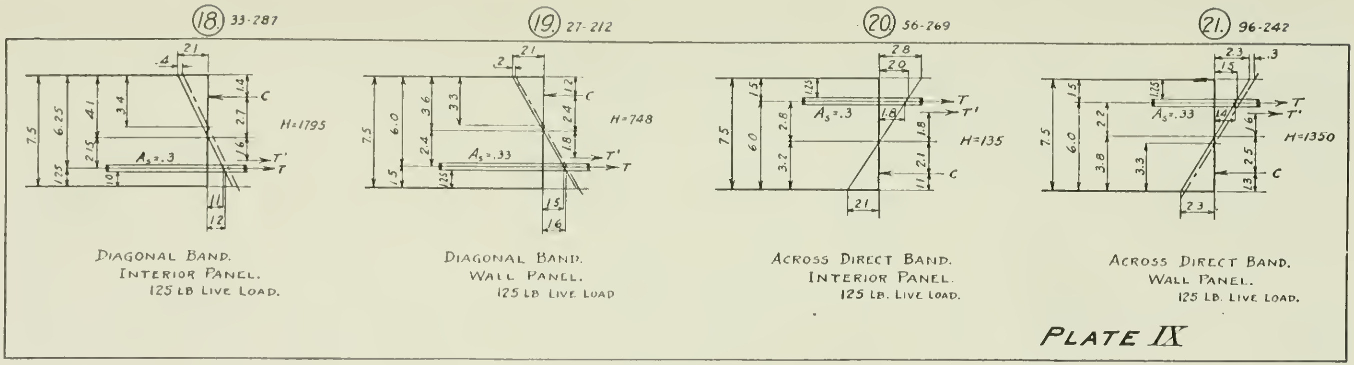
MOMENTS.

For the derivation of bending moment coefficients it is better to consider that all the bending is due to beam action. This is equivalent to saying that the thrusts act at the centres of gravity of the various sections. Plates VI, VII and IX were again made use of in the following manner to determine these coefficients. The thrust existing at each section was divided by the transformed areas of the section, neglecting as in the previous case, all concrete on the tension side of the slab more remote from the neutral surface than the outer fibre of the tensile reinforcement. This gave the average stress due to thrust from which was determined the corresponding deformation. Since the thrust was assumed as being applied at the centre of gravity, the deformation would be uniform over all portions of the cross section.

On each section another deformation line was drawn, parallel to the original and at a distance from it equal to the deformation corresponding to the average stress due to thrust. Thus for each section a new neutral surface was located depending solely on the beam action existing. Values of k and j were then readily computed, and from the formula $M=\frac{e_c}{2}bd^2kj\times 58.7$, the resisting moments in inch-pounds for a strip one foot in width were found for all sections, e_c being the maximum compressive deformation at the section in dial divisions. The constant 58.7 is the stress in the concrete corresponding to a reading of one division on the dial. Since, as may be seen from an inspection of Plates I and II, the stress at the edge of a band of reinforcement is practically the same as that at the centre of a band, the resisting moments for the one-foot strip above mentioned were multiplied by the half panel width, to give the moments for the respective "strips" or "sections" of the various ordinances with which comparison was made.

Bending moments at the various sections are usually expressed as $M=CWL$, in which C is a constant depending upon the location of the section under consideration, W is the total panel load, and L the panel length. Given the values of M, W and L, the coefficient C may be determined from the formula $C=\frac{M}{WL}$. For the interior panel under consideration $WL=125\ (20.25\times 21)\ 20.66\times 12=13{,}170{,}000$ inch-pounds. For the wall panel $WL=125\ (21.0\times 18.7)\ 19.9\times 12=11{,}720{,}000$ or $125\ (18.7\times 18.7)\ 18.7\times 12=9{,}820{,}000$ depending on whether the diagonal or the direct bend is under examination. The coefficients may therefore be found at any point by dividing the moment of a half panel strip by the proper value of WL. Plate X gives in compact form the computations for selected gauge lines, and also a comparison of the constants at the critical points with those prescribed in various ordinances.

The American Concrete Institute specifies a total moment for four "sections" and subdivides it into four parts, giving a minimum require-



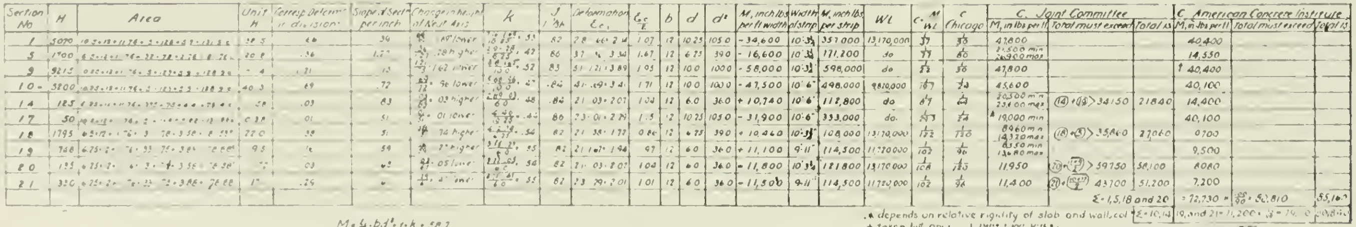
Diagonal Band. Interior Panel. 125 lb Live Load.

Diagonal Band. Wall Panel. 125 lb Live Load

Across Direct Band. Interior Panel. 125 lb. Live Load.

Across Direct Band. Wall Panel. 125 lb. Live Load.

PLATE IX



Section No	H	Area	Unit H	Corresp. Deform. in divisions	Slope of Sect. per inch	Change in height of Neut. Axis	k	j 1-k/3	Deformation ε_c	$\frac{E_c}{E}$	b	d	d²	M, inch lbs per ft width	Width of strip	M, inch lbs per strip	Wl	$C=\frac{M}{Wl}$	C Chicago	C. Joint Committee: M, in lbs per ft	C. Joint Committee: Total must exceed	C. Joint Committee: Total is	C. American Concrete Institute: M, in lbs per ft	C. American Concrete Institute: Total must exceed	C. American Concrete Institute: Total is
1	5070	[illegible]	29.5	.46	.34	[illegible] .49 lower	[illegible] .33	.82	2.8 [illegible] 2.4	1.07	12	10.25	105.0	-34,600	10'3½	357,000	13,170,000	$\frac{1}{37}$	$\frac{1}{38}$	47,800			40,400		
5	1900	[illegible]	20.8	.36	1.2[illegible]	[illegible] .28 higher	[illegible] .42	.86	3.7 [illegible] 3.34	1.67	12	6.25	39.0	-16,600	10'3½	171,200	do	$\frac{1}{77}$	$\frac{1}{80}$	21,500 min 26,900 max			14,550		
9	9215	[illegible]	[illegible]	.31	.13	[illegible] .162 lower	[illegible] .37	.83	3.1 .121 3.89	1.95	12	10.0	100.0	-58,000	10'3½	598,000	do	[illegible]	$\frac{1}{56}$	47,800			† 40,400		
10-	3800	[illegible]	40.3	.69	.72	[illegible] .96 lower	[illegible]	.86	4.1 .69 3.4	1.71	12	10.0	100.0	-47,500	10'6"	498,000	9,810,000	[illegible]	[illegible]	45,600			40,100		
14	125	[illegible]	.58	.03	.83	[illegible] .03 higher	[illegible] .48	.84	2.1 .03 2.07	1.04	12	6.0	36.0	+10,740	10'6"	112,800	do	$\frac{1}{87}$	[illegible]	30,500 min 23,400 max	⑭+(⑲) > 34150	21840	14,400		
17	50	[illegible]	[illegible]	.01	.51	[illegible] .01 lower	[illegible] .44	.86	2.3 .01 2.79	1.5	12	10.25	105.0	-31,900	10'6"	333,000	do.	[illegible]	[illegible]	* 19,000 min			40,100		
18	1795	[illegible]	22.0	.38	.51	[illegible] .74 higher	[illegible] .56	.82	2.1 .38 1.77	0.86	12	6.75	39.0	+10,460	10'3½"	108,000	13,170,000	$\frac{1}{122}$	$\frac{1}{130}$	8960 min 14,320 max	⑱+(⑤) > 35,860	27,060	9700		
19	748	[illegible]	9.5	[illegible]	.59	[illegible] higher	[illegible] .55	.82	2.1 [illegible] 1.94	.97	12	6.0	36.0	+11,100	9'11"	114,500	11,720,000	$\frac{1}{102}$	$\frac{1}{96}$	6550 min 13,680 max			9,500		
20	135	[illegible]	[illegible]	.03	[illegible]	[illegible] .05 lower	[illegible] .56	.82	2.1 .03 2.07	1.04	12	6.0	36.0	-11,800	10'3½	121,800	13,170,000	$\frac{1}{108}$	[illegible]	11,950	⑳+([illegible]) > 59750	58,100	8080		
21	350	[illegible]	17	.26	[illegible]	[illegible] lower	[illegible] .55	.82	2.3 .29 2.01	1.01	12	6.0	36.0	-11,500	9'11"	114,500	11,720,000	$\frac{1}{102}$	$\frac{1}{96}$	11,400	㉑+([illegible]) 43700	51,200	7,200		
																					Σ = 1, 5, 18 and 20		= 72,730 ×	$\frac{85}{90}$ = 50,810	55,160
																						*Σ = 10, 14,	19, and 21 = 71,200 × [illegible] = [illegible]		60,8[illegible]

$M = \frac{1}{2} \cdot bd^2 \cdot j \cdot k \cdot 587$
ε_c being compressive deformation due to bending

* depends on relative rigidity of slab and wall col.
† taken but once [illegible]

PLATE X

ment for each of the four. The aggregate of these four comprises but 90% of the total specified moment, it being assumed that the remaining 10% will be distributed according to the judgment of the designer. Consequently the computed moments have been added and multiplied by $\frac{100}{90}$ for comparison with the sum of the actual moments. Without such an augmentation the American Concrete Institute moments appear small.

On the whole, the coefficients derived correspond very closely with those prescribed by the Chicago Building Ordinance and by the American Concrete Institute. At sections No. 1 and No. 17 the former being adjacent to an unloaded panel and the latter to the wall, the coefficients are almost identical and slightly lower than those prescribed by the Chicago ordinance. At sections No. 9 and 10, the coefficients are again almost identical, but in this case somewhat larger than those prescribed by the Chicago ordinance. Had the load extended indefinitely in all directions, the coefficients for sections No. 1 and 17 would probably have been larger, and those for sections 9 and 10 smaller. At midlength of the direct band in the interior panel, section No. 5, the coefficient is again somewhat larger than that prescribed by the Chicago ordinance, which is again due no doubt to the unloaded condition of the adjoining panel. The influence of the large drop head on the short panel lengths between columns 20 and 21 may be seen in the fact that at section No. 14 the coefficient is considerably smaller than that required by the Chicago ordinance. In general the bending moment coefficients as determined from this test approximated very closely those prescribed by the city of Chicago.

As a practical suggestion for the consideration of thrust in design, it might be feasible to so modify the working stresses in concrete and steel by lowering the former and raising the latter so that these when corrected by the effect of the thrust would approach those values in reinforced concrete design which present practice favours. To illustrate, consider a slab in which sufficient steel is imbedded to take care of tension in bending. Suppose a thrust of 7,200 lbs. per square foot, acting centrally, is anticipated in the constructed slab. This will mean 50 lbs. per square inch compression in the concrete and 750 lbs. per square inch compression in the steel, over and above that which beam action would create. If then instead of stresses of say 750 lbs. per square inch in the concrete and 17,000 lbs. per sq. inch in the steel, which present practice seem to favour, values of 700 lbs. per sq. inch and 17,750 lbs. per sq. inch be adopted for flat slab design, the stresses resulting from the combination of bending and thrust in the finished structure in service would approximate more nearly those which the designer anticipated.

In general the thrust would not affect the thickness of the slab at sections remote from the column, since at these points the percentage of steel is so low that the concrete stresses are quite small, as is verified by the tabulated values. Around the column capital and near the periphery of the drop head on the under surface of the slab, the compression due to beam action will be considerably augmented. That this is not a mere assumption, but an actual fact, is shown by the comparatively high stresses recorded under the heavier loadings. There has been a tendency on the part of some to lower unduly the bending moments and increase the allowable stresses in flat slab design. While this may be satisfactory on sections remote from the columns, great care should be exercised in formulating regulations governing the depth and width of drop panels in order that compressive stresses due to negative moment and thrust may not run too high.

BENDING MOMENTS IN COLUMNS UNDER ECCENTRIC LOADINGS.

On plate III will be found in addition to the deformations obtained on the gauge lines on the slab, those located on the wall and interior columns. It will be observed that for the interior column under an applied live load of 125 lbs. per sq. ft. the average deformations are 2.63 and 2.06 divisions for the lower and upper groups respectively. Similarly for an applied live load of 250 lbs. per sq. ft. these deformations become 4.98 and 5.53 divisions respectively. This column has an external diameter of 26 inches and contains a steel core consisting of a 14-inch, 138 lbs. per ft. Bethlehem *H* section with two $10'' \times \frac{1}{2}''$ and two $10'' \times \frac{3}{5}''$ steel web plates. In addition there are four 1″ diameter round steel rods. The total area of steel is therefore 61.2 square inches, giving a net area of concrete of 469.7 sq. inches. Assuming an elastic modulus for concrete of 2,350,000 lbs. per sq. inch the load producing a dial reading of one division is 469.7×58.7 or 27,500 lbs. for the concrete; and 61.2×750 or 45,700 lbs. for the steel core, making a total of 73,200 lbs. in accordance with the usually accepted theory for the longitudinally reinforced concrete column. The roof slab having been poured during the interval which elapsed between the taking of the initial readings and observing the deformations after the application of the 125 lb. load, its weight must properly be added to the latter. This gives an aggregate load of 200 lbs. per sq. ft. or 82,000 lbs. per panel. The dial reading corresponding to this panel load computed as above should be $\frac{82,000}{73,200} \times 1$ or 1.12 divisions. But the actual deformation corresponding to this load was, as stated above, 2.63 divisions in one case and 2.06 in the other, values approximately twice as great as those computed on a theoretical basis. When, however, the 250 lbs. per sq. ft. was removed from two of the four panels (equivalent to removing 125 lbs. per sq. ft. from four panels), the average recovery on the lower series of gauge lines was .875 divisions as against the .70 divisions arrived at from computation. This is the only instance in the behaviour of this column in which approximate agreement between the observed and the figured deformations occur. A study of the recorded behaviour for wall column No. 21 shows a similar discrepancy between computed and observed deformations to exist.

Consider an indefinitely large series of equal and continuous beam spans all equally and uniformly loaded and similarly constructed. The moments over supports and at midspans approach as limits, 1/12 *WL* and 1/24 *WL* respectively, giving a sum of negative and positive elements of 1/8 *WL* as its limit. This state of affairs would result if all supports were knife edges or the equivalent thereof. It will also be clear that by special or supplementary additions to the loading assumed, the coefficient 1/24 may be increased by diminishing 1/12 to a smaller value and may be decreased by raising 1/12 to a greater value. If in place of these rockerlike supports, columns more or less rigid be substituted, the same statement will hold and moments will remain unchanged, because all tangents to the floor axis are horizontal over supports.

Let it be assumed that the uniformly distributed load on any interior panel is lifted. The tangent to the beam axis over either support adjacent to the unloaded panel is no longer horizontal, but the angle of movement is less than it would be were there neither continuity of beam nor rigidity of colunm to influence the performance. (In that case the bending moment at either support would be zero). Actually this moment is greater than zero, but less than 1/12 *WL* and the restraint supplied by the column plus that afforded by the beam sustaining its dead load only is obviously its equivalent. So it follows that at a panel point for the conditions laid down, the sum of the restraining moments in beam and support will not exceed 1/12 *WL* and may be much less than this value.

This hypothetical condition is doubtless almost never realized, since there is rarely either equality in rigidities or equality between story height and beam span. Moreover, in the flat slab type of construction the comparison is further affected by the fact that the beam is supplanted by a slab whose width is equal to the distance between column centres. It can be shown that the moment at the support in a cantilever of uniform section sustaining an end load varies directly as $\frac{EI\Delta}{L^2}$ where Δ is the end deflection. Since Δ is proportional to the product $L\theta$, θ being the angle of elastic deformation at the junction of column and beam, it follows that the moment of resistance is proportional to $\frac{I}{L}$ where E and θ are constants for the three elements previously considered. A computation, made with this principle in mind, shows that for a flat slab of the proportions of the one tested, the column even if unreinforced could not exceed 20″ in diameter without having a greater value of $\frac{I}{L}$ than the slab; and a 20″ column is the minimum allowable under good practice for the given panel dimensions. This would indicate that column No. 20, owing to its size and high percentage of reinforcement, was restraining the larger portion of the bending moment created by the unsymmetrical load, possibly 12/13 of the whole, half of it above and half of it below the floor line. The lower group of gauge lines on column No. 20 was located above the theoretical line of column contraflexure, a distance equal to one-third the distance of the latter from the ceiling and should therefore indicate about one-third the bending moment in the column at the ceiling line.

Consider now a column of any constant section fixed at the end as in Fig. 11. If a couple M whose plane is vertical, be applied at mid-height, the axis of the column will assume a sinuous form with three points of contraflexure, one of which is located 1/6 of the column height below the top, the second of which is 1/6 of the column height above the bottom, and the third at the point where the couple is applied. Further consideration will show that the bending moments in the column immediately below and immediately above the point of application of the couple are $\frac{M}{2}$ and $-\frac{M}{2}$. Similarly the bending moments at top and bottom are $\frac{M}{4}$ and $-\frac{M}{4}$ respectively. Let now another restraining element, a beam for example be framed at the point of application of the couple and in the plane thereof. This element will be supposed to have a length equal to half the column length. Its rigidity for purposes of discussion will be considered equal to that of the half column and its remote end will be considered fixed. It will be clear that since there are now three restraining elements to oppose the couple M, each will contribute something to the resistance and from the conditions assumed, the contributions of all will be equal.

If instead of the combination described above, there is a series of columns rigidly connected by beams on any floor of a building, the analogy between the hypothetical and the real will be apparent. Let it be imagined that one beam only in the supposed series is loaded. Its ends are obviously restrained more or less where they join the columns

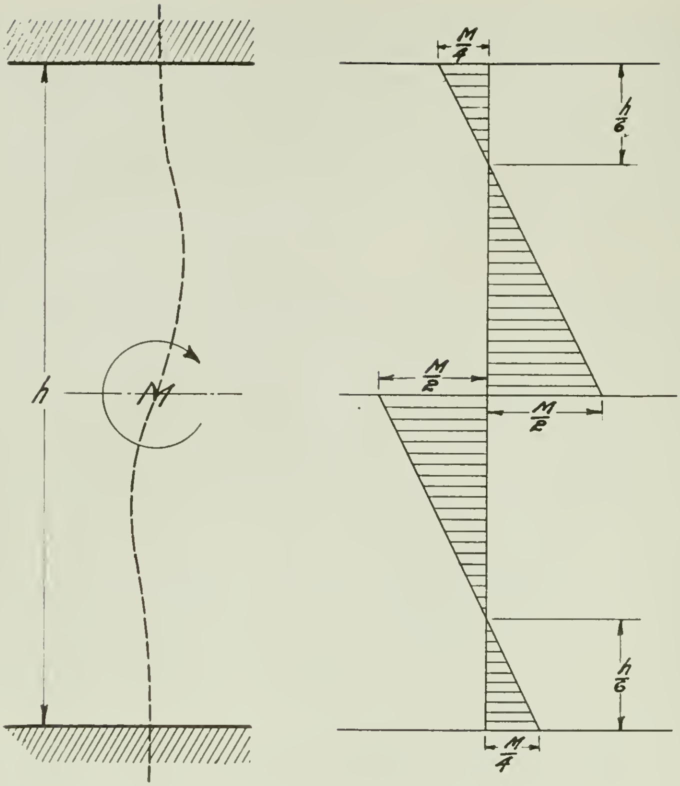


Fig. 11.

and the moment existing at either end will be resisted by three elements, viz., the column above the floor in question, the column below and the beam in the adjacent panel. If all of these possess equal rigidities, using the word in its broad sense, the restraining moments in all three will be equal. If, for example, the bending moment at one extremity of the loaded beam were $\frac{1}{20}WL$ the restraining moment in the column above the floor at the floor line, the restraining moment in the column below the floor at the ceiling line, and the restraining moment in the unloaded beam at its junction with the column might without great error, be taken as $\frac{WL}{60}$.

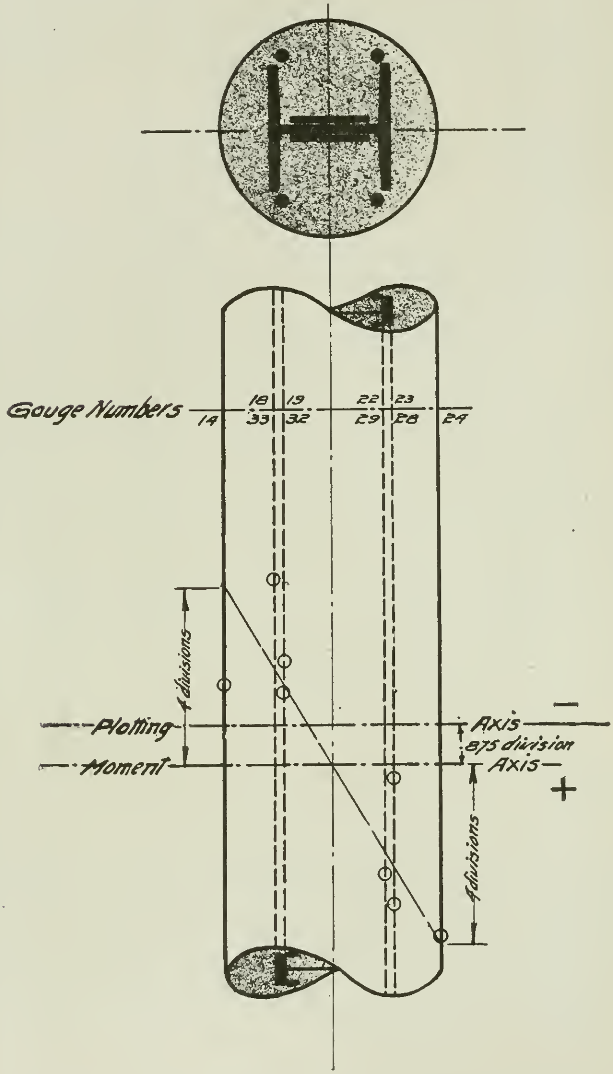
When an attempt was made to check the behaviour of column No. 20 somewhat according to the foregoing, serious difficulties presented themselves as will be presently indicated. Assuming that the concrete had remained intact, it seemed proper to consider its behaviour in flexure by first computing the moment of inertia of its cross-section. This was done by the usual method for heterogeneous sections with the following results:

Moment of inertia of steel area, 23,200 ins.^{4}
Moment of inertia of concrete area, 18,700 ins.^{4}
Moment of inertia of entire area, 41,900 ins.^{4}

On Fig. 12 will be seen a curve purporting to show how the bending strains appear to have varied across a right section of this column on the removal of a load of 250 lbs. per sq. ft. from two of the four contributing panels. This change of loading it will be remembered is the one that produced an average deformation approaching somewhat the computed value. The maximum flexural deformation shown on this graph is 4 divisions, the equivalent of a stress of 232 lbs. per sq. inch in the concrete. The average deformation due to the indicated change in loading, as distinct from bending, is .875 divisions. The bending moment computed from the formula $M=Sf$, is 757,000 inch-pounds of which 337,000 inch-pounds is resisted by the concrete and 420,000 by the metal. Moreover, since the position of the gauge lines on the column is one-third of the distance between the theoretical point of contraflexure and the plane of the floor slab above, the computed maximum bending moment for the column would be three times the value given or 2,271,000 inch-pounds and this takes no account of the nearly equivalent resistance offered by the colunm above the floor. The Chicago flat slab code suggests $\frac{WL}{30}$ as a bending moment to be provided for in column design. For the case in hand this would be equal to 438,000 inch-pounds. Apparently then the steel core itself has provided a resistance to bending much greater than is called for by this regulation and the whole column is supplying one many times in excess of the same requirement and much greater than what seems to be indicated by theory. At the upper series of gauge lines on the same column, a similar state exists. In addition it should be borne in mind that the gauge lines in this series bridged a construction joint and that little if any tension in the concrete could exist there.

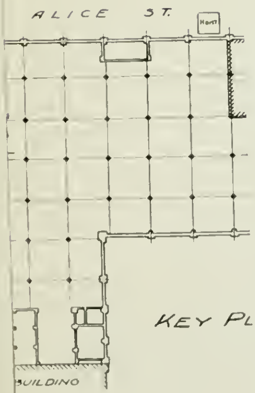
The features then of the column tests is the lack of agreement between the evidence of the extensometer and the facts as they are known. This applies to both the direct stress and the bending phenomena. As possible explanations the shrinkage of the concrete, the existence of unrecorded loads and the inaccuracy of the instrument have suggested themselves.

Mr. F. R. McMillan, in a discussion on the phenomenon of "flow" in concrete under load in Transactions of the American Concrete Institute 1917, exhibited a graph showing the behaviour of concrete columns



COLUMN 20, LOWER SERIES

Fig. 12.



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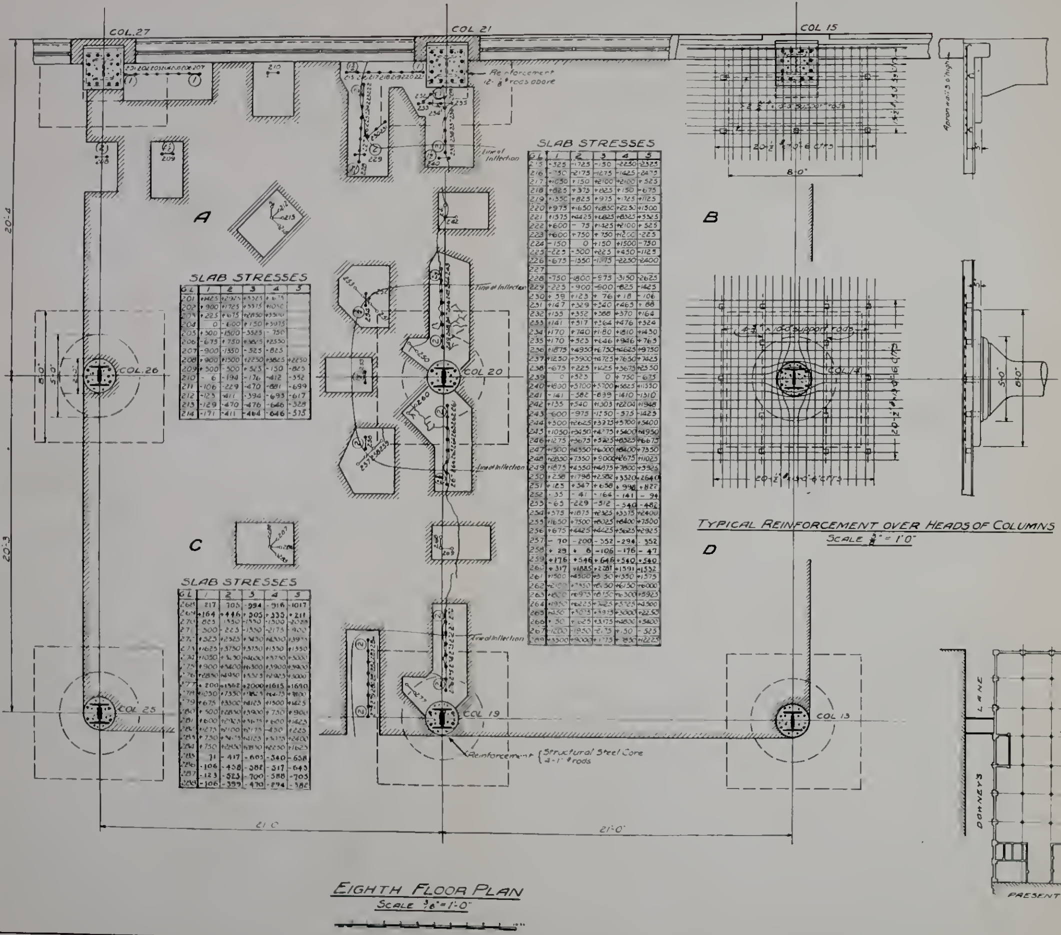


SLAB STRESSES

GL	1	2	3	4	5
201	+1425	+2975	+3325	+675	
202	+900	+1725	+3375	+1050	
203	+225	+675	+2850	+3500	
204	0	-600	+150	+3975	
205	+300	-1500	-3525	-750	
206	-675	+750	+3675	+2550	
207	-900	-1350	-525	-825	
208	+900	+1500	+2250	+3825	+2250
209	+300	-300	+525	-150	-825
210	-6	-194	-176	-412	-352
211	-106	-229	-470	-881	-699
212	-123	-411	-394	-693	-617
213	-129	-470	-476	-646	-328
214	-171	-411	-464	-646	-575

SLAB STRESSES

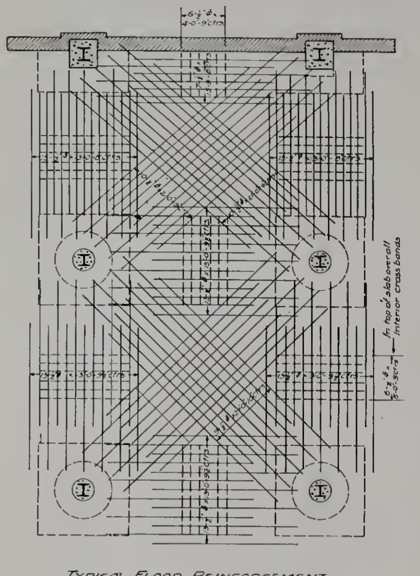
GL	1	2	3	4	5
215	-325	-1725	-150	-2250	-2325
216	-750	-2175	-1275	-1425	-2475
217	+1650	+150	+2100	+2100	+525
218	+825	+375	+825	+150	-675
219	+1350	+825	+975	+1725	+1125
220	+975	+1650	+2850	+2250	+1500
221	+1575	+4425	+6825	+8325	+5325
222	+600	-75	+1425	+2100	+525
223	+600	+750	+750	+1200	-225
224	-150	0	+150	+1500	-750
225	-225	-300	+225	+450	-1125
226	-675	-1350	-1575	-2250	-2400
227					
228	-750	-1800	-975	-3150	-2625
229	-225	-900	-900	-825	-1425
230	+59	+123	+76	+18	-106
231	+147	+329	+240	+465	+88
232	+135	+352	+388	+370	+164
233	+141	+317	+364	+476	+324
234	+170	+740	+1180	+1810	+1430
235	+170	+523	+646	+946	+763
236	+1875	+4950	+6750	+8625	+9750
237	+1250	+3900	+4725	+7650	+7425
238	-675	+225	+1425	+3675	+2550
239	0	+525	0	+750	-675
240	+1800	+5700	+5700	+5825	+11550
241	-141	-582	-839	-1410	-1310
242	+135	+540	+1303	+2204	+1948
243	-600	-975	-1250	-575	-1425
244	+300	+2625	+3375	+5700	+5400
245	+1050	+3450	+4275	+5400	+4950
246	+1275	+3675	+5325	+8325	+6675
247	+1500	+4350	+6000	+8400	+7350
248	+2850	+7350	+9000	+8675	+11025
249	+1875	+4350	+4875	+7800	+5925
250	+258	+1798	+2582	+3320	+2640
251	+123	+547	+658	+998	+822
252	-35	-41	-164	-141	-94
253	-65	-229	-512	-540	-482
254	+375	+1875	+2325	+3375	+2400
255	+1650	+7500	+8025	+8400	+7500
256	+675	+4425	+4425	+5625	+2925
257	-70	-200	-352	-294	-352
258	+29	+6	-106	-176	-47
259	+176	+546	+646	+540	+540
260	+317	+1885	+2281	+1591	+1532
261	+1500	+4500	+5350	+1350	+1575
262	+2100	+7950	+6150	+6150	+6000
263	+1800	+6975	+8150	+6300	+5925
264	+1950	+6225	+7425	+5725	+2500
265	+450	+5075	+4975	+3000	+2250
266	+50	+625	+3175	+4800	+5400
267	-1200	-1950	-2175	+150	-525
269	+3300	+9000	+1775	+850	+2225

SLAB STRESSES

GL	1	2	3	4	5
268	217	705	-994	-916	-1017
269	+164	+446	+305	+335	+211
270	-825	-1350	-1350	-1500	-2025
271	-300	-225	-1350	-2175	-900
272	+325	+2325	+3450	+4350	+3975
273	+1625	+3750	+3750	+1350	+1350
274	+1050	+3450	+4600	+3750	+3000
275	+900	+3400	+6300	+3900	+3900
276	+2850	+4050	+3525	+2925	+3000
277	+200	+1362	+2000	+1615	+1690
278	+1050	+7350	+1825	+4475	+3800
279	+675	+3300	+4125	+1500	+1425
280	+500	+2850	+3900	+750	+900
281	+600	+2325	+3675	+600	+1425
282	+1275	+4100	+2175	-450	+225
283	+750	+3675	+4125	+3075	+2400
284	+750	+2850	+1850	+2250	+1625
285	71	-417	-605	-540	-658
286	-106	-438	-382	-317	-643
287	-123	-523	-700	-588	-705
288	-106	-399	-470	-294	-382

TYPICAL REINFORCEMENT OVER HEADS OF COLUMNS
SCALE ⅜" = 1'0"

EIGHTH FLOOR PLAN
SCALE ⅛" = 1'-0"



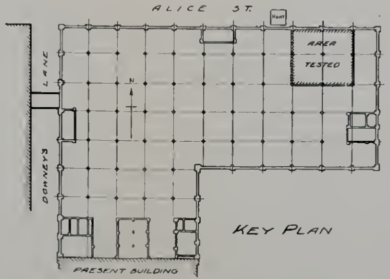
TYPICAL FLOOR REINFORCEMENT
SCALE ¼" = 1'0"

— KEY —

▨ = Loaded area
8" = Gage Line on Concrete
= Gage Line on Reinforcement
Figure in ○ = Distance from face of steel to surface of concrete
7½" = Slab thickness
5" = Depth of drop heads

— TABLES —
(Live Load stresses only)
G L = Gage Line number
1 = Stress, 4 panels loaded 125#/□ft.
2 = Stress, 4 panels loaded 250#/□ft.
3 = Stress, 4 panels loaded 250#/□ft 5 days later
4 = Stress, panels A&B only loaded 250#/□ft
5 = Load removed
+ = Tension
− = Compression

Design live load 125 lbs per sq ft.
E_s = 30,000,000.
E_c = 2,350,000.



KEY PLAN

PLATE I

REPORT OF EXTENSOMETER TEST
ON 4 WAY FLAT SLAB REINFORCED CONCRETE BUILDING
— ALICE ST., TORONTO. —
— OWNED BY —
THE T. EATON C^O LIMITED.

TESTED BY
CITY ARCHITECT'S DEPT.
TORONTO, CANADA.
-AND-
UNIVERSITY OF TORONTO
JUNE 25 - AUG. 3 - 1917

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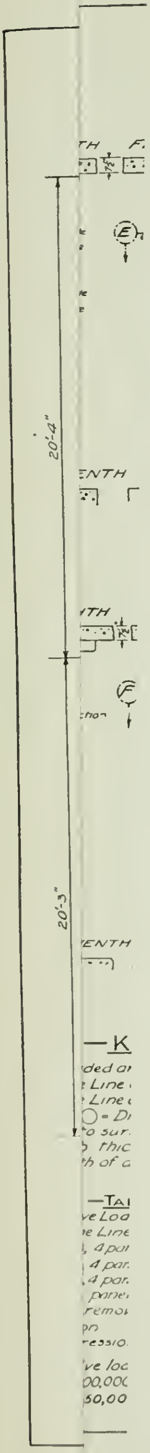
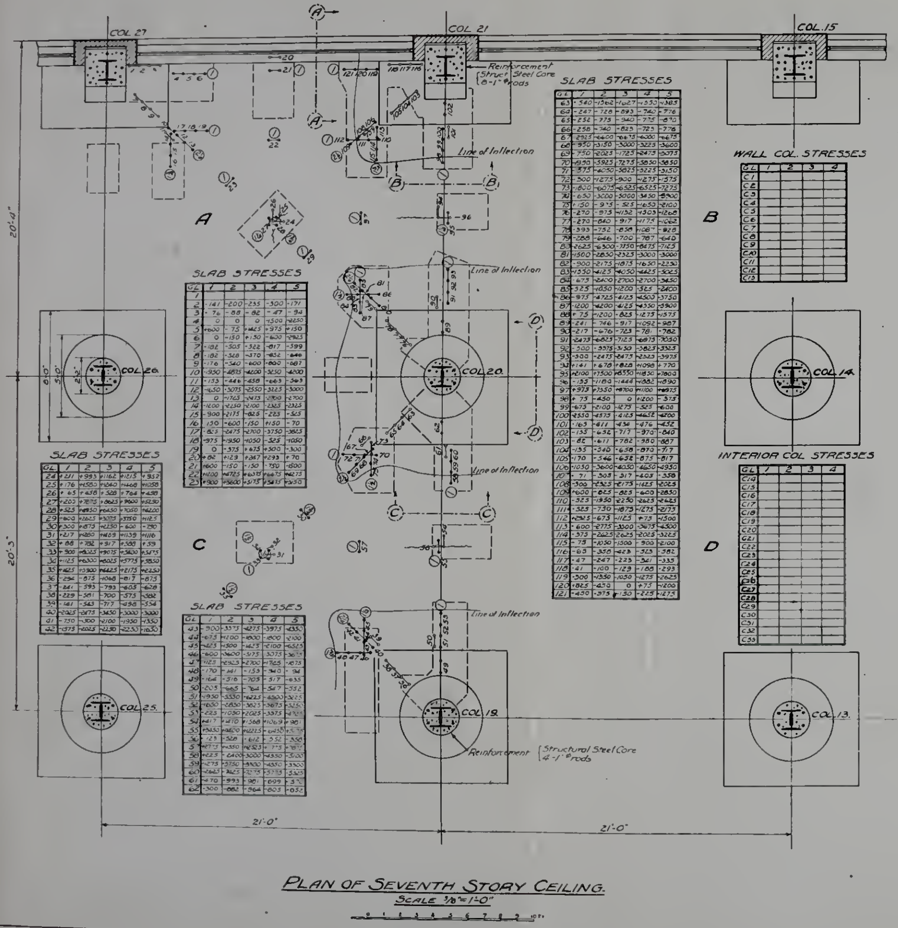




PLAN OF SEVENTH STORY CEILING.
SCALE 3/16"=1'-0"

SLAB STRESSES

GL	1	2	3	4	5
1					
2	-141	-200	-255	-300	-171
3	-76	-88	-82	-47	-94
4	0	0	0	+500	-2250
5	+600	-75	+1425	+975	+150
6	0	-150	+150	-600	-2925
7	-182	-505	-522	-817	-599
8	-182	-528	-570	-832	-646
9	-176	-540	-600	-600	-687
10	-1950	-4875	-4200	-5250	-4200
11	-153	-446	-458	-663	-565
12	-1650	-3075	-2550	-3225	-3000
13	0	-1725	-2475	-2700	-2700
14	-1200	-2250	-2100	-2325	-2325
15	-900	-2175	-825	-225	-525
16	150	-600	-150	+150	-70
17	-825	-2475	-2700	-3750	-3825
18	-975	-1950	-1050	-525	-1050
19	0	-375	+675	+500	-300
20	+82	+129	+247	+293	+70
21	+600	-150	-150	-750	-1500
22	+1200	+4725	+6375	+6475	+4275
23	+900	+5600	+5175	+5475	+5150

SLAB STRESSES

GL	1	2	3	4	5
24	+211	+993	+1162	+1215	+952
25	+176	+1550	+1260	+1468	+1058
26	+65	+458	+528	+764	+458
27	+200	+7275	+6625	+9600	+5250
28	+525	+6950	+6450	+7050	+4200
29	+600	+2625	+3075	+3750	+1125
30	+300	+1875	+2250	-600	-750
31	+217	+1280	+1469	+1139	+1116
32	+88	+782	+917	+588	+59
33	+900	+6025	+9075	+5400	+5475
34	+1125	+6300	+6025	+5775	+5850
35	+825	+3900	+4425	+2175	+2250
36	-294	-875	-1068	-817	-875
37	-241	-593	-793	-605	-628
38	-229	-581	-700	-575	-582
39	-141	-563	-717	-498	-554
40	-2025	-2475	-3450	-3000	-3000
41	-750	-300	-2100	-1950	-1350
42	-1575	-2025	-2250	-2250	-1650

SLAB STRESSES

GL	1	2	3	4	5
43	-900	-3375	-4275	-3975	-4350
44	-675	-1200	-1800	-1800	-2100
45	-225	-1500	-1425	-2100	-6525
46	-600	-3600	-3175	3075	-3675
47	-1125	-2925	-2700	-1725	-1875
48	-170	-141	-153	-340	-94
49	-164	-516	-705	-517	-635
50	-205	-665	-764	-517	-552
51	-1950	-3550	-4225	-4500	-3225
52	-1650	-2850	-3825	-3675	-5250
53	-225	-1050	-2025	-3375	[illegible]
54	+417	+1410	+1568	+1069	+981
55	+5550	+4200	+12225	-6450	[illegible]
56	-125	-528	-612	-552	-556
57	+2775	+4350	+2523	+775	[illegible]
58	+225	-2400	-3000	-4350	-3100
59	-1275	-3750	-3900	-4350	-3300
60	-2625	-3225	-2775	-5775	-5325
61	-470	-993	-981	-699	[illegible]
62	-300	-882	-964	-805	-1052

SLAB STRESSES

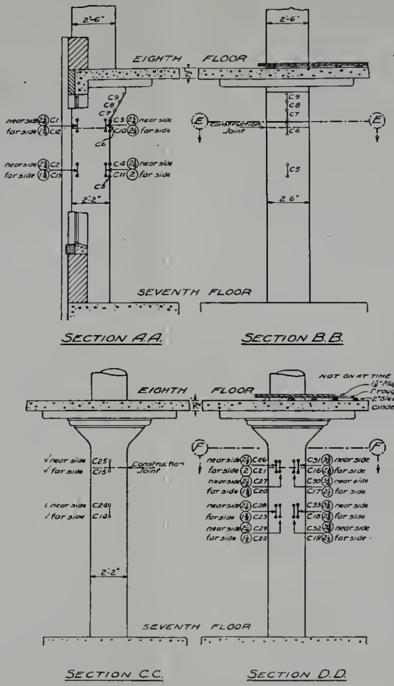
GL	1	2	3	4	5
63	-540	-1562	-1627	-1550	-1385
64	-247	-728	-893	-740	-776
65	-252	-775	-940	-775	-870
66	-258	-740	-825	-725	-776
67	-2325	-6600	-6675	-6000	-6675
68	-950	-3150	-3000	-3225	-3600
69	-750	-2025	-1725	-2475	-3075
70	-1950	-5925	-7275	-5850	-5850
71	-575	-4050	-5825	-3225	-3150
72	-500	-1275	-900	-1275	-1575
73	-1800	-6075	-6525	-6525	-7275
74	-650	-3000	-5000	-3450	-3900
75	+150	-975	-525	-1650	-2100
76	-270	-975	-1152	-1503	-1268
77	-270	-840	-917	-1175	-1062
78	-393	-752	-858	-1087	-928
79	-288	-646	-700	-787	-640
80	-2625	-6300	-7350	-8475	-7125
81	-1500	-2850	-2325	-3000	-3000
82	-900	-2175	-1875	-1650	-2250
83	-1550	-4125	-4050	-4425	-5025
84	-675	-2400	-2700	-2700	-3450
85	-325	-1050	-1200	-525	-2400
86	-975	-4725	-4125	-4500	-3750
87	-1200	-4200	-4125	-4350	-5900
88	+75	-1200	-825	-1275	-1575
89	-241	-746	-917	-1092	-987
90	-217	-676	-723	-781	-782
91	-2475	-6825	-7125	-6975	-7050
92	-500	-3375	-3150	-3825	-3325
93	-900	-2475	-2475	-2525	-3975
94	+141	+678	+828	+1098	+770
95	+2100	+7500	+6550	+1850	+7800
96	-135	-1180	-1444	-1882	-1890
97	+973	+7350	+8700	+1100	+6975
98	+75	-450	0	+1200	-375
99	-675	-2100	-1275	-525	-600
100	-2550	-4575	-4125	-4652	-4200
101	-165	-411	-430	-476	-452
102	-135	-652	-717	-970	-840
103	-82	-611	-782	-980	-887
104	-135	-540	-658	-870	-717
105	-170	-546	-652	-875	-817
106	-1050	-3600	-4050	-4650	-4950
107	-71	-505	-517	-405	-358
108	-300	-2325	-2175	-1425	-2025
109	+600	-825	-825	-600	-2850
110	-325	-1950	-2250	-2625	-2625
111	-325	-750	-1875	-1275	-2175
112	+2325	-675	-1125	+75	-1500
113	-600	-2775	-3300	-3675	-4500
114	-375	-2625	-2625	-2025	-3225
115	-75	-1050	-1500	-900	-2100
116	-65	-350	-423	-323	-382
117	-47	-247	-223	-341	-335
118	-41	-100	-129	-188	-293
119	-300	-1350	-1050	-1275	-2625
120	+825	-450	0	+75	-1200
121	-450	-375	-150	-225	-1275

WALL COL. STRESSES

GL	1	2	3	4
C1				
C2				
C3				
C4				
C5				
C6				
C7				
C8				
C9				
C10				
C11				
C12				
C13				

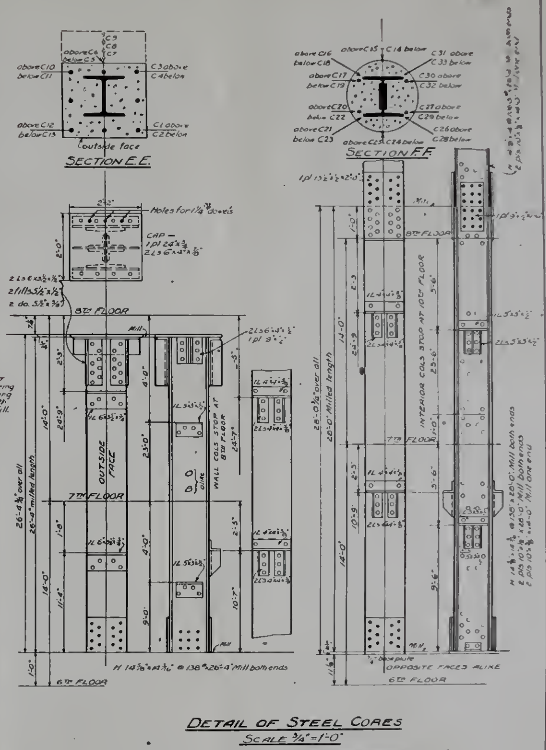
INTERIOR COL STRESSES

GL	1	2	3	4
C14				
C15				
C16				
C17				
C18				
C19				
C20				
C21				
C22				
C23				
C24				
C25				
C26				
C27				
C28				
C29				
C30				
C31				
C32				
C33				


2'-6"
2'-6"
EIGHTH FLOOR
SEVENTH FLOOR
near side C1 far side C12
near side C2 far side C3
C3 near side C10 far side
C4 near side C11 far side
Construction Joint
2'-2"
2'-6"
NOT ON AT TIME OF TEST
1/2" Maple Flooring
Rough flooring
2" Sleepers with cinder conc fill.
near side C25 far side C15
near side C24 far side C14
2'-2"
near side C26 far side C21
near side C27 far side C20
near side C28 far side C23
near side C22
C31 near side C16 far side
C30 near side C17 far side
C33 near side C18 far side
C32 near side C19 far side

SECTION A.A.   SECTION B.B.

SECTION C.C.   SECTION D.D.



DETAIL OF STEEL CORES
SCALE 3/4"=1'-0"

—KEY—

- ▨ = Loaded area
- Gage Line on Concrete
- Gage Line on Reinforcement
- Figure in ○ = Distance from face of steel to surface of concrete
- 7½" = Slab thickness
- 5" = Depth of drop heads

—TABLES—
(Live Load stresses only)

GL = Gage Line number
1 = Stress, 4 panels loaded 125#/□ft
2 = Stress, 4 panels loaded 250#/□ft
3 = Stress, 4 panels loaded 250#/□ft 3 days later
4 = Stress, panels A & B only loaded 250#/□ft
5 = Load removed
+ = Tension
- = Compression

Design live load 125 lbs per sq ft.
E_s = 30,000,000
E_c = 2,350,000

PLATE II

REPORT OF EXTENSOMETER TEST
ON 4 WAY FLAT SLAB REINFORCED CONCRETE BUILDING
— ALICE ST., TORONTO —
— OWNED BY —
THE T. EATON CO. LIMITED

TESTED BY
CITY ARCHITECT'S DEPT
TORONTO, CANADA
— AND —
UNIVERSITY OF TORONTO
JUNE 25-AUG 3-19[illegible]

2	3	4	5	
-4·2		-5·1	-4·1	
-3·1		-9·9	-0·1	
-3·3		-5·8	-3·8	
-6·2		-5·0	-5·2	
-8·0		-3·2	-2·2	
-5·2		-5·9	-3·6	
-7·1		+1·5	-1·7	
-7·9		+0·9	-5·1	
-3·6		-0·3	-1·8	
-4·0		0·0		
-4·6		+0·1	-1·1	
-7·7		+17·8	+12·5	
-4·4		+0·6	-1·5	
-5·8		+8·2	-2·8	
-5·5		-4·3	-4·0	
-4·8		-7·2	-4·2	
-5·0		-8·2	-6·0	
-6·5		-7·9	-4·9	
-6·2		-9·4	-5·2	

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DEFORMATIONS IN 8" GAUGE LENGTH

1Unit = $\frac{1}{3000}$" n = 12.76
1Unit = 750 #/□" = Stress in Steel – E_s = 30,000,000
1Unit = 58.75 #/□" = Stress in Concrete – E_c = 2,350,000

G.L.	1	2	3	4	5
1	abandoned #/□ pipe line				
2	-2.4	-3.4	-4.0	-5.1	-3.0
3	-1.3	-1.5	-1.4	-0.8	-1.6
4	0.0	0.0	0.0	2.0	-3.0
5	+0.8	-0.1	+1.9	+1.3	+0.2
6	0.0	-0.2	+0.2	-0.8	-3.9
7	-3.1	-8.6	-8.9	-13.9	-10.2
8	-3.1	-9.0	-9.7	-14.5	-11.0
9	-3.0	-9.2	-10.2	-15.0	-11.7
10	-2.6	-6.5	-5.6	-7.0	-5.6
11	-2.6	-7.6	-1.8	-11.3	-9.6
12	-2.2	-4.1	-3.4	-4.3	-4.0
13	0.0	-2.3	-3.3	-3.6	-3.6
14	-1.6	-3.4		-3.1	-3.1
15	-1.2	-2.9		-0.3	-0.7
16	-0.2	-0.8	-0.2	+0.2	-1.2
17	-1.1	-3.3	-3.6	-5.0	-5.1
18	-1.3	-2.6	-1.4	-0.7	-1.4
19	0.0	-0.5	+0.9	+0.4	-0.4
20	+1.4	+2.2	+4.2	+5.0	+1.2
21	+0.8	-0.2	-0.2	-1.0	-2.0
22	+1.6	+6.3	+8.5	+8.9	+5.7
23	+1.2	+4.8	+6.9	+7.3	+4.2
24	+3.6	+16.9	+19.8	+20.7	+16.2
25	+3.0	+18.0	+21.1	+25.0	+18.0
26	+1.1	+7.8	+9.0	+13.0	+7.8
27	+1.6	+9.7	+11.5	+12.3	+7.0
28	+0.7	+6.6	+8.6	+9.4	+5.6
29	+0.8	+3.5	+4.1	+5.0	+1.5
30	+0.4	+2.5	+3.0	-0.8	-1.0
31	+3.7	+21.8	+25.0	+19.4	+19.0
32	+1.5	+13.3	+15.6	+10.0	+10.0
33	+1.2	+10.7	+12.1	+7.2	+7.3
34	+1.5	+8.4	+10.7	+7.7	+7.8
35	+1.9	+5.2	+5.9	+2.9	+3.0
36	-5.0	-14.9	-18.2	-13.9	-14.9
37	-4.1	-10.1	-13.3	-10.3	-10.7
38	-3.9	-9.9	-11.9	-9.8	-9.9
39	-2.4	-9.3	-12.2	-8.5	-9.1
40	-2.7	-3.3	-4.6	-4.0	-4.0
41	-1.0	-2.0	-2.8	-2.6	-1.8
42	-2.1	-2.7	-3.0	-3.0	-2.2
43	-1.2	-4.3	-5.7	-5.3	-5.8
44	-0.9	-1.0	-2.4	-2.4	-2.8
45	-1.9	-2.0	-1.9	-2.8	-8.7
46	-0.8	-4.8	-6.9	-4.1	-4.9
47	-1.5	-3.9	-3.6	-2.3	-2.5
48	-2.9	-2.4	-2.6	-1.6	-1.6
49	-2.8	-8.8	-12.0	-8.8	-10.8
50	-3.5	-11.3	-13.0	-9.3	-9.4
51	-2.6	-7.4	-8.3	-6.0	-4.3
52	-2.2	-3.8	-5.1	-4.9	-7.0
53	-0.3	-1.4	-2.7	-4.5	-6.3
54	+7.1	+24.0	+26.7	+18.2	+16.7
55	+4.6	+15.2	+16.3	+8.6	+6.9
56	-2.1	-9.0	-10.4	-6.0	-6.1
57	+3.7	+15.4	+16.7	+15.7	+10.5
58	+0.3	-3.2	-4.0	-5.8	-6.8
59	-1.7	-5.0	-4.4	-5.8	-4.4
60	-3.5	-9.9	-9.7	-7.8	-7.1
61	-8.0	-16.9	-16.7	-11.9	-9.7

G.L.	1	2	3	4	5
62	-5.1	-15.0	-16.4	-13.7	-14.5
63	-9.2	-23.2	-27.7	-23.0	-23.6
64	-4.2	-12.4	-15.0	-12.6	-13.2
65	-4.3	-13.2	-16.0	-13.2	-14.8
66	-4.4	-12.6	-14.0	-12.3	-13.2
67	-3.9	-8.8	-8.9	-8.0	-8.9
68	-2.6	-4.2	-4.0	-4.3	-4.8
69	-1.0	-2.7	-2.3	-3.3	-4.1
70	-2.6	-7.9	-9.7	-7.8	-7.8
71	-2.1	-5.4	-5.1	-4.3	-4.2
72	-0.4	-1.7	-1.2	-1.7	-2.1
73	-2.4	-8.1	-0.7	-8.7	-9.7
74	-2.2	-4.0	-4.0	-4.6	-5.2
75	+0.2	-1.3	-0.7	-2.2	-2.8
76	-4.6	-16.6	-19.3	-25.8	-21.6
77	-4.6	-14.3	-15.6	-20.0	-18.1
78	-6.7	-12.8	-14.6	-18.5	-15.8
79	-4.9	-11.0	-11.9	-13.4	-10.9
80	-3.5	-8.4	-9.8	-11.3	-9.5
81	-2.0	-3.8	-3.1	-4.0	-4.0
82	-1.2	-2.9	-2.5	-2.2	-3.0
83	-1.8	-5.5	-5.4	-5.9	-6.7
84	-0.9	-3.2	-3.6	-3.6	-4.6
85	-0.7	-1.4	-1.6	-0.7	-3.2
86	-1.3	-6.3	-5.5	-6.0	-5.0
87	-1.6	-5.6	-5.5	-5.8	-5.2
88	+0.1	-1.6	-1.1	-1.7	-2.1
89	-4.1	-12.7	-15.6	-18.6	-16.8
90	-3.7	-11.5	-12.3	-13.3	-13.3
91	-3.3	-9.1	-9.7	-9.3	-9.4
92	-0.4	-4.5		-5.1	-7.1
93	-0.4	-3.3	-3.3	-3.1	-5.3
94	+2.4	+11.7	+14.1	+18.7	+13.1
95	+2.8	+10.0	+11.4	+15.8	+10.4
96	-2.3	-20.1	-24.6	-22.1	-32.2
97	+1.3	+9.8	+11.6	+14.8	+9.3
98	+0.1	-0.6	0.0	+1.6	-0.5
99	-0.9	-2.8	-1.7	-0.7	-0.8
100	-3.4	-6.1	-5.5	-6.2	-5.6
101	-2.0	-7.0	-7.4	-8.1	-7.7
102	-2.3	-11.1	-12.2	-16.5	-14.3
103	-1.4	-10.4	-13.3	-16.7	-15.1
104	-2.3	-9.2	-11.2	-14.8	-12.2
105	-2.9	-9.3	-11.1	-14.9	-13.9
106	-1.4	-4.8	-5.4	-6.2	-6.6
107	-1.2	-5.2	-5.4	-6.9	-6.1
108	-0.4	-3.1	-2.9	-1.9	-2.7
109	+0.8	-1.1	-1.1	-0.8	-3.8
110	-0.7	-2.6	-3.0	-3.5	-3.5
111	-0.7	-1.0	-2.5	-1.7	-2.9
112	+3.9	-0.9	-1.5	+0.1	-2.0
113	-0.8	-3.7	-4.4	-4.9	-6.0
114	-0.3	-3.5	-3.5	-2.7	-4.3
115	-0.1	-1.4	-2.0	-1.2	-2.8
116	-1.1	-6.1	-7.2	-8.9	-9.9
117	-0.8	-4.2	-3.8	-5.8	-5.7
118	-0.7	-1.7	-2.2	-3.2	-5.0
119	-0.4	-1.8	-1.4	-1.7	-3.5
120	+1.1	-0.6	0.0	+0.1	-1.6
121	-0.6	-0.5	-0.2	-0.3	-1.7

G.L.	1	2	3	4	5
201	+1.9	+3.9	+7.1	+0.9	
202	+1.2	+2.3	+4.5	+1.4	
203	+0.3	+0.9	+3.8	+4.4	
204	0.0	-0.8	+0.2	+5.3	
205	+0.4	-2.0	-4.7	-1.0	
206	-0.9	+1.0	+5.1	+3.4	
207	-1.2	-1.8	-0.7	-1.1	
208	+1.2	+2.0	+3.0	+5.1	+3.0
209	+0.4	-0.4	+0.7	-0.2	-1.1
210	-0.1	-3.3	-3.0	-7.0	-6.0
211	-1.8	-3.9	-8.0	-15.0	-11.9
212	-2.1	-7.0	-6.7	-11.8	-10.5
213	-2.2	-8.0	-8.1	-11.0	-9.0
214	-3.0	-7.0	-7.9	-11.0	-9.8
215	-0.7	-2.3	-0.2	-3.0	-3.1
216	-1.0	-2.9	-1.7	-1.9	-3.3
217	+1.4	+0.2	+2.8	+2.8	+0.7
218	+1.1	+0.5	+1.1	+0.2	-0.9
219	+1.8	+1.1	+1.3	+2.3	+1.5
220	+1.3	+2.2	+3.8	+3.0	+2.0
221	+2.1	+5.9	+9.1	+11.1	+7.1
222	+0.8	-0.1	+1.9	+2.8	+0.7
223	+0.8	+1.0	+1.0	+1.6	-0.3
224	-0.2	-0.0	+0.2	+2.0	-1.0
225	-0.3	-0.4	+0.3	+0.6	-1.5
226	-0.9	-1.8	-2.1	-3.0	-3.2
227	0.0	-2.1	-1.9	-2.9	-3.5
228	-1.0	-2.4	-1.3	-4.2	-3.6
229	-0.3	-1.2	-1.2	-1.1	
230	+1.0	+2.1	+1.3	+0.3	-1.8
231	+2.5	+5.6	+5.8	+7.9	+1.5
232	+2.3	+6.0	+6.6	+6.3	+2.8
233	+2.4	+5.4	+6.2	+8.1	+5.5
234	+2.9	+12.6	+20.1	+30.9	+23.9
235	+2.9	+8.9	+11.0	+16.1	+13.0
236	+2.5	+6.6	+9.0	+15.5	+12.9
237	+1.6	+5.2	+6.3	+10.2	+9.9
238	-0.3	+0.3	+1.9	+4.9	+3.4
239	0.0	+0.7	+0.0	+1.0	-0.9
240	+2.4	+7.6	+7.6	+21.1	+15.4
241	-2.4	-9.9	-15.3	-24.0	-22.3
242	+2.3	+9.2	+22.2	+37.6	+33.0
243	-0.8	-1.3	-1.8	-0.5	-1.9
244	+0.4	+3.5	+5.3	+7.6	+7.2

G.L.	1	2	3	4	5
245	+1.4	+4.6	+5.7	+7.2	+6.6
246	+1.7	+4.9	+7.9	+11.1	+8.9
247	+2.0	+5.8	+8.0	+11.2	+9.8
248	+3.8	+9.8	+12.0	+16.9	+14.7
249	+2.5	+5.8	+6.5	+10.4	+7.9
250	+4.4	+30.6	+44.0	+56.5	+45.0
251	+2.1	+9.3	+11.2	+17.0	+14.0
252	-0.6	-0.7	-2.8	-2.4	-1.6
253	-1.1	-3.9	-8.7	-9.2	-8.2
254	+0.5	+2.5	+3.1	+4.5	+3.2
255	+2.2	+10.0	+10.7	+11.2	+10.0
256	+0.9	+5.9	+5.9	+7.5	+3.9
257	-1.2	-3.4	-6.0	-5.0	-6.0
258	+0.5	+0.1	-1.8	-3.0	-0.8
259	+3.0	+9.3	+11.0	+9.2	+9.2
260	+5.4	+32.1	+30.9	+27.1	+26.1
261	+2.0	+6.0	+5.0	+1.8	+2.1
262	+2.8	+9.8	+11.0	+8.2	+8.0
263	+2.4	+9.3	+11.6	+8.4	+7.0
264	+2.6	+8.3	+9.9	+7.7	+6.0
265	+0.6	+4.1	+5.3	+4.0	+3.0
266	+0.2	+2.2	+4.5	+6.4	+7.2
267	-1.6	-2.6	-2.9	+0.2	-0.7
268	-3.7	-12.0	-16.9	-15.6	-17.3
269	+2.8	+7.6	+8.6	+5.7	+3.6
270	-1.1	-1.8	-1.8	-2.0	-2.7
271	-0.4	-0.3	-1.8	-2.9	-1.2
272	+0.7	+3.1	+4.6	+5.8	+5.3
273	+2.2	+5.0	+5.0	+1.8	+1.8
274	+1.4	+4.6	+6.2	+5.0	+4.0
275	+1.2	+7.2	+8.4	+5.2	+5.2
276	+3.8	+6.6	+7.1	+3.9	+4.0
277	+3.4	+23.2	+34.1	+27.5	+28.8
278	+1.4	+9.8	+13.1	+8.9	+10.4
279	+0.9	+4.4	+5.5	+2.0	+1.9
280	+0.4	+3.8	+5.2	+1.0	+1.2
281	+0.8	+3.9	+4.9	+0.8	+1.9
282	+1.7	+2.8	+2.9	-0.6	+0.3
283	+1.0	+4.9	+5.5	+4.1	+3.2
284	+1.0	+3.8	+3.8	+3.0	+2.2
285	-1.2	-7.1	-10.3	-9.2	-11.2
286	-1.8	-7.8	-9.9	-8.8	-10.9
287	-2.1	-8.9	-11.9	-10.0	-12.0
288	-1.8	-6.8		-5.0	-6.5
289	+4.4	+12.0	+15.7	+23.8	+16.3

GL	1	2	3	4	5
C14	-1.9	-4.2		-5.1	-4.1
C15	-1.0	-3.1		-9.9	-0.1
C16	-1.0	-3.3		-5.0	-3.8
C17	-3.0	-6.2		-5.0	-5.2
C18	-2.8	-8.0		-3.2	-2.2
C19	-3.0	-5.2		-5.9	-3.6
C20	-2.7	-7.1		+1.5	-1.7
C21	-2.0	-7.9		+0.9	-5.1
C22	-2.4	-3.6		-0.3	-1.8
C23	-3.0	-4.0		0.0	
C24	-2.8	-4.6		+0.1	-1.1
C25	-3.0	-7.7		+17.0	+12.5
C26	-1.9	-4.4		+0.6	-1.5
C27	-1.7	-5.8		+8.2	-2.8
C28	-2.9	-5.5		-4.3	-4.0
C29					
C30	-2.1	-4.8		-7.2	-4.2
C31	+2.2	-5.0		-8.2	-6.0
C32	-2.7	-6.5		-7.9	-4.9
C33	-2.2	-6.2		-9.4	-5.2

PLATE III

under load, in which the relative deformation or "flow" at 60 days following construction, varied from 1 in 10,000 to 2 in 10,000. Moreover this deformation seemed to be more or less proportional to the age of the member up to a six months' limit. Considère asserts that the shrinkage of mortars in air will vary from 3 in 10,000 to 5 in 10,000, although no time interval is given. So far as the evidence of the extensometer is concerned, it would seem that "flow" and shrinkage would be similar.

If shrinkage be restrained by the presence of a metal core (the non-shrinking element) the envelop of concrete will be in tension and the core in compression. This of course assumes that a perfect bond between the two materials exists. In addition to this the core will have to sustain the major portion of the applied load. If the restraint be equal to 1 in 20,000, it would account for a tension in the concrete envelope of nearly 120 lbs. per sq. inch and the resulting and, in the aggregate, equal compression thrown into the steel would be sufficient to account for some of the excessive deformation previously referred to. On the removal of the 250 lbs. per sq. ft. load from two of the four panels, it might with some show of reason be assumed that because of the greater age of the concrete and the diminished loading the tendency to shrink would partially disappear. This assumption receives colour from the fact that the mean changes in deformation observed agree roughly with the computed value as stated previously, the former being .875 and 1.1 divisions for the lower and upper series respectively and the latter .70.

Another circumstance which would operate to produce a discrepancy between the observed and the computed readings, was the fact that during the test the material required for the brick curtain walls was being elevated to the floors where it was to be used. The practice was to pile it on the floor adjacent to the hoist until such time as the workmen would require it. As the hoist was located in the panel immediately to the west of those under test, it is conceivable that some of the material which had been hoisted was placed on floor area above and tributary to column 20, and that its presence there was evidenced in the large readings obtained. This circumstance is one, the influence of which cannot be estimated in any quantitative way. Even though this additional loading on the upper floors were present, it will be conceded that it would operate almost entirely as a centric load on the column and that all manifestations of eccentricity in the column and of bending therein could not properly be attributed thereto.

Finally it might be suggested that the element of error in observation is much greater here than elsewhere because of the cramped position of the observer and because the axis of the instrument must be held vertically.

The writers do not believe that shrinkage of concrete about the steel core would create therein such compressive stresses as the data sheets call for. Certainly this could not occur at a construction joint. The possibility of loads of material on upper floors could affect average deformations only and not the bending stresses, since the loads resulting therefrom would of necessity be centric. The irregular character of these results is possibly largely due to inaccuracies consequent on using a high clearance instrument in a vertical position and by an observer working in a cramped situation. The consistency in results elsewhere shows that when the instrument is used in the horizontal position flexibility due to high clearance may be overcome by skilful handling.

SUMMARY.

1. There is much evidence that arch action occurs in the slab, resulting in an excess of compression over tension at sections lying between columns.

2. The necessity of providing reinforcement to take care of negative moments transverse to direct lines joining columns is established.

3. The existence of tensile stresses in the concrete tends to lower the stress in the reinforcement, the amount of the tension in the concrete varying in different parts of the slab.

4. This variation is dependent upon the quality of the concrete and upon the deformation existing. In those portions of the slab where the tensile deformations are small a large proportion of the stress may be resisted by the concrete.

5. In the tests upon the Eaton Building a tensile value thought to be consistent was assigned to the concrete, and from the observed deformations the tensions and compressions at all sections were computed and the excess compressions found.

6. Making an adjustment to allow for the thrusts or excess compressions gave the deformations due to bending moment only, and moments for strips one foot in width were computed from the revised data.

7. A comparison of these values with similar values calculated according to various ordinances lead to the following conclusions:

(*a*) The bending moment coefficients prescribed in the Chicago ordinance are approximately correct as far as slabs of the span and live load of the size tested are concerned, being perhaps slightly conservative.

(*b*) The American Concrete Institute coefficients approximate those found in the tests.

(*c*) The Joint Committee requirements appear too conservative for ordinary practice.

8. To provide approximately for the phenomenon of thrust it is suggested that in specifications for the design of the flat slab, the working stress for concrete be lower and that for steel higher than those recommended for flexure uncombined with arch action.

9. The indicated stresses in the columns were very irregular and could not be reconciled with known facts. This may have been due to the fact that the extensometer was of the high clearance type and of necessity had to be supported in a vertical position when observations on columns were being made.

AN IMPROVISED VIBROMETER.

BY G. R. ANDERSON, M.A.

The writer was recently called upon to determine the amount of vibration produced by the machinery of a small munitions plant and transmitted through the earth to neighbouring houses. The case arose out of the action brought by a physician to compel the plant to cease operations on the ground that the vibration was so great that it interfered with his work in the consulting room. There was no time to procure any special form of instrument designed to measure ground vibrations and therefore something had to be improvised.

The only piece of apparatus in the laboratory that looked promising was a common spectrometer and this was accordingly pressed into the service. The central table ordinarily used for supporting a prism or grating was removed and in its stead a circular glass trough of about six inches in diameter was placed centrally on the divided plate of the instrument; this trough contained mercury. The telescope and collimator of the instrument were then tilted up so that each subtended an angle of about 45° to the surface of the mercury. When the collimator slit was illuminated a narrow band of light appeared in the field of the telescope by reflection from the surface of the mercury. A small camera

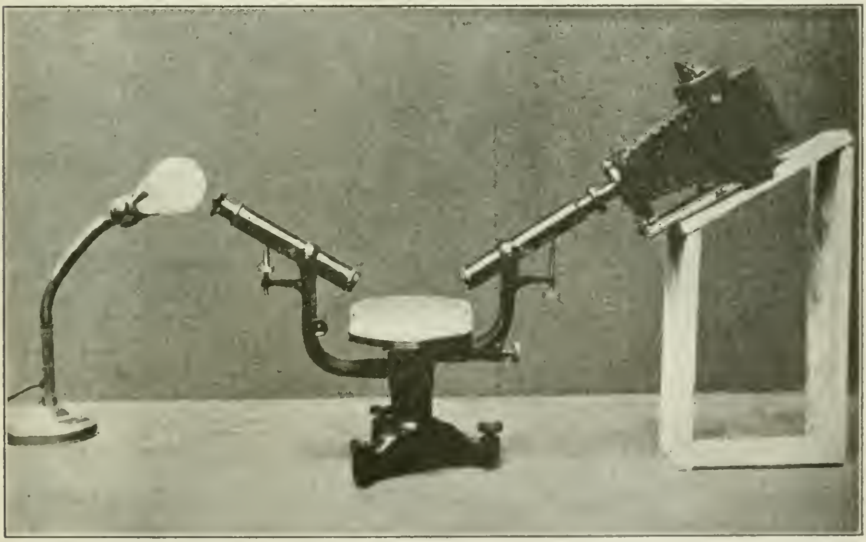


FIG. 1.

with the lens removed was connected with the ocular end of the telescope to register any disturbances of the surface of the mercury photographically. Figure 1 shows a general view of the assembled apparatus.

The position selected for observation was in a narrow passage midway between the two buildings; the instrument was placed on a concrete sidewalk in this lane at a distance of about 80 feet from the centre of the street. Observations were begun during the night when traffic was slack and continued up to the time of commencing operations in the factory. The instrument proved very sensitive and the results throughout several series of observations were entirely consistent.

Figure 2 exhibits 10 records all of one series, obtained between 3.00 a.m. and 7.30 a.m., the details of which are:

1. Normal appearance when there was no disturbance.
2. A car passing along the street at right angles to the one where the observations were being taken and distant some 300 to 400 feet.
3. A car passing the position of observation.
4. A car crossing the intersection of the two streets.
5. A milk wagon coming along the street where observations were being made and distant about half a block.
6. The same milk wagon passing the point of observation.
7. The starting of the machinery on the second floor of the factory, no street traffic.
8. The machinery on the third floor also started, the factory now in full operation.
9. A heavy street car brought suddenly to a stop in front of the point of observation by having the brakes jammed on hard, factory running.

The magnitude of any disturbance is measured by the amplitude of oscillation, and the broadening of the light band being proportional to the amplitude of the ripples on the surface of the mercury, a means of estimating the relative magnitude of the various disturbances is afforded by the records. The measurement of the foregoing series gives the following results:

Record	Width of Band	Difference from Normal	Record	Width of Band	Difference from Normal
1	2.6	0	6	3.7	1.1
2	3.5	.9	7	2.85	.25
3	6.3	3.7	8	2.95	.35
4	5.1	2.5	9	10.7	8.1
5	2.8	.2			

The differences from normal in the above table indicate the relative magnitudes of the disturbances; from which it would appear that the operation of the machinery in this particular plant produced much less vibration than the ordinary street traffic.

It may be observed that such an instrument is readily calibrated by submitting it to a regular vibration of known magnitude, but inasmuch as relative results only were required in this case and as the apparatus was merely improvised for the particular work in hand, no calibration was made. The results therefore remain relative.

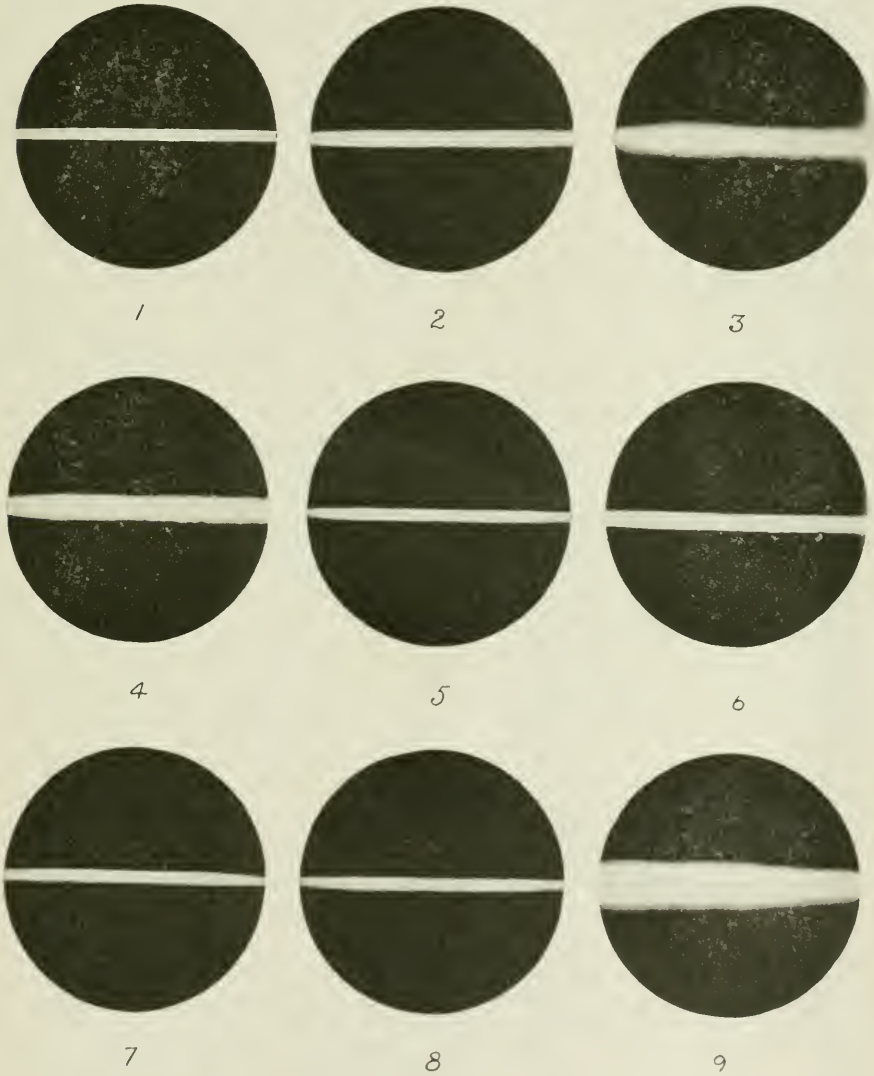


FIG. 2.

PRELIMINARY REPORT ON AN INVESTIGATION INTO THE CONCENTRATION OF GRAPHITE FROM SOME ONTARIO ORES.

BY H. E. T. HAULTAIN, C.E., M.I.M.M., F. C. DYER, B.A.Sc. AND J. T. KING, B.A.Sc.

The beneficiation of graphite ores is recognized as one of the difficult problems of the art of ore-dressing. Unlike many minerals, graphite has no method of concentration that might be called standard; each mill has its own method, the mills between them employing nearly every known means of concentration, in addition to a number of carefully guarded secret processes.

The difficulty in the production of graphite arises from the nature of the graphite itself, and from its mineral associations, which difficulties are complicated by exacting market standards.

Graphite occurs in two forms; crystalline and amorphous. Either form may occur as a massive variety, or as a disseminated mineral. Most of the Ontario graphite ores are of the crystalline and disseminated type, the graphite occurring as flakes varying in size from one-half inch in diameter down to dust. In Ontario are large deposits of graphite bearing rocks, yet in 1917 only about 4,000 tons of graphite were marketed of which approximately one-third was of the best or crucible grade. This investigation was made to elucidate the principles underlying the concentration of graphite and possibly to render some of the at present valueless deposits into commercial propositions.

It is estimated that more than one-half of the graphite consumed in the world is used for the manufacture of crucibles. As graphite suitable for crucible manufacture commands a very much higher price than other grades, therefore this investigation is limited to the production of crucible-grade graphite.

The specific gravity of naturally occurring graphite varies from 2.1 to 2.9 according to its purity, an average being 2.6; the specific gravity of the most commonly associated minerals is from 2.6 to 3, consequently complete separation of the graphite from its mineral associates by gravity methods alone is impossible.

The cleavage of graphite is so perfect that it splits readily into flat scales. In wet gravity concentration methods this flakeyness causes the migration of the graphite to be extremely erratic and a process depending on the flakeyness is apt to be complicated by the presence of mica or chlorite, both flakey minerals and commonly found with graphite.

A screen analysis of one of these lots showed that the sizes coarser than 8 mesh contained practically no free flake, the 8/10 mesh a few and the 10/14 mesh and smaller sizes contained much free flake. The proportion of attached particles diminished with the decrease in the size of particles until at 65 mesh the true middlings were comparatively few.

DETAILS OF SOME OF THE TESTS.

For convenience of reference the tests are numbered consecutively, though only a very few of the experiments are detailed here.

EXPERIMENT No. 1.—One kilo of the ore was ground wet in two stages until all the gangue passed 10 mesh. During grinding 4 drops of coal oil were added to the pulp in the mill and the oiled pulp was then floated without extra oiling. The resulting graphite assayed 65% carbon. The tails were reoiled with 4 more drops of oil and refloated until the finer sands were clean, most of the remaining visible graphite being in thick pieces coarser than 14 mesh. The resulting float was very dirty being largely made up of true middling.

EXPERIMENT No. 2.—Like No. 1 except that only one-half of the amount of oil was added to the mill during grinding. The results were much the same as in the first trial.

EXPERIMENT No. 3.—The grinding was done as before but with no oil except the traces remaining in the crusher from the previous grindings. On flotation the first unoiled float looked clean, and assayed 79.3% as against 65.0% of the first test. These three tests showed that even small quantities of oil greatly increase the amount of gangue lifted, which, in view of the subsequent crushing, is undesirable. Therefore all future grindings were made without any oil.

These tests had produced a quantity of material containing considerable gangue and tests were made to get an idea for a suitable treatment for such dirty concentrates.

EXPERIMENT No. 4.—The low grade oiled floats from Experiment 1 were reground wet without further oil until the gangue passed 65 mesh and then screened, the oversize being a clean looking flake. The undersize was refloated giving a concentrate that was not particularly clean.

EXPERIMENT No. 5.—The very dirty second float from Experiment 3 was reground and re-screened as before at 65 mesh, the oversize again looking quite clean. The undersize mixed with a quantity of tailings was refloated resulting in a clean tailing but a dirty concentrate. This concentrate was reground till the gangue passed 80 mesh and then screened giving a clean flake and a tailing from which repeated flotations failed to obtain a clean product. The assays of these combined experiments are given in Tables 1 and 2. These results indicated what later trials confirmed, that reflotation alone is not sufficient to clean a concentrate and that regrinding followed by screening will give a clean flake.

TABLE 1 (Combined results of Experiments 1 and 4).

Size Mesh	Material Used	Product Obtained	Weight	Assay. % Carbon	Remarks
−10	ore		1,000	6.9	
+65	1st float	Flake	36	87.	Material from Experiment 1
+65	2nd float	Flake	20.3	84.5	Material from Experiment 1
−65		Flake	10.5	68.0	
		Tailing	862	.76	

TABLE 2 (Combined results of Experiments 3 and 5).

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Remarks
	Ore		1,000	6.9	Materials from Experiment 3
+65	1st float	Flake	39.2	79.3	
+80	2nd float	Flake	37.8	86.4	
−80		Flake	8.0	59.3	
		Tailing	852	.55	
		Loss	73		

EXPERIMENT NO. 6.—Experiments on other ores at other times had shown that cleaner concentrates could be obtained from skin flotation by grading the material into several sizes and treating each size separately. Quality of product being the main essential in the production of graphite, accordingly trial was made of material graded by screening into four lots and each lot treated separately as follows: The ore was crushed without oil until the gangue passed 14 mesh, the oversize was clean flake and was reserved without further treatment. The next size 14/48 mesh was oiled with coal oil and floated. The first float was clean except for a few attached particles which were liberated by a light regrind and reflotation; the tailing was almost pure mica. A second flotation of the sands with more coal oil and a third with a mixture of coal oil and rosin oil gave dirty floats which were reground and screened at 80 mesh giving a flake and an undersize which was added to the rest of the −80 material. The size 48/80 was treated similarly. The combined undersize through 80 mesh was floated once without oil and once with oil though no particular attempt was made to obtain a very clean product. The result of the treatment of the whole lot is given in Table 3 the products being combined for assay.

TABLE 3 (Combined results of 4 lots of Experiment 6).

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Remarks
	Ore		2,000	6.9	
+14		Flake	5		Contained no gangue
14/48		Ore	1,000		Most of the flake in this size
48/80		Ore	600		
−80		Ore	395		Includes finest slime
+48	Screened ore products as above	Flake	83.8	91.8	+14 flake added
48/80		Flake	29.0	85.6	
−80		Flake	12.4	74.2	
		Tailings	1,874	1.4	

This gives a recovery of 112 lbs. per ton of ore, of graphite flake of 90% carbon and +80 mesh size. This is a larger recovery and a higher grade of flake than was obtained in tests without grading, and is almost 74% of the total graphite of the ore recovered as a high grade crucible flake.

EXPERIMENT NO. 7.—The ore of this test was low grade with the same hard siliceous gangue as the ore of the previous test. Both it and the ore of the next test were considered to be of doubtful commercial value. This experiment and experiment 8 were intended as trials in the recleaning of flake. The flotation procedure was the same as in the previous test. The result is given in Table 4.

TABLE 4.

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Remarks
−14	Ore		2,000	4.4	
+80		Flake	96	67.4	This assay is much lower than the appearance indicated
−80		Middlings	31.5	31.9	
+200		Tailings	1,514	.4	
−200		Slimes	358	1.9	
+80	Flake		96	67.4	
+80		Flake	56	88.1	This flake looked clean under the microscope
80/150		Middling	14	59.1	
−150		Dirt	26		This was a low grade brown product largely of mica

In experiment No. 8 the ore was like that of No. 7 except that the original graphite flakes were smaller, most of the flakes passing 28 mesh. The procedure was as before except that flotation was added as part of the recleaning process. Table 5 gives the extraction before recleaning.

TABLE 5.

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Remarks
	Ore		1,350	4.7	
+80		Flake	61	68.1	
—80		Middlings	46.5	20.7	
+200		Tailings	1,147	.9	
−200		Slime	96	2.6	Not treated

The regrinding and screening through 80 mesh of this +80 mesh flake raised the assay to 80% carbon, a second regrinding brought the assay to 85%; but at the expense of heavy loss in quantity only 35 grams remaining from the original 61. A second test was made on the ore, seeking to avoid the necessity of such excessive grinding. The ore was graded as before and floated giving a +48 mesh flake and a middling; this middling was reground and refloated the floats being added to the first flake, and the concentrate was used for the recleaning test. A light regrind to crack the true middlings was followed by screening, giving a +48 mesh flake assaying 85% carbon and a 48/80 mesh flake assaying 71% carbon. An attempt was made to separate the 85% flake into two parts by flotation one of which should be of sufficiently high grade as not to require recleaning. This was not found to be possible, the difference in assay between the floats and the sinks being so small in every case as to be negligible. Regrinding the sinks and screening gave a product assaying 88.43% carbon while similar treatment of the floats gave a product assaying 88.40% carbon. These two last when reground and screened gave a product assaying 94% carbon. This shows that regrinding is a necessary part of the recleaning and that reflotation alone is not sufficient.

Screening, particularly of fine sizes, is not a desirable process and grading can often be better performed by classification when used as a preliminary to other processes. In the case of graphite, classification would appear to have a further advantage over screening in that it might be possible to remove a large part of the purer flake from the coarser sizes of gangue to the finer sizes of gangue and thus avoid a considerable loss of flake in the grinding of the coarser portion of the gangue, which portion is the larger part of a properly ground graphite ore. In some cases it might be possible to remove a part of the gangue directly as tailing without treatment, though on the ores tried this was not possible.

Some of the advantages of classification over screening for graphite ores are shown by experiment No. 9. A quantity of the ore as used in the first experiments was crushed in a ball mill until the gangue passed 8 mesh. This pulp was divided by a hydraulic hindered settling classifier into lots, each lot being treated by flotation, followed by regrinding and screening as the method of recleaning. The results are shown in Table 6. This table shows well the increase in the amount of dirt floated with the increase in the amount of oil, also the assay of the first float is higher than anything obtained either without grading or with screening as a means of grading. A screen analysis of some of the products shows that following grading by classification, screening alone will remove a large part of the dirt. See Table 7. As shown by this experiment H.S. classification followed by screening of some of the products is capable of giving a considerable quantity of flake that does not require regrinding; which means a saving of the larger and more valuable flake. In this particular case by combining the first float of the 35/80 mesh lot with the clean flake obtained from screening the second and third floats of the same lot (product 8, Table 6 and products 7 and 9, Table 7) was obtained a flake assaying 92% carbon and in quantity approximately one-third of the total recovery expected from this ore.

TABLE 6.

Gangue Size Mesh	Material Used	Number	Product Obtained	Flotation Oil per Kilo	Weight	Assay % of Carbon	Remarks
	Ore			None	24,554	6.9	
8/35	Ore	1		None	5,679		
		2	1st float	None	24		C.O. = coal oil R.O. = rosin oil
						76.7	Some middlings, no free dirt
		3	2nd "	2 drops C.O.	109	86.7	Much true middlings
		4	3rd "	None extra	10.5	64.0	Much true middlings
		5	4th "	6 C.O. 4 R.O.	348.5	39.2	Very dirty
		6	Tailing		5,187	1.4	Contained thick lumps of graphite
35/80	Ore	7		None	6,614		
		8	1st float	None	228	96.2	Very clean flake
		9	2nd "	2 drops C.O.	314	68.6	Much true middlings
		10	3rd "	6 C.O. 2 R.O.	300	22.1	Very dirty
		11	Tailing		5,772	1.0	
80/300	Ore	12		None	7,119		
		13	1st float	None	713	81.5	Flake fairly clean to the eye
		14	2nd "	2 drops C.O.	212	41.4	Dirty
		15	3rd "	6 C.O. 2 R.O.	155	29.4	Very dirty
		16	Tailing		6,139	.8	
−300	Ore	17			5,142		Not treated

TABLE 7 (Screenings of products of Table 6).

Material Used (from Table 6)	Number *	Product Obtained	Size Mesh	Weight	Assay % Carbon	Remarks
No. 2, 3, 4 and 5 All 8/35 floats	1					All these products contained so much middlings that screening would not give a clean flake
	2					
	3					
	4					
No. 8 1st 35/80 float	5	Clean flake		228	96.2	This was very clean except a few grains in the 65/80 mesh size
No. 9 2nd float	6			314	68.6	
	7	Clean flake	+28	143	88.9	Clean to the eye
		middlings	28/35	171	51.7	Dirt present mostly as true middlings
No. 10 3rd float	8			300	22.1	
	9	Clean flake	+28	52	83.2	Good flake
	10	Middlings	28/35	58	19.3	Mostly true middlings
	11	Dirt	−35	190	6.2	Largely composed of mica (The distinction between these three products was very marked)
No. 13 1st float 80/300	12			713	81.5	These were not assayed but the difference in appearance of these three products was almost very pronounced
	13	Clean flake	+65			
	14	Middlings	65/100			
	15	Dirt	−100			

The ore used in the previous tests was a siliceous ore of which the feldspar and quartz gangue minerals are not easily floated, while the previous tests show that the free biotite present did not present any difficulties with the method of treatment adopted. A test (Experiment 10) similar to the last was made on an ore of a different type, being an altered limestone, the carbonates of which are much more easily floated than the silicates, consequently it was considered that it might be more difficult to obtain a high grade flake without considerable regrinding and loss of flake. The actual work showed that the loss was not as great as expected; that while the floated graphite was not quite as high grade as that from the other ore the greater ease with which calcite particles could be crushed made possible a large recovery of a high grade flake. The details of the extraction are given in Table 9 and a summary of the recovery in Table 8. The original ore assayed 14.6% carbon, the recovery of crucible flake was 232 lbs. per ton of graphite assaying 90% carbon which is a recovery of 71.7% of the total carbon present in the ore. When allowance is made for the original flake of smaller size than 80 mesh and for the inevitable losses in grinding due to the breaking up of the larger flake it will be seen that this is a very good recovery. A treatment of some lumps of the original ore with acids and subsequent screening of the flake showed that approximately 3 per cent. of the original flake was less than 80 mesh in size which makes the recovery of possible crucible flake 74 per cent.

TABLE 8.

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Remarks
−14	Ore		1,940	14.6	Mesh size of gangue only
+80		Crucible flake	224.8	90.5	Total of all +80 flake
−80		Flake	111.2	20.3	No attempt at cleaning
		Tailings	1,604	2.0	Slimes included

TABLE 9

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Flotation Oil	Remarks
−10	Ore		1,910	14.5		Removed by screening 100% clean to the eye
+10		Flake	13	89.2	None	
10/28	Ore		825			
		1st flake	70.0	76.7	None	H.S. classifier product
		2nd float	61.5	47.3	2 drops C.O.	Few true middlings. 99% clean to the eye
		3rd "	112.5	10.8	6 C.O. 2 R.O.	
		Tailing	582.0	2.4		Contains much fine true middling
Recleaning of 10/28 2nd float						
10/28	Float		60.5	47.3		Reground and screened through 48 mesh undersize added to the 28/80 lot. All sizes smaller than 48 mesh were very dirty
+48		Flake	32.5	93.5		
−48		Middling	28.0			
28/80	Ore		832			All −80 mesh material removed by screening
		1st flake	122.0	70.9	None	
		2nd float	70.0	55.7	2 drops C.O.	
		Tailing	640.0	2.7		Contains much fine true middling

TABLE 9. *Continued.*

Recleaning of 10/28 and 28/80 1st flake combined

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Flotation Oil	Remarks
10/28	Flake		192			Reground and screened through 80 mesh undersize added to final −80 flake
+80		Flake	142.8	93.9		
−80		Flake	49.2			
Recleaning of 10/28 and 28/80 mesh last floats combined						
10/80	Float		182.5			
+80		Flake	36.5	77.8	None	Reground and screened through 80 mesh undersize added to next lot for flotation, oversize floated and sinks added to the final tailings. The sinks were mostly mica
+80		Tailing				
−80		Middling				
80/300	Ore		390.5			
		1st float	42.0	35.9	None	No attempt made at recleaning
		2nd "	20.0	7.1	2 drops C.O.	
		Tailing	328.5	.53		
−300	Slimes		53.5	4.9		Not treated

In all the tests made only rarely was it found possible to obtain a concentrate of graphite of such high grade that no further cleaning was necessary. Also it was found that reflotation alone is not sufficient and that a combination of screening and flotation while applicable sometimes was generally useless without some regrinding of the flake.

The usual procedure in the recleaning of graphite is to grind dry between buhr stones. This method is open to the objection of being very destructive of the larger flake on account of the twisting and abrading character of the grinding action. For this reason in some mills an attempt has been made to substitute fine rolls for the buhr stones. Rolls fine enough to grip flake graphite are expensive and costly in upkeep and are not well adapted for wet crushing. A rolling action is better than a twisting one because the flakes will split and roll out flat rather than break thus liberating any enclosed mica without much reduction in the size of the flakes. Wet grinding of flake in addition to the advantages accruing to a continuously wet process has possibly a further advantage in that by dry grinding methods flakes of mica may be simply smeared with graphite and thus hidden instead of being removed. The wet grinding tests showed a possible additional advantage of wet grinding over dry grinding in the preservation of the character of the surface of the flake. This will be referred to later.

The regrinding of the flake and middlings of the previous tests was done in the small pot and ball mill described before. This mill combined the rolling action of a roll and the washing action of a considerable quantity of water, the water preventing the smearing action of the graphite and also to some extent acting as a classifier in removing the lighter flake from the grinding face, the lighter flake presumably not requiring any regrinding. The absence of smearing was well shown in one experiment, of which the details are not given, in which a clean looking coarse flake assaying about 75% and sized between 10 mesh and 48 mesh was reground and refloated. The resulting graphite flake assayed over 90% while the tailing was pure mica, the separation being very clean and distinct. The absence of the smearing action on the reground material means that it is possible to return such a product into the general circuit of the mill without the dirt being refloated by the graphite smears and reappearing in the concentrate.

The small pot and ball mill is suitable for only small quantities and trial was made of a larger mill built more after the style of commercial machines. This mill is a small tube mill 12 inches diameter and 18 inches long with continuous feed and central discharge. In operation it was charged with about 50 ¼ inch diameter steel rods. This weight of rods is only a small part of a full charge of rods for the mill but it was found quite sufficient as selective grinding is desired, only the heavier flakes and the coarser middlings requiring treatment. To aid the selective action a considerable excess of water was used. This mill used intermittently did the regrinding of the products of Table 9 and the results were considered satisfactory both as to the effectiveness of grinding and the preservation of the character of the flake.

A quantity of dirty graphite and middlings had accumulated from the previous work and this was made use of to get an idea as to the operation of the mill used continuously. The middlings were first run through the mill and then screened at 100 mesh, the undersize was brown and contained only a small part of the graphite and assayed between 4% and 5% carbon. The oversize was added to a lot of flake and this reground in the mill. The mill was started with water only and then the flake fed in at a rate of 50 lbs. per hour: the mill was stopped one minute after the last of the graphite was in and the mill emptied. The results showed that the capacity of the mill on such a graphite material is large and the grinding efficient even with as small a rod load as was used. In this particular instance the raising of the assay of the

+80 mesh graphite from 79.5% to 95% carbon shows that the mill was operated much under capacity and the graphite consequently much over ground, resulting in an unnecessary destruction of flake. Owing to an accident the complete returns cannot be given but Table 10 gives a summary of the results of the last regrinding.

TABLE 10.

Size Mesh	Material Used	Product Obtained	Weight	Assay % Carbon	Remarks
10 100	Original flake		4,920	70.6	
+80		Flake	3,670	79.5	Screen sizing test before regrinding
−80		Flake	1,250	64.3	
+80		Flake	1,915	95.0	Screen sizing test after regrinding
−80		Flake	3,005	56.7	

Confirmatory evidence of the impossibility of avoiding regrinding, apart from the regrinding of gangue which may be visible as middlings, is given by physical examination of some of the flake. Repeatedly in making these tests graphite products were obtained that to the eye and to the feel of the fingers were clean and in some cases the microscope failed to show the presence of any dirt, and yet the assays were often disappointingly low. This invisible dirt consists of some minute particles of quartz, etc., embedded in the flake and some iron compounds, very often as oxide, distributed throughout the flake, but by far the larger part of the dirt consists of flakes of mica sandwiched between flakes of graphite. The iron rusts are not usually in sufficient quantity to be of great importance provided the mica can be efficiently removed. The distributed dirt can only be removed by chemical means while these tests show that the mica can be comparatively easily removed by proper means of regrinding. The intimate relationship of the mica and the graphite is well shown by Figures 1 and 2. Figure 1 is a flake from product No. 2, Table 6. This was hand picked to be 100% pure to the eye aided by a glass. It was placed on an alundum plate and burned. Figure 2 is a photo of the remaining mica after combustion for 35 minutes in an atmosphere of oxygen. In each case the mottled background is the alundum plate on which the flake was placed for burning.

American crucible manufacturers, for the manufacture of their graphite crucibles, prefer a form of graphite imported from Ceylon rather than the flakey product of American origin. Of the graphite they use, only about 20% is of domestic origin, the remainder being imported. This preference is partly due to the at one time unreliability of the domestic grades and production, but the manufacturers also state

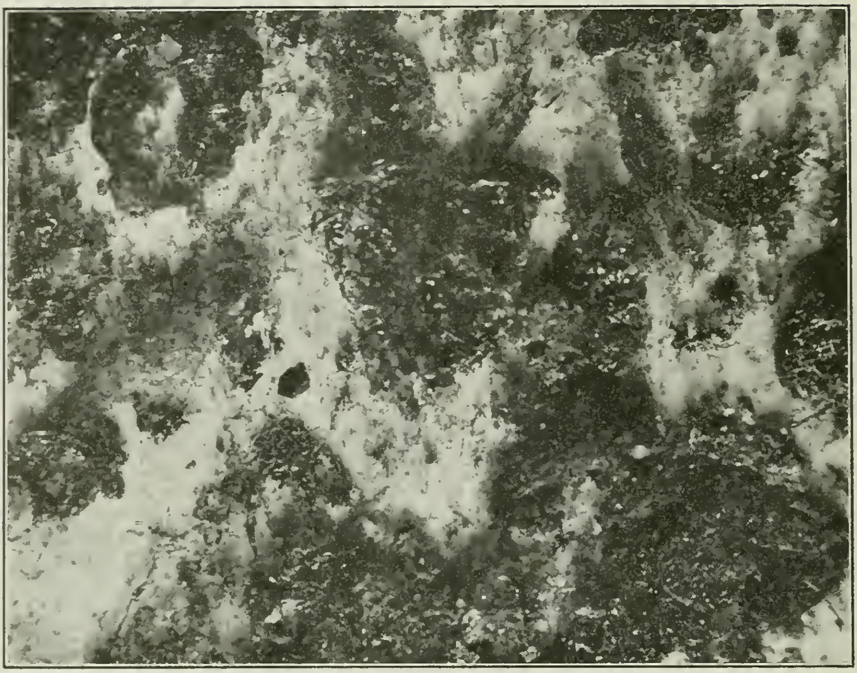


FIG. 1.

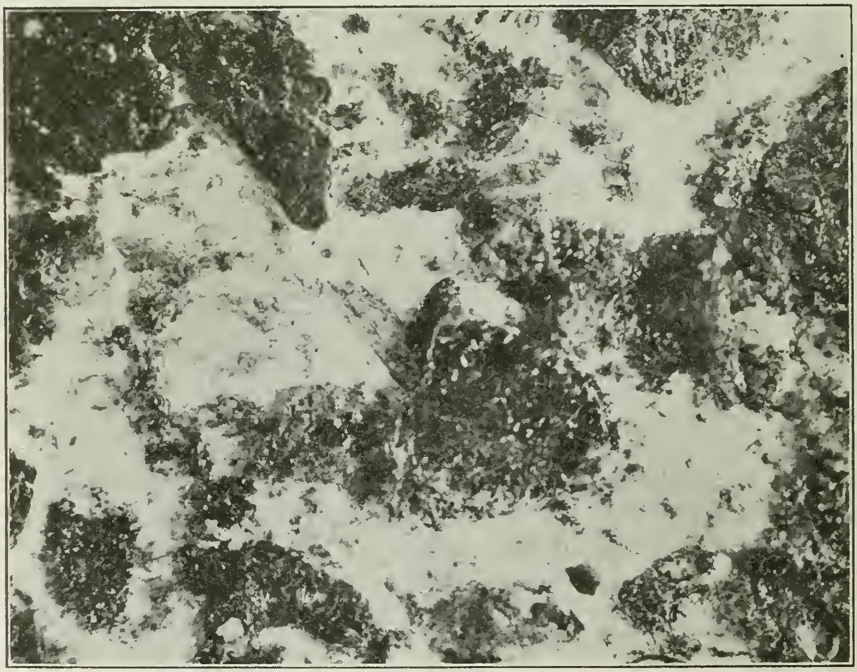


FIG. 2.

that Ceylon graphite makes a better crucible than the domestic flake of the same grade and chemical composition. This difference must then be due to the physical character of the flake, probably to the thickness of the flake and to the nature of its surface. With many graphites the thickness of the flake, and with all graphites the character of the surface, depends on the nature of the regrinding process during cleaning. Naturally by reducing the regrinding to a minimum the original thickness of the flake will be best preserved but the nature of the regrinding by preserving or destroying the natural surface structures of the flake may have a most important effect on the future use of domestic flake. Next to the refractory qualities of a crucible is its strength and toughness, and given the right kind of clay as the bonding agent, the strength will depend on the bond between the clay and the graphite, which bond will be best if the naturally occurring roughness of the flake are preserved. Graphite is crystalline and the crystalline character expresses itself on the surface of the flake in a variety of cleavage lines and fascets, resulting in a distinctly rough surface which is the ideal surface for bonding purposes. Figure 3 is a photo of a raw underground flake and shows the irregularities on the natural surface. Ceylon graphite, which requires little grinding before use, retains these irregularities and consequently makes a good bond with the clay. By the common method of grinding graphite between buhr stones the roughnesses are largely smoothed away resulting in an inferior bonding property and therefore an inferior crucible. Figure 4 is a photo from one of the graphites on the market, note the smoothed faces and rounded edges. Compare this with figure 5 a photo of some of the wet ground flake taken from the ball mill. Figure 6 is of a similar flake given a light regrind in a rotary laboratory grinder. Figure 7 is of the wet ground flake of Table 1. Figure 8 is of some of the wet ground flake assaying 95% carbon of Table 10. Even excessive regrinding in a wet rod mill does not destroy the surface structures as is shown by the graphite of Table 8 (Figure 9) of which 43% of the flake was ground away in raising the assay from 68% to 85% carbon. Inspection of these figures shows clearly the differences in surface structure due to different grinding operations.

In these tests skin flotation was used because of its convenience and ease of manipulation, but the essentials of the system of treatment based on the results obtained will apply also to other systems employing other processes of flotation.

From the results of the experiments a system of concentration for graphite ores may be outlined. The details are subject to variation and arrangement to suit the peculiarities of various ores but in a general way the scheme is as follows:

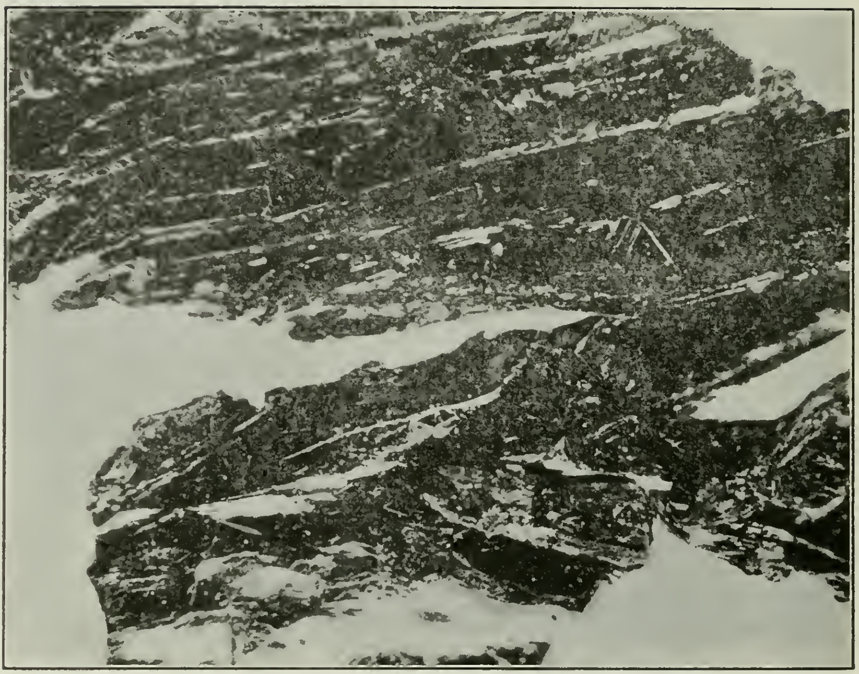


FIG. 3.

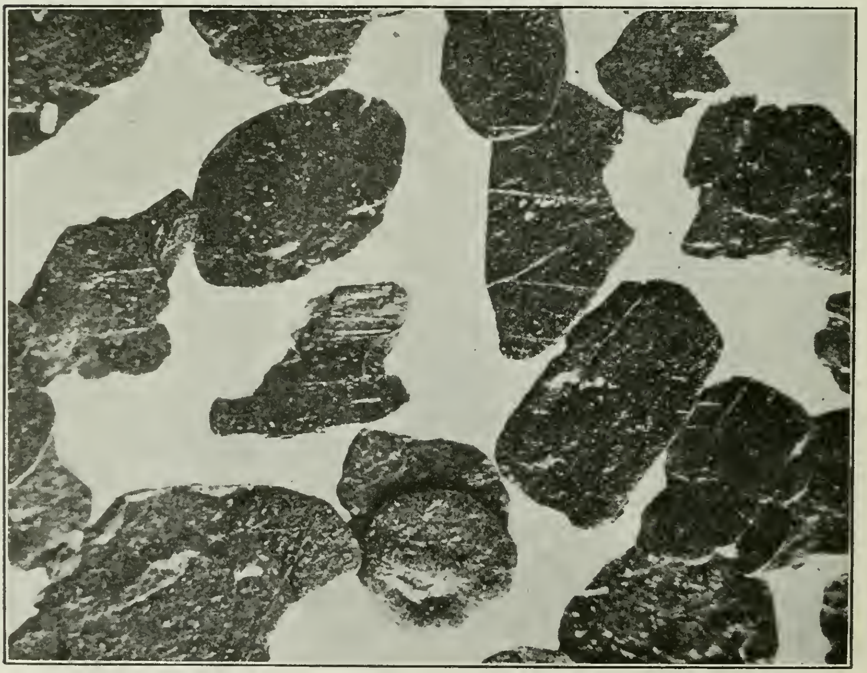


FIG. 4.

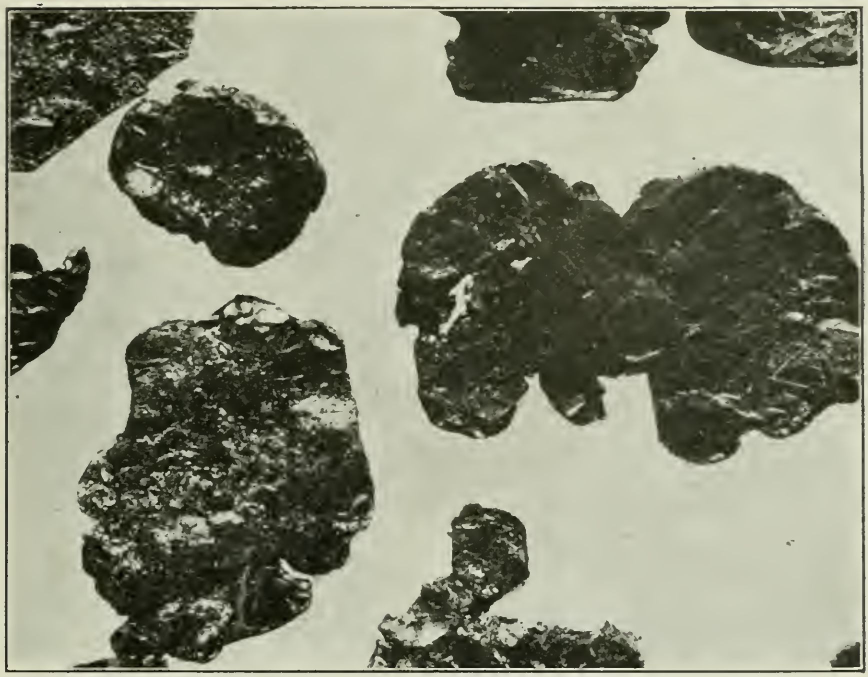


FIG. 5.

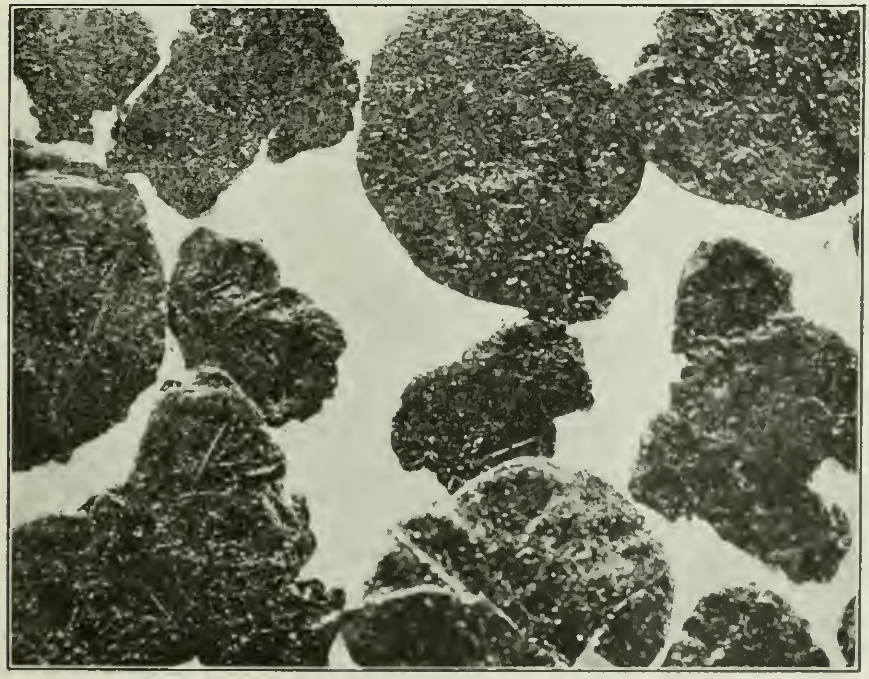


FIG. 6.

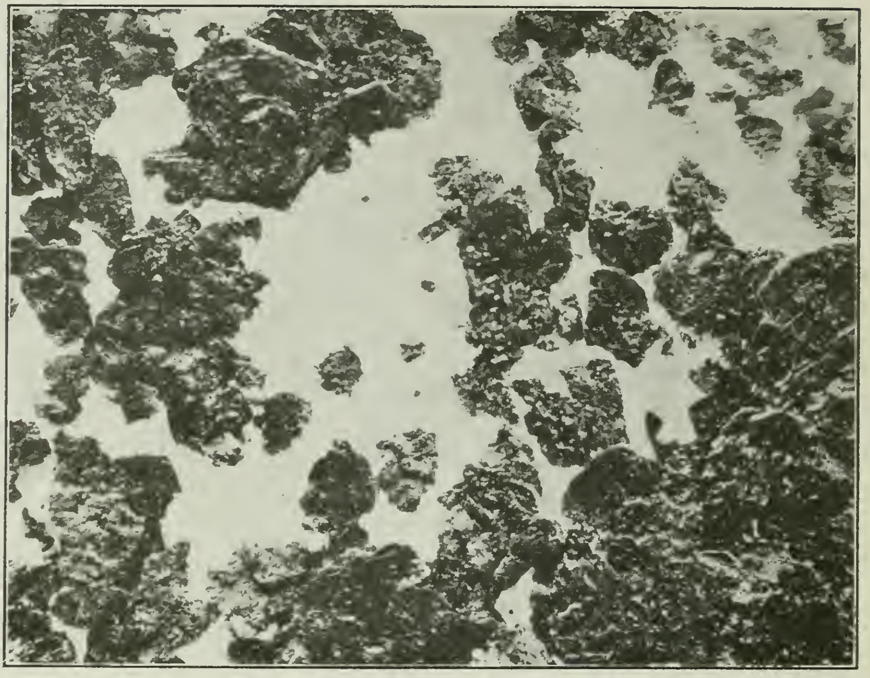


FIG. 7.

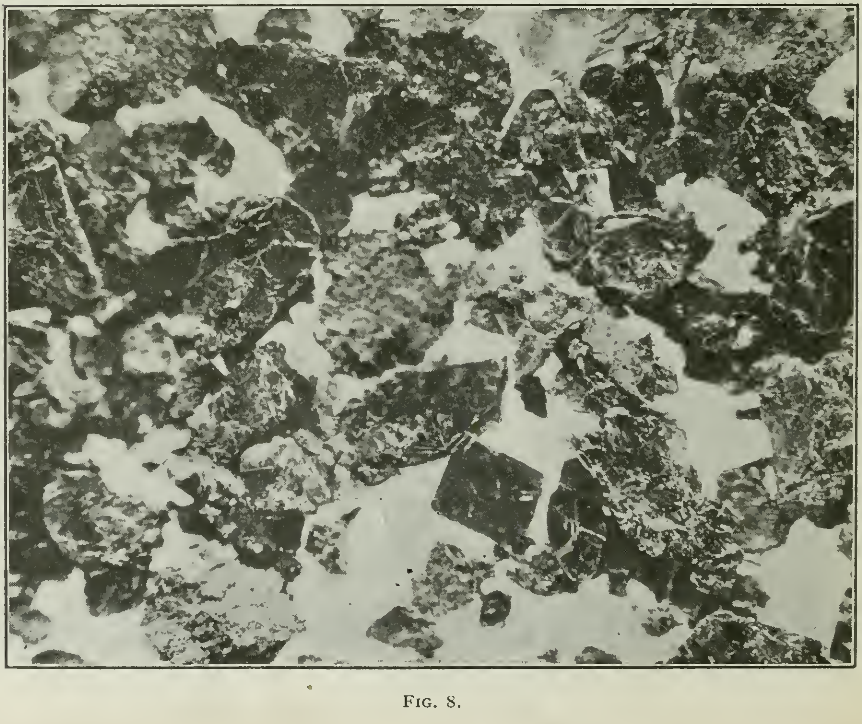


FIG. 8.

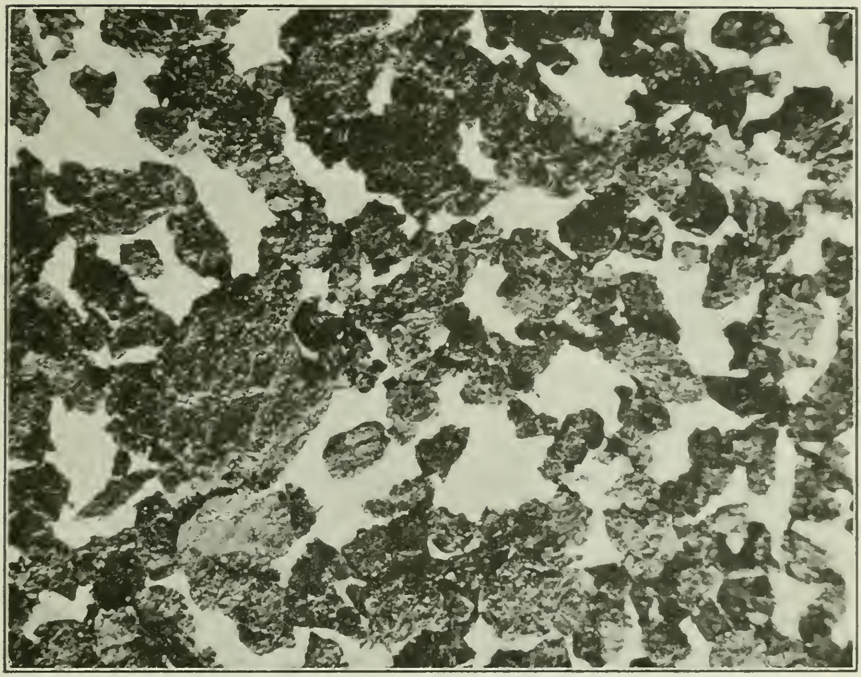


FIG. 9.

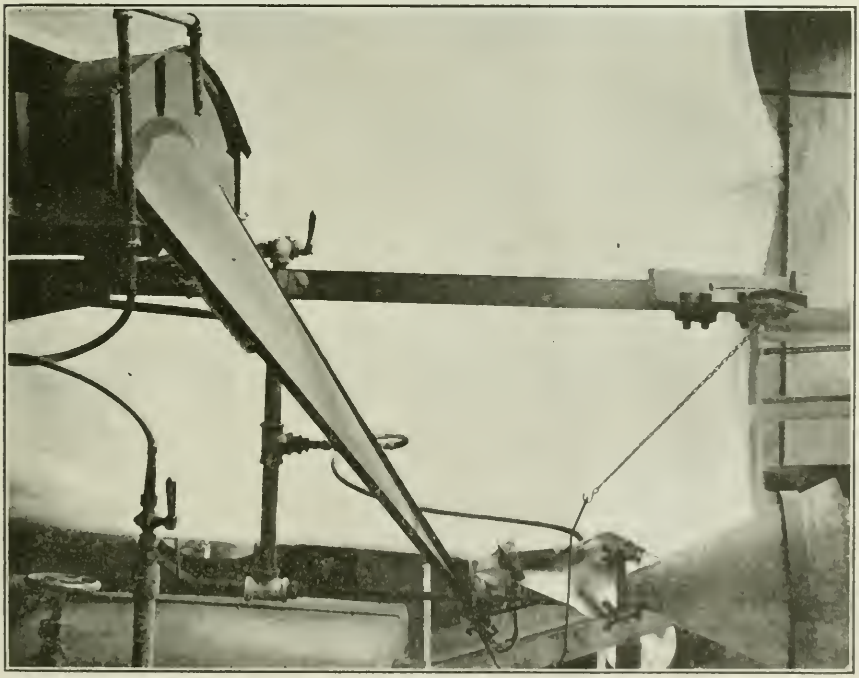


FIG. 10.

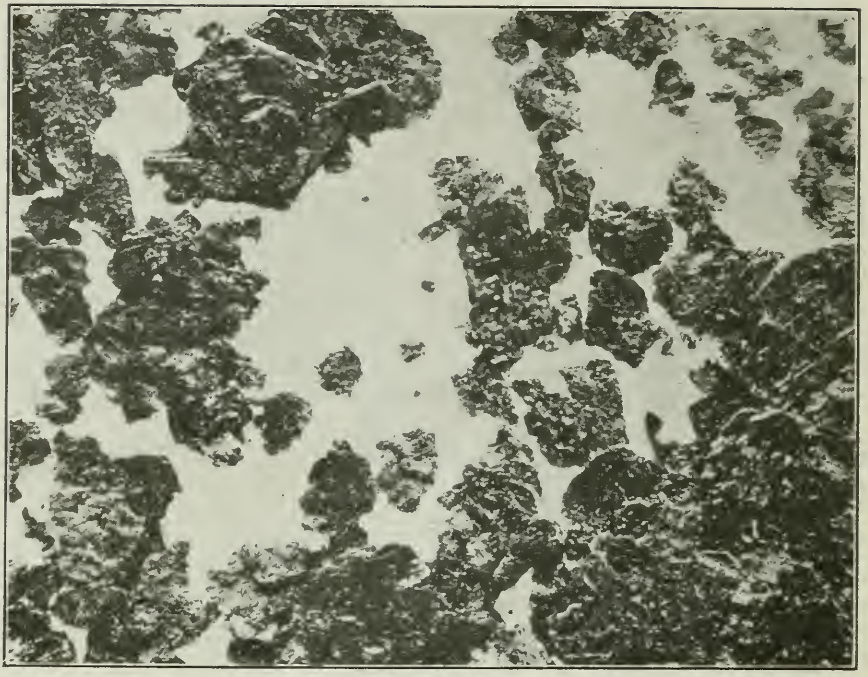


Fig. 7.

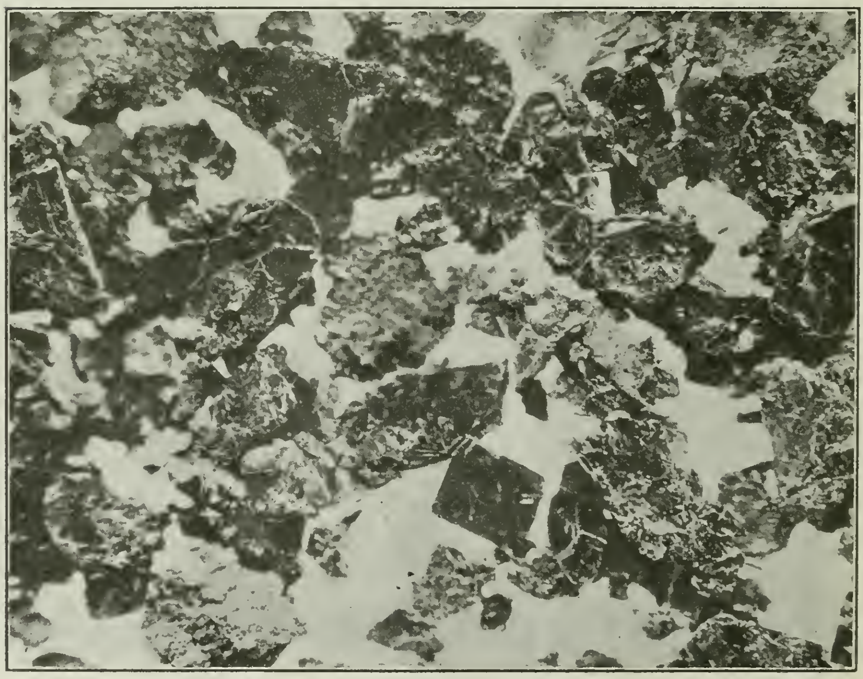


Fig. 8.

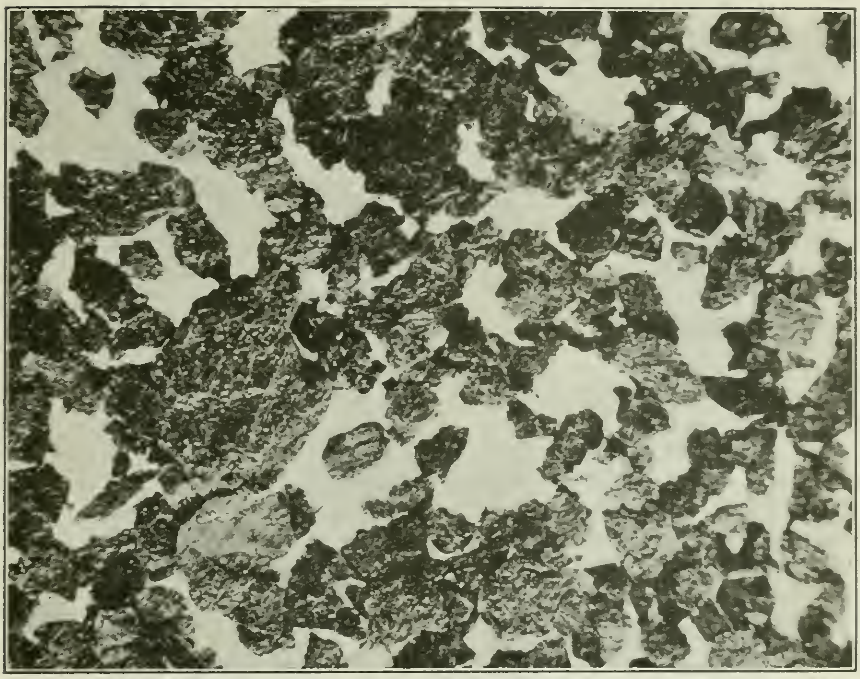


FIG. 9.

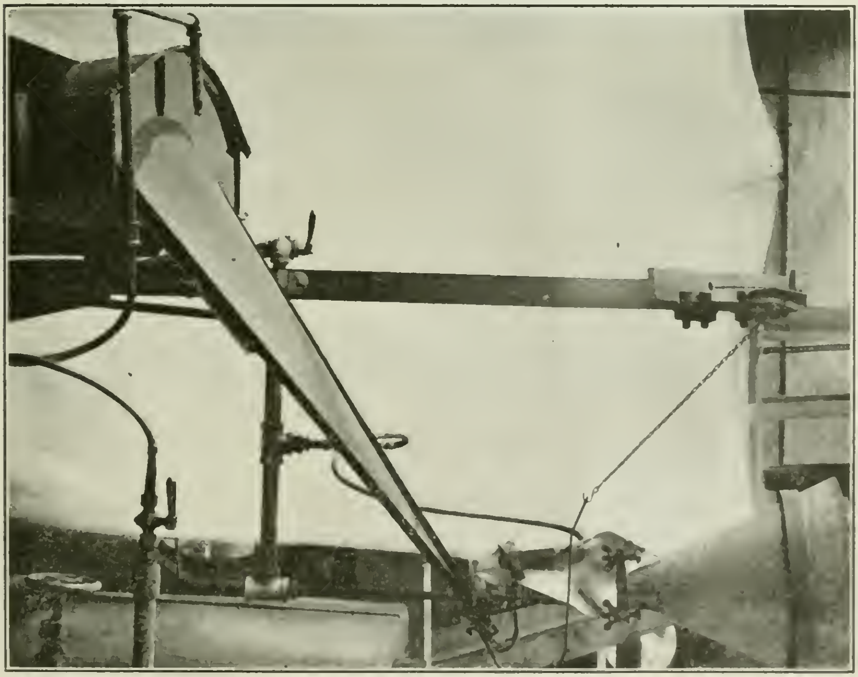


FIG. 10.

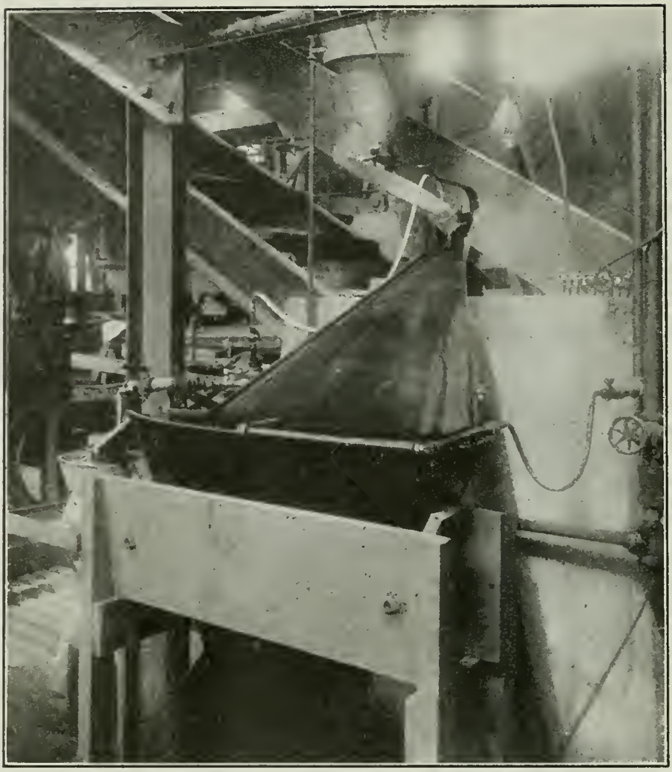


FIG. 11.

1. Crush wet in a rod mill or a ball mill using an excess of water and the minimum of crushing compatible with the liberation of the maximum of coarse flake.

2. Remove the large flake from the mill oversize by screening. With the smaller-flake ores this is not advisable.

3. Divide the sands by Hindered-settling classification into several grades.

4. Float each grade separately and by stages, regulating the quantity of oil to the requirements of each stage, making concentrates middlings, and waste.

5. Regrind the concentrates and the middlings separately in a rod mill with excess of water.

6. Screen wet making a high grade flake and a dirty undersize to be returned for flotation.

This system offers some decided advantages over the older processes. The whole process is wet, eliminating the dust nuisance and avoiding the necessity of drying large tonnages of rock, saving the cost of heat and the handling required in the drying operation, the only drying that will remain to be done will be of the small proportion of finished graphite.

The minimum destruction of flake due to its rapid removal and the minimum of crushing required combine to produce a maximum of coarse flake from this method of crushing. The absence of smearing helps to keep the production of floatable middling at the lowest point.

Separation into grades while not absolutely necessary is an advantage in that it renders possible the flotation of some high-carbon flake that may not require regrinding at all, and also because the sizing of the material helps the separation of graphite and mica in the subsequent flotation process, where the elimination of free mica is practically complete. Flotation by stages permits the adjustment of the amount of oil added so that after the cleaner graphite is removed almost any desired amount of middlings may be obtained leaving to be discharged as waste by far the largest portion of the ore as a coarse sand.

Perhaps the most striking difference between this system and the older systems is in the character of the final product due to the substitution of wet grinding for dry grinding not only in the original crushing but in the cleaning operation on the recovered graphite. Not only does the wet grinding, as proposed, split the flake, liberating the mica clean and in good condition for separation, and also doing some selective-grinding as between graphite and dirt, but it does this work on the flake without destroying the original surface characteristics of the graphite flakes and so preserves its bonding qualities. The graphite flake produced by wet grinding in connection with wet screening is very different in appearance from the dry ground flake. The flake is thicker, it has a larger proportion of the coarser sizes of flakes, and is more brilliant in appearance than dry ground flake because of its crystalline faces and the more efficient removal of the fine dirt by the wet screening. Graphite produced by this system should prove to be the equal or the superior of any flake at present produced.

The process as outlined is simple and does not call for an elaborate flow sheet. Any seeming complication due to flotation by stages is more apparent than real, and would be more than offset by the added recovery of only a few pounds of the high grade flake from each ton of ore.

NEW AND RAPID APPARATUS FOR ELECTRO CHEMICAL ANALYSES.

By J. T. King, B.A.Sc.

It has long been recognized in electro chemical analysis that agitation of the electrolyte hastens deposition of the metals. In an effort to keep apace with the modern demand of more speed in analysis, many mechanical devices and artifices have been introduced, employing the agitation principle. The best known types of these employ rotating anode, rotating cathode, rotating paddle, solenoid, and air streams. Analyses which formerly required many hours, can now be satisfactorily performed in a few minutes. This saving in time has so revolutionized the usefulness of many electrolytic methods, that the old objection of the time required can no longer in fairness be held against them.

It is unfortunate that many operators are prejudiced against the use of rapid plating apparatus. Undoubtedly this attitude has arisen because of the complexity of some of the apparatus that have been introduced. Often the feature of convenience to the operator has been sadly neglected, is some apparatus by the use of a multiplicity of motors with a complexity of switches, and instructions for the use thereof. The apparatus are not sufficiently simple and fool proof.

While observing some of the apparatus employing moving electrolytes in operation, it occurred to the writer, that stirring an electrolyte by rotating its containing beaker, would be a more suitable and efficient method of agitation than others then in use. A careful search of the literature showed that no apparatus employing this principle had been described. The only reference to it having been tried, was found in Smith's Electro-Analysis. In mentioning the work performed by Klobukow (1886), one of the earliest investigators in rapid electro analysis, Smith states, "in his efforts to contrive mechanical devices, he rotated the cathode, and then the anode; indeed he even held the electrodes stationary while moving the electrolyte". Before experiments were begun to ascertain the suitability of the new method of securing agitation, it was felt that provided it was efficient little would be gained unless simple and convenient apparatus could be designed to apply it in every day use.

In the Assaying Laboratories of the Department of Mining Engineering at the University of Toronto, the "Guess Haultain Electrolytic

Outfit" ((*Trans.* A.I.M.E. Vol. 36) is used. This apparatus is simple in design, and is standard among apparatus employing still electrolytes. Three electrodes are used, a central flag shaped cathode, and two narrow anodes on each side of the cathode and spaced equidistant from it. These are suspended in aluminum holders above. As platinum but 0.001 inch in thickness is used for the electrodes, a set will not weigh over two grams. This novel arrangement of the electrodes, permits of distribution of current to both sides of the cathode, and its whole area is utilized for plating. Copper determinations on ores can be made in three hours, lead in two, as good deposits are obtained with high current densities, employing the "nitro compound" recommended. The apparatus has many commendable features, notably high plating speed, small platinum cost, general simplicity and convenience of design. It seemed that the rotating beaker idea could be successfully applied to this design, thereby retaining its best features, with the added advantage of speed.

EXPERIMENTAL DESIGN.

The apparatus was modified as in Fig. A. Here three electrodes per cell were used, the cells being in series. A belt moving over the idlers provides means of revolving the upper part of the stands, on which the beakers stood in a cup, one quarter inch in depth. The lower part of

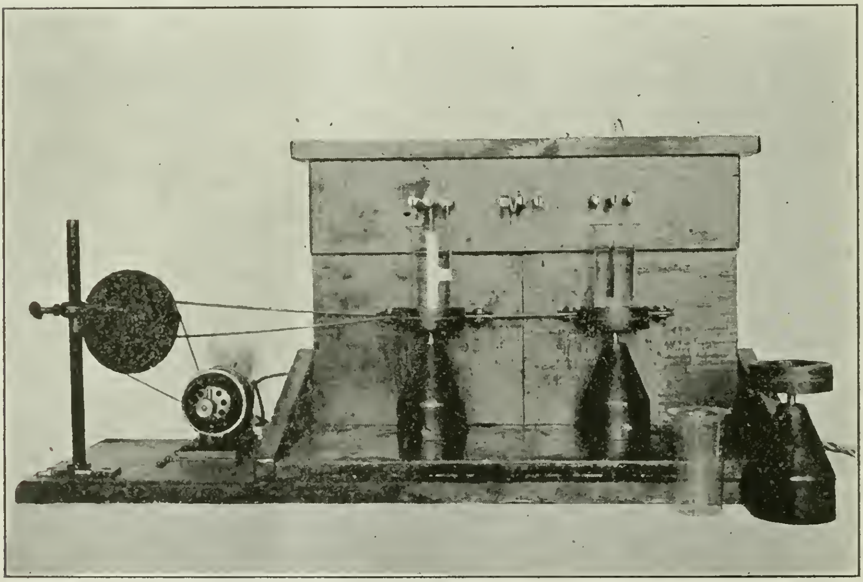


FIG. A.

the stands were weighted with lead, to impart stability when the upper part was in contact with the belt. R.P.M. over sixty could not be used, as the beakers wobbled badly because of the small lateral support given by the cups. To overcome this the cups were increased in depth, when it was found that interference with the electrodes, rendered the operation of placing the beakers in position, and their removal difficult. The cells were short circuited by inserting a metal plug between the electrode holders. Considerable speed and dexterity were essential in removing a cell with its stand, and introducing the plug, so that the current would not be broken while any of the cathodes with their plating were in contact with the acid solutions. Without rotation it required ten hours to plate one gram of copper using N.D.$_{100}$ 3 amperes. The modified form reduced this time to two and a half hours.

Profiting by the defects and limitations of the above apparatus, another was made shown in Fig. B. The beakers were supported in cups but these were fixed centrally under the electrodes above, and rotated by a belt from beneath. The cups were drilled a little large, and in an annular recess inside, rubber tubing was placed to grip the beakers and take care of variations in their diameters. The electrode

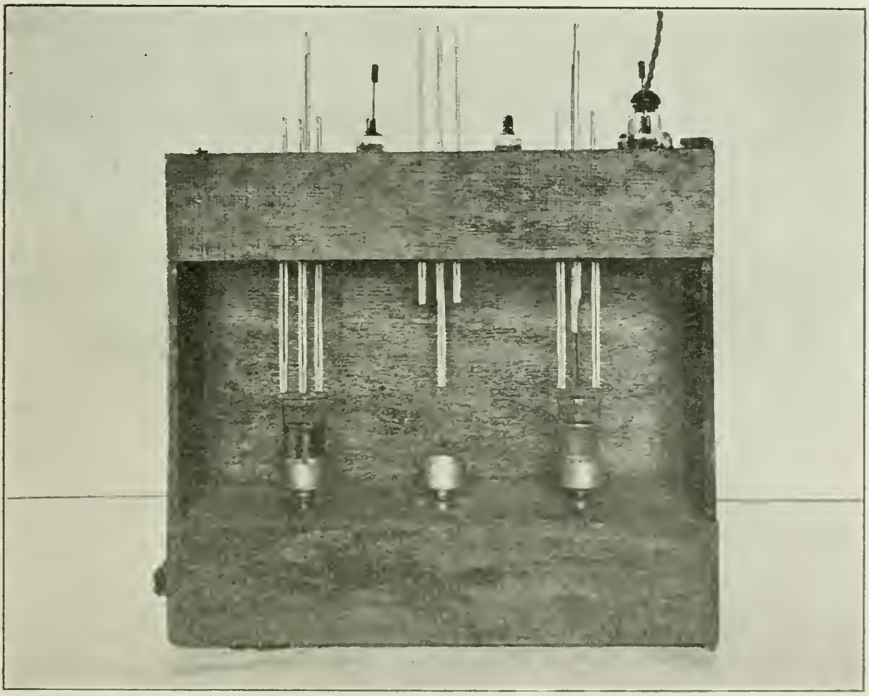


FIG. B.

holders were movable vertically, electric contact being made in the upper compartment. These were of clamp form, and were a marked improvement over the rigid split holders, as they accommodated themselves to the variations in the thickness of the electrodes. Besides the three electrodes formerly used, a fourth or auxiliary one was introduced, placed behind the cathode and in parallel with it. At the end of an electrolysis, before the cathode was raised from its electrolyte, the auxiliary electrode was lowered into the solution. In this way none of the cells in the series were short circuited by the removal of the cathode, and current passed to it as long as it was in the electrolyte.

Good deposits of copper were obtained with as high as N.D.$_{100}$ 8 amperes, and one gram plated in one and a half hours. Rotating speeds up to about two hundred per minute were practical, above this the flat cathode acted too much as a baffle, and splattered the solution out of the beaker. With the higher current density, and higher rotating speed used here, it was expected that plating would be more rapid. It was noted that the plating was much heavier at the sides of the cathode than at the centre, its whole area was not efficiently used. This was probably because the agitation in the beaker was more complete away from the central zone of the solution. The experiments showed that the flat form of electrode was not adapted to the method of agitation used. Cylindrical gauze cathodes were tried and found to be particularly adapted to the rotating beaker idea. Their use eliminated the necessity of a third electrode, as one anode placed in the centre was sufficient. High rotating speeds could be used without the ill effects previously mentioned and uniform plating over the whole surface obtained. The apparatus would do excellent work in the hands of a sympathetic operator but was not sufficiently practical for general use.

DESCRIPTION OF FINAL DESIGN.

The final form of apparatus developed is shown in Fig. 1. Here many features have been incorporated to simplify the operation of rapid plating devices. The chief features which distinguish this from others, besides the novel method of stirring the electrolyte are the system of wiring, and the form of electrode holders. The fig. shows four cell units marked *A*, *B*, *C* and *D*. *A* and *B* are in operation, *C* is being lowered from its electrodes, *D* is at rest. In the left hand compartment are housed a motor, its rheostat, a pilot lamp, ammeter, switch, and two loose pulleys. A belt from the motor passes over the pulleys to the open or cell compartment, along the front and rear of four idler pulleys 1, Fig. 4, and around an end pulley. These are of aluminum with steel shafts running on a ball in the bronze bearings, supported on a cross frame. The five pulleys are spaced equidistant from the units in front. Four perpendicular wooden guide posts 2, see Fig. 4, attached to the base of the cabinet are bored longitudinally and open to the front in a narrow slot 3. On each side of this slot are fastened upper and lower aluminum face plates 4 and 5. In the hole of the guide posts are long hollow cylindrical bearing tubes 6, of aluminium. From the lower part of these a threaded rod 7, extends forward through slot 3. On each rod in order from the face plate are aluminum washer 8, known as the sliding contact, a rubber washer 9, and a nut 10. The bearing tubes are movable up and down the guide posts, and are held in any desired position by the tension of the rubber as adjusted by the nut.

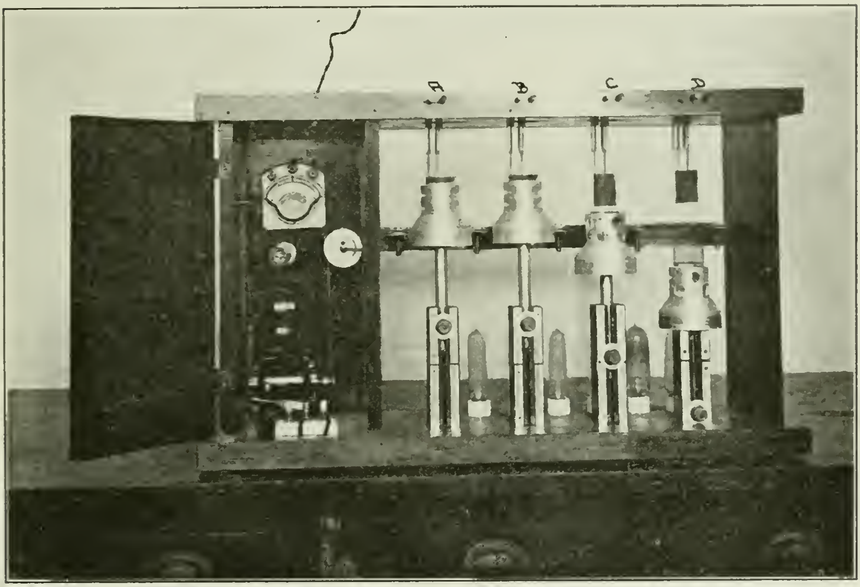


FIG. 1.

Steel spindles turning on a ball in the lower of two bearings of the tubes support the beaker cups 11, above. The cups have two rows of inwardly projecting rubber plugs, arranged to frictionally engage the beakers, thus centering them and accommodating any variations in their diameters that may occur. Above the cups are the electrode holders, one 12, over the spindle axis or centre of the beaker and one 13, offset to the left. These are of broad clamp form, the outside jaw of each being stationary, the other movable. After passing up into the hollow top of the cabinet, these turn at right angles in wooden guides. A coil spring holds the jaws together. Push buttons 14, and 15, operate the movable jaws when desired. The electrical wiring is enclosed in hollow compartments with removable lids at the base and back of the cabinet.

Fig. 2 shows the electrolysis circuit diagrammatically, beakers are shown to correspond to the beaker positions on Fig. 1. The connection

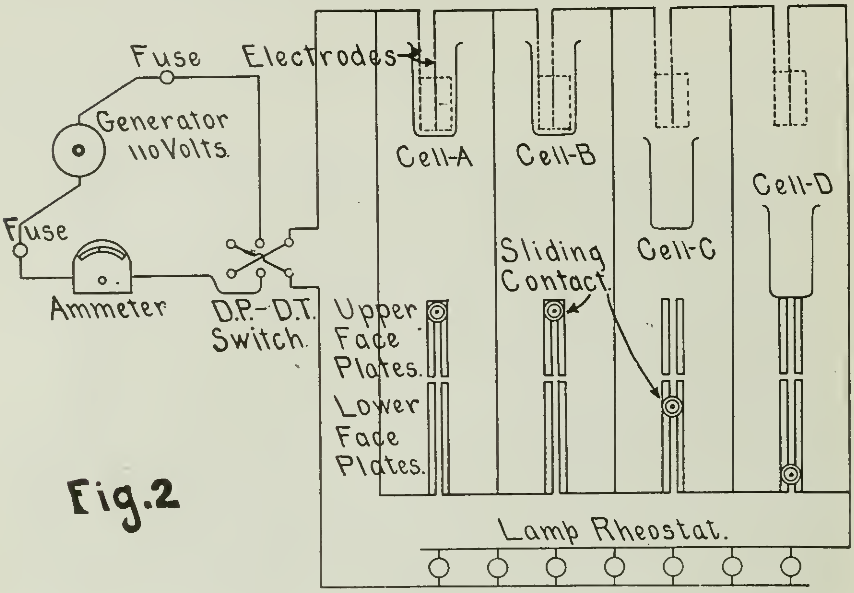


Fig. 2

to the 110 colt D.C. lighting circuit used, and the lamp rheostat to control the cell current are shown in Fig. 3, a rear view of the apparatus.

OPERATION.

To follow the operation of the apparatus assume that the cups are in the lower position, as is cup *D* of Fig. 1. Beakers ready for electrolysis are in the cups, electrodes are in position, the belt is in motion at the desired speed as controlled by the motor rheostat. The current adjusted by the lamp rheostat, will pass across the lower face plates through the sliding contacts. To operate any cell, it is raised by lifting the sliding contact nut until the electrodes are immersed in the electrolyte. On the way up the cone face of the cup engages the belt and rotation begins. At the moment the sliding contact passes from the lower to the upper face plates, which are insulated, the electrodes will have met the electrolyte, current then passes through the cell. Any other cell may be put in operation similarly.

To terminate electrolysis on any cell, it is lowered to the starting position. The operator with a beaker of wash water in one hand lowers the sliding contact with the other hand, as shown in Fig. 4, transferring the water to the plated electrode quickly. The electrode is disengaged by pressing its button. While the cell is being lowered the current continues to pass through the electrolyte until the moment the solution leaves the electrodes, when the sliding contact has engaged the lower

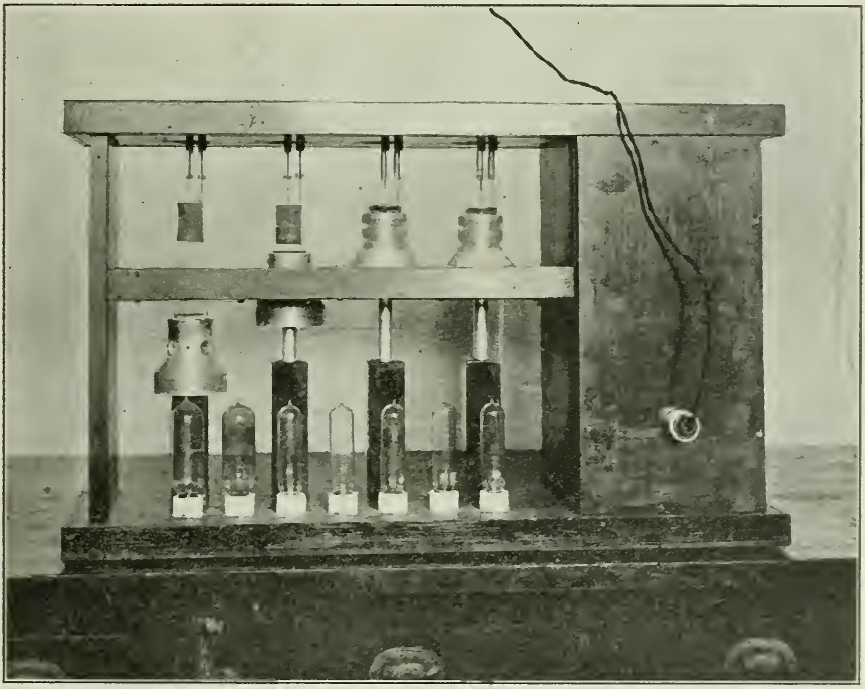


FIG. 3.

face plates. Then the current passes through the lower circuit of the cell. Thus the current is not broken in other cells that may be in operation, they are protected from resolution, and current is on the electrode being removed as long as it is in contact with its electrolyte. The procedure is extremely simple, convenient, speedy, and positive.

The usual manner of fastening electrodes to their holders is by screws, chucks, slots, and similar devices. Some of these are ruinous to the platinum wire that is used, and are difficult to clean when attacked by vapours from the electrolyte. They often do not give sufficient area of contact, causing burning of the electrode and consequent loss in weight. The contact area of the holders here is broad and smooth. The tension on the springs is adjusted, so that while firm hold is exerted on the electrodes, it is not sufficient to injure them in any way. The faces are easily accessible for any cleaning that may be necessary, so that a perfect contact is assured. The push buttons provide an easy and quick way of inserting the electrodes and removing them. The set of four cathodes may be easily removed, rinsed in two changes of water, then two of alcohol, and set to dry hanging over the lamps of the rheostat in eighty seconds.

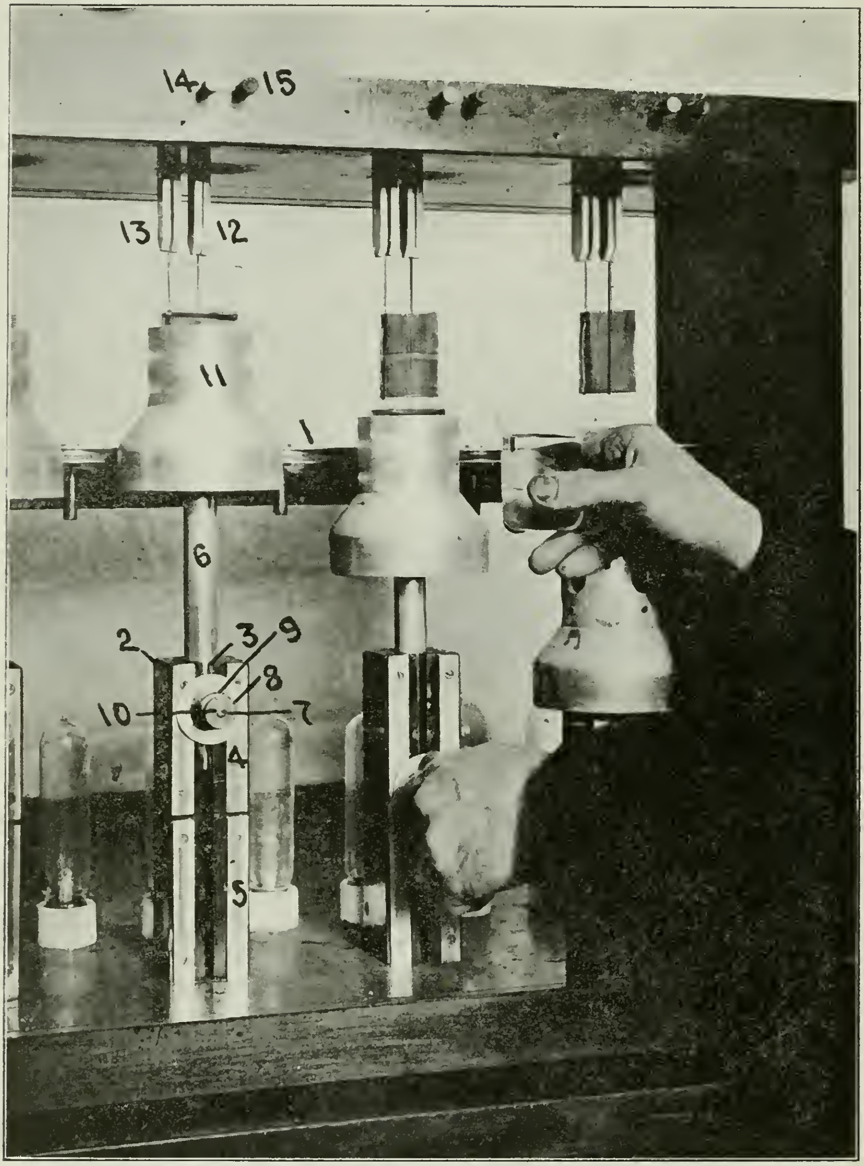


FIG. 4.

The apparatus runs very smoothly, requiring a minimum of attention from the operator. The ammeter and motor being housed in a separate compartment are amply protected from fumes and dust, and aluminium has been utilized wherever practicable in the exposed metal parts. The 1/32 H.P. motor used would operate many more units, so little power is required for rotation. It is balanced on a hinge underneath, thus accommodating itself to any changes occurring in the belt position when units are started or stopped.

TESTS FOR SPEED AND ACCURACY.

In order to ascertain the speed of plating and the accuracy of the work, a number of analyses were made on copper solutions of known strength, and on unknown brasses. In order to determine the copper content of the copper wire used to make up the known solutions, duplicate analyses by electrolyses were made with gauze electrodes and still electrolyte. The method followed was that given by Heath in his "Copper Analyses", for "The Standard Five Gram Electrolytic Assay". These gave 4.9906 and 4.9903 grams, or 99.81% copper.

The known solutions were made as follows: About 20 grams of the wire were carefully weighed, heated on a water bath with 40 c.c. nitric acid, 40 c.c. sulphuric acid, and 100 c.c. of water, until all the wire was in solution and nitrous fumes were driven off. When cool, the cover glasses and sides were washed down and the solution diluted to one litre at 15° C. For the tests aliquot parts of 50 c.c. were pipetted at once, using a pipette agreeing with the flask, and transferred to the electrolysis beakers. Fifteen c.c. of a saturated solution of potassium nitrate, and 25 c.c. of water were added, making 90 c.c. of solution. This volume filled the beakers (new form, tall Fry) over half full, leaving over one inch clear at the top.

On several occasions while working with other apparatus it was observed that when higher current densities and an electrolyte containing nitric acid were used, the copper deposit actually dissolved off the cathode while current was flowing, faster than the current flowing could plate it In order to overcome this and retain the same character of deposit obtained at lower current densities, an equivalent amount of potassium nitrate was substituted for the most of the nitric acid. When this was done the deposits were satiny, pink, and adherent.

Cylindrical cathodes, 2 inches high and $1\frac{3}{8}$ inches diameter, of 28 mesh, 28 gauge copper gauze, were used. The stems were No. 16 copper wire, pushed through the top and bottom of the narrow lap of the gauze, the bottom end was turned back over the gauze and pressed to it. The upper end of the stems were formed into an open loop, about one half inch in diameter. These afforded large area of contact with the holders, and were easily hung over the lamps for drying after being washed.

These cathodes are cheap, simply and quickly made, and with the exception of requiring cleaning in bright dip at least every second plating, are very satisfactory. The immersion area was calculated to be about 100 square centimetres. This particular gauze was adopted after a series of experiments with various sizes and mesh had shown that this was the best, having sufficient rigidity, convenient weight (about 12 grams), and withstood the cleaning operations better than the others. For anodes strips of platinum foil 0.2 inch wide and 0.001 inch thick were used.

Speeds of rotation of about 500 per minute were used. As the tops of the gauze electrodes are set at the level of the electrolyte any tendency to mount up the sides is counteracted and no splattering or throwing of the solution occurs. Table 1 gives the results of tests using 3 amperes current and a voltage drop of 3.7 per cell.

TABLE 1

Time Minutes	Copper Deposited Grams	Copper in Solution Grams	Temperature of Electrolyte °C.
0	0.0000	0.9981	19
5	0.3231	0.6750	23
10	0.6252	0.3729	
15	0.8952	0.1029	
20	0.9904	0.0077	
25	0.9971	0.0010	
30	0.9979	0.0002	
34	0.9982	0.0001	
35	0.9980	0.0001	40
36	0.9981	0.0000	
40	0.9980	0.0001	
50	0.9981	0.0000	
60	0.9981	0.0000	

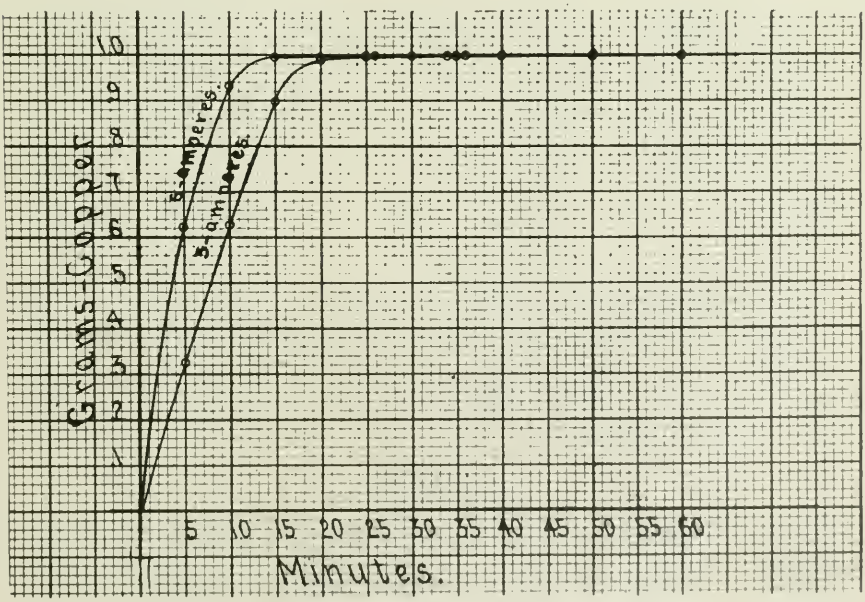
To confirm the completeness of deposition as indicated by weighing, the depleted electrolytes were tested colormetrically, with a saturated solution of hydrogen sulphide water, against equal volumes of solution of the same acid concentration as the electrolyte, and containing 0.002% copper. The usual spot plate test was found to be not sufficiently delicate. Tests after 34 minutes plating time showed complete deposition of copper though not indicated by the balance sensitive to 0.00005 grams. The first wash waters were similarly tested, and in no case where complete deposition was indicated by the balance was any copper found in them.

Table 2 gives the results using 6 amperes current and voltage per cell of 4.1, also at 9 amperes and 4.9 volts.

TABLE 2.

Amperes	Time Minutes	Copper Deposited Grams	Copper in Solution Grams	Temperature of Electrolyte °C.
6	0	0.0000	0.9984	19
	5	0.6235	0.3759	
	10	0.9372	0.0612	
	15	0.9960	0.0024	51
	21	0.9977	0.0007	57
	25	0.9982	0.0002	
	26	0.9984	0.0000	
	30	0.9984	0.0000	63
	35	0.9983	0.0001	67
9	0	0.0000	0.9984	19
	15	0.9983	0.0001	
	16	0.9985	−0.0001	
	17	0.9984	0.0000	71

The deposits of copper without exception were a beautiful pink, satiny, adherent, and firm. These tests show the capability of the apparatus in rapidly plating these large amounts of copper quantitatively, showing that with 3, 6 and 9 amperes the amounts taken are plated in approximately 35, 26, and 16 minutes, respectively, within very close limits of experimental error. The following curves show the results plotted from Tables 1 and 2.



CURVES

This apparatus is ideal for plating the copper of brasses and alloys, allowing of assured deposition on one gram in 40 minutes with 4 amperes, though 30 minutes is usually sufficient. Unknown brasses *A*, *B*, and *C* were analysed for copper. To overcome the difficulty of selecting representative one gram samples, ten grams were taken, treated with 25 c.c. nitric acid and water 75 c.c. to end of brown fumes, 35 c.c. sulphuric acid were added and heated until all the copper was in solution, diluted, filtered and filter washed. The filtrate and washings were brought up to 500 c.c. with water; 50 c.c. aliquots pipetted, 15 c.c. potassium nitrate and 25 c.c. water were added in beaker. Electrolysis was continued 40 minutes with 4 amperes current. Table 3 gives the results of the work on brasses, together with the results on the same brasses by other operators on this and other stirred electrolyte apparatus.

TABLE 3.

Brass	Operator	Apparatus	Copper %
Drillings A	W	Magnetic stirring	61.31
	B	" "	61.36
	G	" "	61.20
	G	" "	61.20
	Mc	Rotating cathode	61.30
	H	Air stirred	61.12
	K	King	61.33
	K	"	61.36
	K	"	61.35
Drillings B	D	Rotating cathode	62.82
	R	Dish cathode	60.68
	K	King	62.90
	K	"	62.83
	K	"	62.85
Cuttings C	F	King	67.64
	F	"	67.65
	K	"	67.59
	K	"	67.60
	K	"	67.56

The apparatus has been thoroughly tested in two laboratories of this University, and has been subjected to the rigors of constant use in a commercial laboratory for over a year, working on the determination of copper and lead in alloys and ores, where it has given excellent satisfaction in every respect.

SUMMARY.

Electro analytical methods using still and stirred electrolytes are discussed.

Results of experimental work with apparatus employing both principles are given and explained.

An apparatus of new design, exhibiting a new method of agitating the electrolyte (stirring by rotating the containing beaker) is described, together with preliminary apparatus leading to the adoption of the apparatus in its final form.

The operation of the apparatus and the method of carrying out an analysis are given in detail.

Tests carried out with the view of determining the accuracy and speed of the new apparatus are described and results tabulated.

A table showing results of experiments on three different brasses, obtained by this and other methods of stirring and carried out by different operators is included.

In conclusion the writer wishes to express acknowledgment of his appreciation, for many helpful suggestions from the staff of the Department of Mining Engineering, and to Professor J. T. Burt-Gerrans of the Electrochemical Department for data furnished, and tests made.

A RESEARCH ON A 4-INCH SINGLE STAGE TURBINE PUMP.

BY J. H. PARKIN, B.A. SC.

PURPOSE OF RESEARCH.

The research was carried out to determine the characteristics of the pump and to investigate the effect of the exit guide ring and of the speed on the characteristics.

The characteristics of a pump are curves showing the variation in head, power, and efficiency with variations in the discharge. The curves are ordinarily plotted for the normal speed only. In this research, the characteristics were determined at a number of speeds and the results compared with the theoretical results calculated from equations.

The effect of the exit guide ring on a pump's characteristics is generally understood, but actual comparative results from a single pump with and without the ring are not common although of considerable interest.

Little actual performance data on small, low head centrifugal pumps (under 6 in. and 50 ft. head) is available for reference compared with the large amount on pumps of larger size. This is doubtless due to monetary considerations. Large pumps, having large initial and operating costs, are generally of more or less special design to meet separate requirements, since the quantities involved are large, and a slight difference in design and resulting operation may mean a considerable saving in cost, either initial or operating. Carefully conducted acceptance trials are therefore run.

On the other hand, small pumps have relatively low initial and operating costs, and are generally manufactured in stock sizes, the pump being chosen from curves and data supplied by the manufacturer, of stock size giving results nearest to the requirements. The nice differences considered in the case of the large pump do not enter into the question and acceptance trials are not often made. While the manufacturer doubtless runs rating tests the results are made available only in such form as to be practically valueless.

PART I. TEST RESULTS.

DESCRIPTION OF THE PUMP.

The pump was of the horizontal belt driven, single stage type with single 4 inch suction and 4 inch discharge, designed to operate at 1,400 r.p.m. giving 0.7 c.f.s. at 45 feet head. The general design of the pump may be seen in Fig. 1, 2 and 3. The single 10 inch runner was of bronze

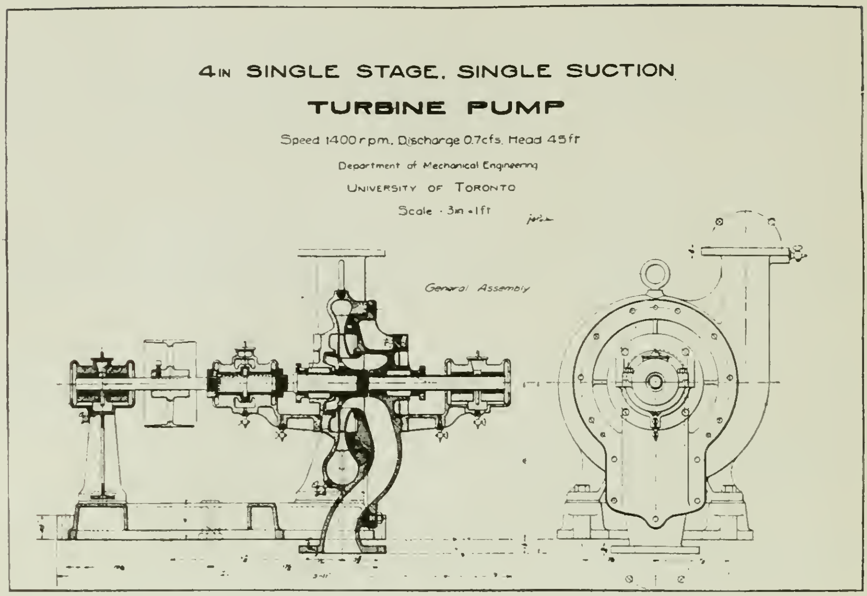
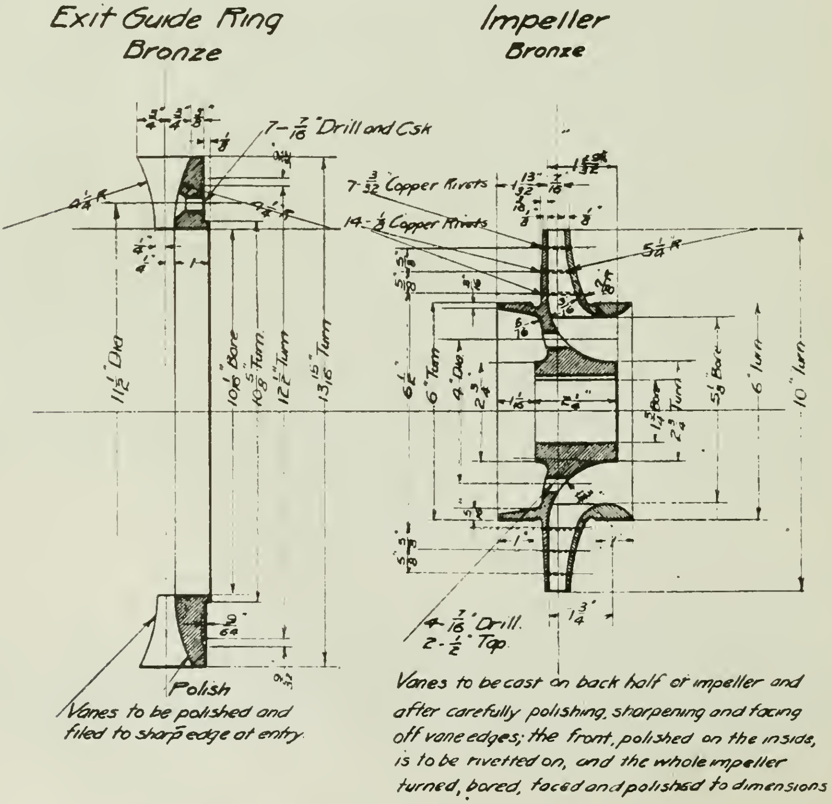

4IN SINGLE STAGE. SINGLE SUCTION
TURBINE PUMP
Speed 1400 r.p.m. Discharge 0.7cfs. Head 45ft
Department of Mechanical Engineering
UNIVERSITY OF TORONTO
Scale · 3in = 1ft
General Assembly

FIG. 1.



Vanes to be polished and filed to sharp edge at entry.

Vanes to be cast on back half of impeller and after carefully polishing, sharpening and facing off vane edges; the front, polished on the inside, is to be rivetted on, and the whole impeller turned, bored, faced and polished to dimensions

FIG. 2.

with eight vanes, all faces being finely finished and polished, the runner was made in two separate parts, to facilitate this polishing, and afterwards rivetted together. The outside stationary guide ring, also of bronze, is 14 inch outside diameter with seven finely polished guides.

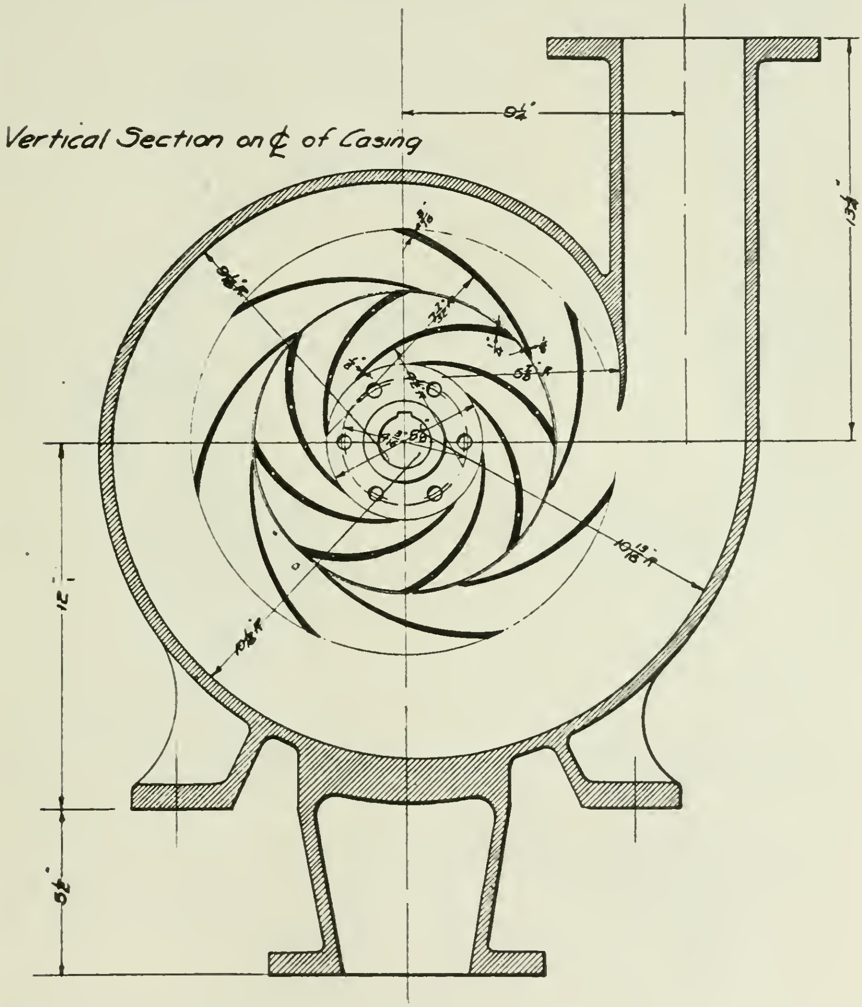


FIG. 3.

There is provided another stationary ring without guides by the use of which the pump may be operated purely as a centrifugal pump. The balancing of end thrust is by the Jaeger system with a water cooled thrust bearing as a precautionary measure. The suction entrance is of convenient form to facilitate removal of exit guide ring and interior examination.

LAYOUT OF THE TEST APPARATUS.

The layout of the pump, motor, piping and various gauges for the test are shown in Fig. 4.

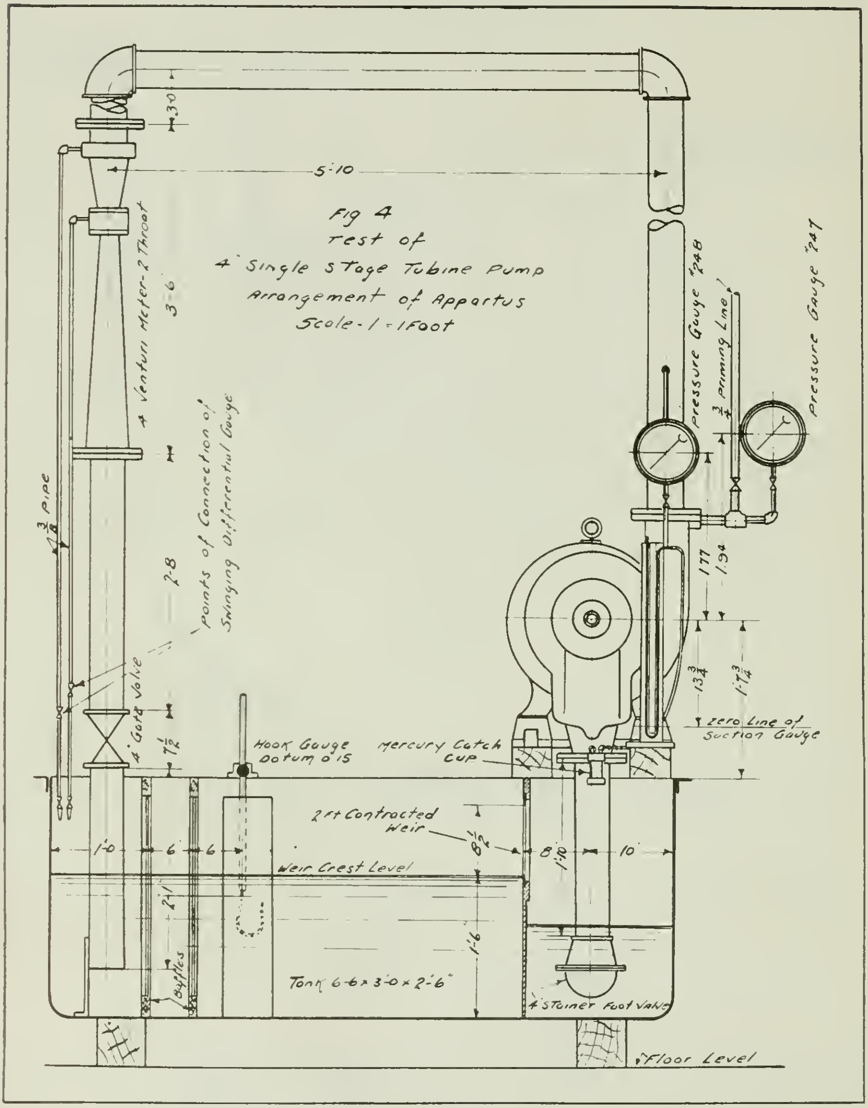


FIG. 4.

A D.C. shunt motor operating at 220 volts, with field rheostat speed control drove the pump through a 5 inch leather belt at about a 9 : 10 speed ratio. Two ammeters, a high and a low reading (100^A and 15^A)

were connected in the line so that either one could be used at will, and a voltmeter was connected directly across the brushes.

The pump was set up, on a steel tank containing a two foot contracted weir. The supply was drawn from the tank on one side of the weir, through a common 4 inch strainer foot valve and discharged through 4 inch W.I. pipe, a Venturi meter and gate valve into the tank on the other side of the weir, thence through a pair of wooden baffles (to remove turbulence) and over the weir.

A hook gauge reading to 0.01 inches was used to read heads on the weir and a special form of swinging differential gauge for the Venturi Meter.

Head Corrections.

In calculating the total operating head of the pump from the readings of the pressure and suction gauges, these readings were corrected to equivalent readings at the centre of the pump and added. The velocity heads being equal may be neglected.

Calculation of Discharge.

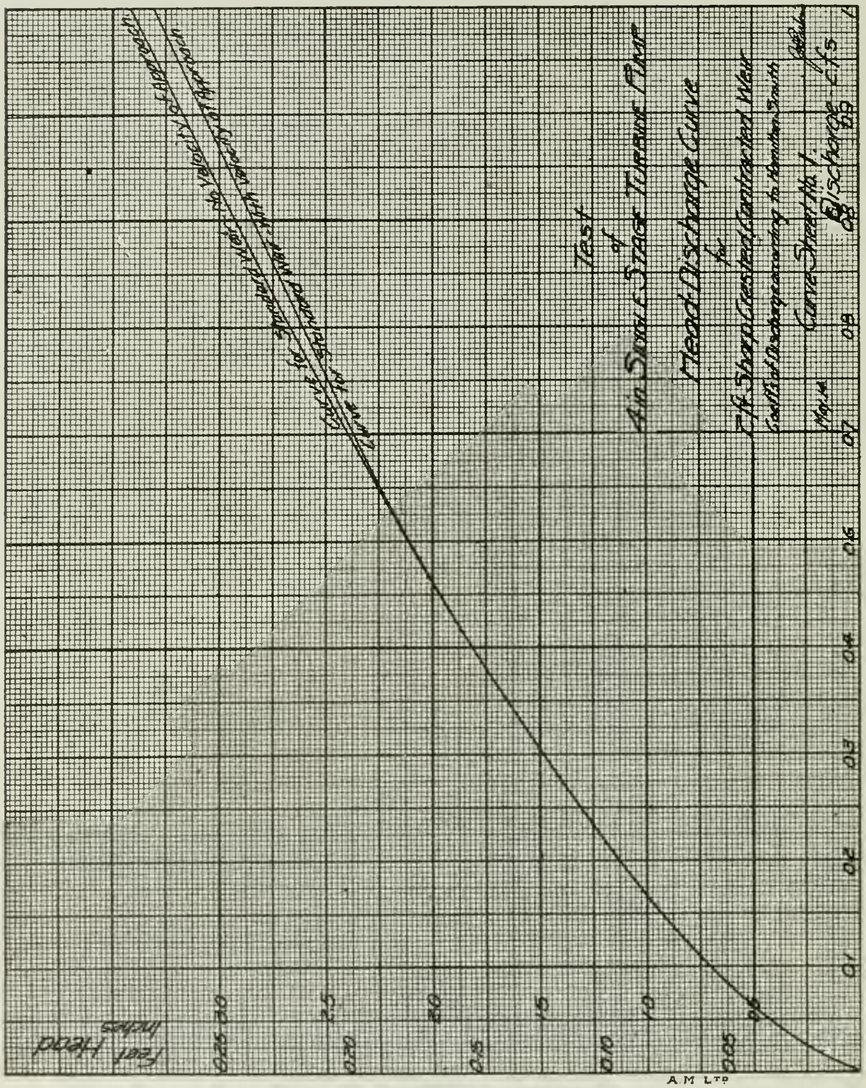
The discharge was determined by two methods, namely Venturi meter and weir. The Venturi meter formula $Q = A\,Cd\sqrt{2g(H_1 - H_2)}$ in the case of this meter (2 inch throat and $Cd = 0.98$) reduces to $Q = 0.1754\sqrt{H}$ where H is pressure difference in inches of mercury. The Hamilton Smith weir formula $Q = \frac{2}{3}Cd\,b\sqrt{2g}\;H^{\frac{3}{2}}$ was used for the contracted weir and reduced to $Q = 10.69\,Cd\,H^{\frac{3}{2}}$ for this weir. A head discharge curve was plotted to facilitate the work (see curve sheet 1). It was found that for heads on the weir greater than two inches the velocity of approach affected the discharge and a correction was necessary as shown.

Comparing the results by the two methods it was found that they agreed quite closely at all discharges over 0.1 c.f.s. Below this the meter was found to be erratic. This discharge corresponds to a meter throat velocity of about 3.5 feet per second.

Subdivision of Power Input.

In order to better analyse the pump performance with regard to the various losses that occur, the input power delivered to the belt by the motor was subdivided and the portions lost in mechanical losses, in hydraulic losses, and usefully employed in pumping water were determined. The purely mechanical losses include those due to bearing and gland friction, disc friction and belt losses; and the hydraulic losses consist of losses due to shock and friction.

The input power was readily found from the electrical measurements on the motor, the mechanical losses by running the pump empty and finding the input at various speeds on the assumption that these losses varied directly as the speed and were independent of the load.



CURVE SHEET No. 1.

The difference between the input and the mechanical losses represents the power actually delivered to the water. The power output or energy of the discharge was found directly from the total head and discharge. The power to the water less the output power is the power lost in hydraulic losses.

DETERMINATION OF EFFICIENCY.

The total or net efficiency of the pump is the ratio of the output power to the power delivered to the belt. This efficiency includes the efficiency of the belt drive. It seems proper to include this latter because the pump is belt driven, the type of drive is a part of the pump itself and the losses introduced by it must always be reckoned on. The mechanical efficiency of the pump is the ratio of the power delivered to the water, to the power input. The ratio of the net power output to the power delivered to the water is the hydraulic efficiency. The product of mechanical and hydraulic efficiencies is the total efficiency.

CORRECTION OF RESULTS TO COMMON SPEEDS.

The various results are corrected to a series of common speeds for curve plotting by making use of the following well known fundamental relations relating to water turbines and centrifugal pumps, namely:

1. The head varies as the square of the speed, hence

$$H=H_1\left(\frac{N}{N_1}\right)^2.$$

2. The discharge varies as the square root of the head, hence

$$Q=Q_1\sqrt{\frac{H}{H_1}}.$$

3. The power varies as the three-halves power of the head, hence

$$P=P_1\left(\frac{H}{H_1}\right)^{\frac{3}{2}}.$$

In the above N_1, H_1, Q_1, and P_1 are the actual test results and N, H, Q and P the corrected values.

The results of each test were thus all reduced to a common speed and then plotted on the various curve sheets.

This correction was not a necessity in this particular test as the difference between the test speeds and the even speeds, 1,000, 1,100 r.p.m., etc., at which results were desired was in no case large. The error arising due to assuming the results as those at the even speeds being less than those likely to arise due to the natural errors occurring in test work.

RESULTS.

These results are as shown on the accompanying tables and curves.

Table Nos. 1 and 2 show in detail the results of the pump test.

Table No. 3 gives the principal results corrected to constant speeds.

Table No 1

Test of 4 in Single Stage Turbine Pump
— Test Data —

Speeds of 1000, 1100 and 1200

No	Pressure		Suction		Total	Venturi		Weir		Mean	Pump	— Power —				- Efficiency —			No
	Gauge	Head	Gauge	Head	Head	Gauge	Disch	Head	Disch	Disch	Speed	Input	Frictn	To Water	Output	Mech	Hyd'c	Net	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2.01	3 95	10 05	11 27	15.24	16 80	0 719	2 65	0718	0 719	995	1420	351	1069	683	75 2	64 0	48 1	1
2	10 00	11 94	8 00	9 03	20 97	12 12	0 610	2 40	0.610	0 610	995	1372	351	1021	798	74 4	78 1	58 1	2
3	14 85	16 79	6 40	7 28	24 07	8 70	0 517	2 14	0 513	0 515	995	1317	351	966	774	73 3	80 1	58 8	3
4	19 40	21 34	5 00	5 75	27 09	5.55	0 413	1 85	0 410	0 411	1000	1248	352	896	694	71 8	77 4	55.6	4
5	21 90	23 84	4 00	4 66	28 50	3 40	0 323	1 58	0 329	0 326	1000	1057	352	705	579	66 7	82 1	54 8	5
6	22 25	24 19	3 40	4 02	28.21	2 06	0.252	1 32	0 255	0 253	1004	970	354	616	445	63 5	72 2	45.9	6
7	21 75	23 69	3 00	3 58	27.27	1 28	0 198	1 11	0 195	0 196	1004	904	354	550	334	60 8	60 7	36 9	7
8	20 22	22 16	2 60	3 14	25 30	0 56	0 131	0 81	0 120	0125	1005	792	355	437	198	55.1	45.3	25.0	8
9																			9
10																			10
										1100 R P M									
11	3 20	5 14	11 80	13.10	18 24	19 80	0 780	2 79	0.780	0 780	1093	1743	376	1367	887	78 4	64 9	50 9	11
12	13 80	15 74	8 90	10.00	25 74	15.00	0 679	2 55	0.678	0 679	1105	1705	379	1326	1088	77.1	82 1	63 8	12
13	17.96	19 90	7 20	8 06	27 96	11.34	0 590	2 35	0 591	0 591	1090	1656	375	1281	1030	77 5	80.4	62 2	13
14	21 92	23 86	6 20	6 99	30.85	8.46	0 510	2 10	0 498	0 504	1090	1575	375	1200	969	76.2	80.7	61 5	14
15	25.60	27 54	5 10	5 79	33 33	5.50	0 412	1 84	0 408	0 410	1100	1458	378	1080	861	74 1	78 8	58 4	15
16	27 80	29 74	4.10	4 70	34 40	3 60	0 333	1 60	0 333	0 333	1100	1330	378	952	714	71 6	75 0	53 7	16
17	28 10	30 04	3 40	3.94	33 98	2 38	0 270	1.34	0 260	0 265	1100	1175	378	797	561	67.9	70 4	47 8	17
18	27 50	29.44	3 00	3 51	32 95	1 16	0 189	1 09	0.191	0 190	1105	1050	379	671	396	63 9	58 9	37 7	18
19	26 52	28 26	2 60	3 06	31.32	0.56	0 131	0.83	0.127	0 129	1109	925	380	545	252	58 9	46 2	27 2	19
20	13 00	14 94	2 60	3 06	18 00	0.40	0 111	0 72	0 103	0 107	1110	922	381	541	120	58 7	22 1	13 0	20
										1200 R.P.M									
21	5 00	6 94	12 80	14 20	21.14	—	—	2 90	0 826	0 826	1190	2117	400	1717	1089	81 1	63 5	51 4	21
22	16 00	17.94	10 40	11.57	29 51	17 60	0 735	2 69	0 735	0 735	1200	2082	403	1679	1351	80 6	80.5	64 9	22
23	21 50	23 44	8 10	9 05	32 49	12 50	0 620	2 41	0 616	0.618	1186	1987	399	1588	1252	80 0	78 8	63 0	23
24	27.20	29 14	6 24	7 04	36 18	8 50	0 511	2 13	0 510	0 511	1187	1867	399	1468	1152	78 6	78 5	61 7	24
25	31.00	32 94	4 94	5.61	38 55	5 70	0.418	1 86	0 415	0 416	1190	1735	400	1335	1000	77 0	74 9	57 6	25
26	33 60	35 54	3 90	4 48	40.02	3 75	0 339	1 61	0 337	0 338	1195	1605	401	1204	844	75.1	70 1	52 6	26
27	33 30	35 24	3 40	3 94	39 18	2 21	0 261	1 36	0 264	0 262	1195	1433	401	1032	641	72 1	62 1	44 8	27
28	31 80	33 74	2 80	3 29	37.03	1.14	0 187	1 10	0 193	0 190	1200	1282	403	879	439	68 5	49 9	34 2	28
29	30 40	32 34	2 60	3 06	35 40	0.38	0 108	0.86	0 132	0 120	1195	1109	401	708	265	63 9	37 4	23 9	29
30	13 50	15 44	2 60	3 06	18 50	0 26	0 089	0 55	0 072	0 072	1195	1072	401	671	81	62.6	12.1	7.5	30

TABLE NO. 1.

Table No 2

Test of 4 in Single Stage Turbine Pump
— Test Data —

Speeds of 1300, 1400 and 1500

No	Pressure		Suction		Total	Venturi		Weir		Mean	Pump	— Power —				Efficiency —			No
	Gauge	Head	Gauge	Head	Head	Gauge	Disch.	Head	Disch	Disch.	Speed	Input	Frictn	To Water	Output	Mech	Hyd	Net	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
31	8 00	9 94	13 60	15.13	25.07	—	—	2 98	0 859	0 859	1295	2560	430	2130	1331	83 2	62 5	52 0	31
32	20 60	22 54	11 00	12 30	34 84	18 60	0 756	2 74	0.756	0.756	1300	2532	430	2099	1642	82 8	78 3	64 8	32
33	27 20	29 14	8 70	9 79	38 93	13 80	0 651	2 49	0 651	0 651	1300	2483	433	2050	1580	82 5	77 1	63 6	33
34	33.40	35 34	6 90	7 75	43 09	9.70	0 546	2.22	0 541	0.544	1300	2360	433	1927	1482	81 6	75 9	61.9	34
35	38 90	40 84	5.00	5.79	46 63	6 20	0 437	1 92	0 435	0.436	1300	2178	433	1745	1269	80 1	72 7	58 2	35
36	40 40	42 34	4.00	4 70	47 04	4.10	0 355	1.67	0 355	0 355	1300	2075	433	1642	1041	79 2	63 4	50.2	36
37	39 50	41 44	3 30	3 90	45.34	2 42	0 273	1 42	0.280	0.280	1300	1802	433	1369	780	75.9	57 0	43.2	37
38	38 00	39 94	2 80	3 36	43 30	1.19	0 191	1 11	0.195	0.195	1300	1553	433	1120	520	72 1	46 4	33 5	38
39	36 20	38 14	2.60	3 06	41 20	0 37	0 106	0.87	0 134	0.134	1300	1368	433	930	308	68 2	33 5	22 6	39
40	19 00	20 94	2 60	3 06	24 00	0.33	0.101	0 67	0.094	0.094	1295	1340	430	910	145	67 9	15 9	10 8	40
										1400 R.P.M									
41	11 70	13 64	14 30	15 82	29 46	—	—	3.05	0 892	0 892	1400	3065	471	2594	1639	84 7	63.2	53 5	41
42	24 20	26 14	12 00	13 30	39 44	21 10	0 805	2.85	0.805	0.805	1410	3075	476	2599	1980	84 5	76.2	64 3	42
43	32 70	34 64	9 50	10 59	45 23	16 10	0.704	2 60	0.698	0.701	1410	3028	476	2552	1978	84 5	77.5	65.3	43
44	40 00	41.94	7 80	8 61	50 55	11.40	0 592	2.35	0.591	0.592	1410	2883	476	2407	1866	83 5	77.5	64 7	44
45	45.00	46 94	6 20	6.98	53 92	8 00	0 496	2.10	0 498	0 497	1410	2735	476	2259	1670	82 5	74 0	61.0	45
46	48 20	50.14	4 90	5.56	55 70	5.65	0 417	1 85	0.410	0.414	1410	2580	476	2104	1438	81 6	68 3	55.8	46
47	48 80	49 94	3.90	4 48	54 42	3 65	0 335	1.61	0.336	0.336	1410	2295	476	1819	1139	79 2	62.6	49 6	47
48	46 00	47 94	3 30	3 82	51.76	2.16	0 258	1.35	0 261	0 260	1400	1970	471	1499	839	76.1	56.0	42.6	48
49	45 00	46 94	2.90	3 37	50.31	1 34	0 203	1.10	0190	0 196	1400	1797	471	1326	596	73 8	44.9	33.2	49
50	42 00	43 94	2.60	3 06	47.00	0.64	0.140	0.85	0 132	0 136	1400	1583	471	1112	398	70 2	35.8	25.1	50
										1500 R.P.M									
51	14 50	16 44	15 50	17.15	33 59	—	—	3.125	0 923	0.923	1500	3720	523	3197	1932	86 0	60.5	51 9	51
52	31.00	32 94	12.10	13.26	46 10	21.10	0 805	2.85	0 805	0 805	1500	3735	523	3212	2315	86 0	72.1	62 0	52
53	42 50	44 44	9 70	10 81	55 25	15.60	0.693	2.60	0 698	0 696	1500	3550	523	3027	2400	85.3	79 2	67 6	53
54	47 80	49.74	7.60	8.50	58.24	11.50	0.594	2 35	0 591	0 593	1500	3315	523	2792	2158	84 4	77 1	65.0	54
55	52 60	54 37	6 10	6 88	61.25	8 00	0 496	2.10	0 498	0 497	1500	3185	523	2662	1898	83 6	71 2	59 6	55
56	56 50	58 27	5 10	5.79	64.06	5.80	0 422	1 87	0 416	0 419	1500	2835	523	2312	1673	81 5	72 4	59 0	56
57	54 80	56.57	4.00	4.59	61.16	3.50	0.328	1.60	0 334	0.331	1490	2550	517	2033	1262	79.7	62 1	49 5	57
58	52.60	54 37	3.30	3.82	58.19	2.16	0.258	1.35	0.261	0 260	1490	2285	517	1768	944	77.3	53 4	41.3	58
59	51 00	52.77	2.90	3.37	56 14	1.20	0 192	1.10	0.190	0.191	1500	2140	523	1617	669	75 6	41.3	31.3	59
60	45.80	47.74	2.60	3.06	50.80	0 64	0.140	0.85	0.132	0.136	1500	1955	523	1432	430	73.3	30.0	22.0	60

TABLE NO. 2.

Test of 4-in Single Stage Turbine Pump

Table No 3. — Test Data Corrected to Constant Speeds —

No	Test Speed	Speed N	Head H	Disch. Q	Power P	Effy E
1	2	3	4	5	6	7
1	995	1000	15 40	0 722	1435	481
2	995		21 10	0 613	1388	581
3	995		24 30	0 517	1330	58 8
4	1000		27 09	0 411	1248	55 6
5	1000		28 50	0 326	1057	54 8
6	1004		28 00	0.252	958	459
7	1004		27 00	0.195	890	36 9
8	1005		25 05	0 124	780	25 0
9						
10						
			1100 RPM			
11	1093	1100	18 45	0 785	1775	50 9
12	1105		25 50	0 676	1682	63 8
13	1090		28 45	0 596	1700	62 2
14	1090		31 40	0 503	1617	61 5
15	1100		33 33	0 410	1458	58 4
16	1100		34 40	0 333	1330	53 7
17	1100		33 98	0 265	1175	47 8
18	1105		32 60	0.189	1034	37 7
19	1109		30 80	0 128	903	27 2
20	1110		17 70	0 106	899	13 0
			1200 RPM			
21	1190	1200	21 50	0.835	2168	51 4
22	1200		29 51	0.733	2082	64 9
23	1186		33 20	0.625	2053	63 0
24	1187		37 10	0 517	1938	61 7
25	1190		39 10	0 419	1772	57 6
26	1195		40.30	0.339	1622	52 6
27	1195		39 50	0 263	1449	44 8
28	1200		37 03	0.190	1282	34 2
29	1195		35.65	0.120	1120	23 9
30	1195		18 65	0.072	1085	7.5

No	Test Speed	Speed N	Head H	Disch. Q	Power P	Effy E
1	2	3	4	5	6	7
31	1295	1300	25.30	0 862	2585	52 0
32	1300		34 84	0 756	2532	64 8
33	1300		38 93	0 651	2483	63 6
34	1300		43 09	0.544	2360	61 9
35	1300		46 63	0 436	2178	58 2
36	1300		47 04	0 355	2075	50 2
37	1300		45 34	0 276	1802	43 2
38	1300		43 30	0 193	1553	33 5
39	1300		41 20	0.120	1363	22 6
40	1295		24 15	0 097	1353	10 8
			1400 RPM			
41	1400	1400	29 46	0.892	3065	53 5
42	1410		38 80	0 799	3005	64 3
43	1410		44 50	0 696	2965	65 3
44	1410		49 80	0 588	2820	64 7
45	1410		53 10	0 494	2675	61 0
46	1410		55 00	0 411	2545	55 8
47	1410		53 70	0 334	2245	49 6
48	1400		51.76	0 260	1970	42 6
49	1400		50.31	0 196	1797	35 2
50	1400		46 97	0 136	1583	25.1
			1500 RPM			
51	1500	1500	33 59	0 923	3720	51 9
52	1500		46 10	0 805	3735	62 0
53	1500		56 07	0 696	3550	68 6
54	1500		58 24	0 593	3315	65.0
55	1500		61. 25	0 497	3185	59 6
56	1500		64 06	0 419	2835	59 0
57	1490		62 10	0 333	2605	49 5
58	1490		59 00	0 262	2335	41 3
59	1500		56 14	0 191	2140	31 3
60	1500		50 77	0 136	1955	22 0

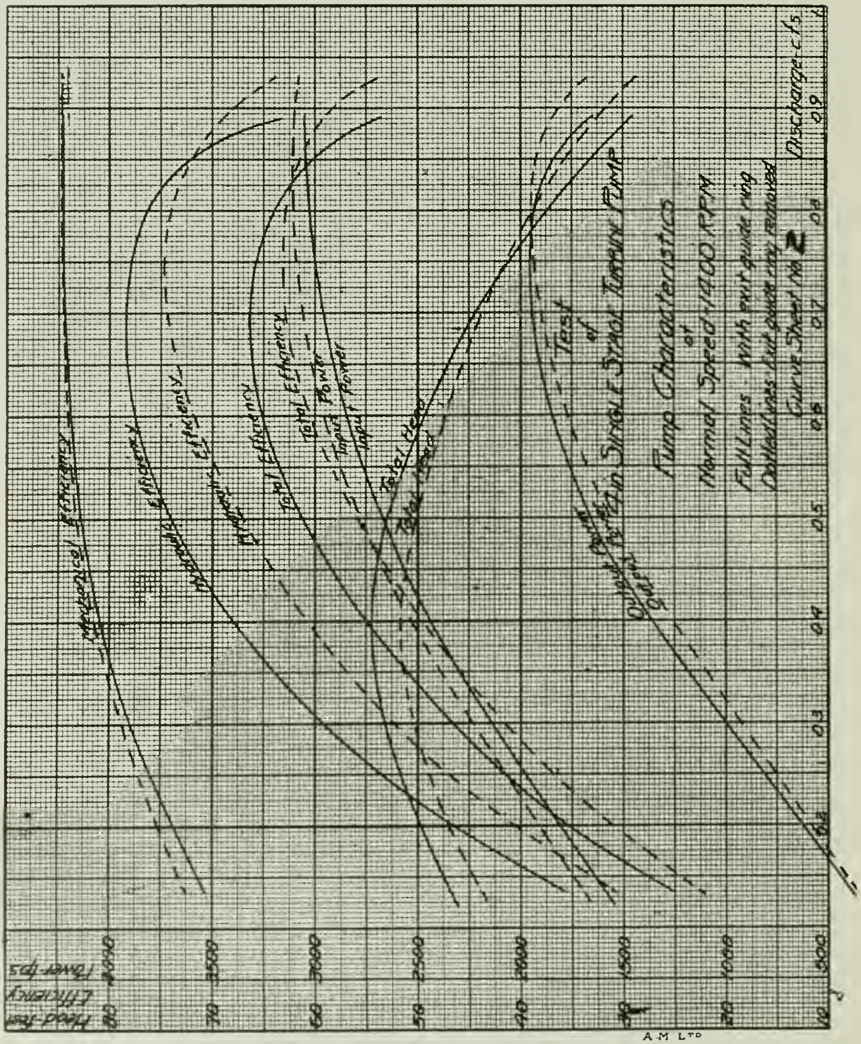
TABLE NO. 3.

DISCUSSION OF RESULTS.

The characteristics of the pump at normal speed are plotted together on curve sheet (2). Examining these, it is seen that the pump comes very close to the designed requirements, namely 0.7 c.f.s. at 45 feet head and 1,400 R.P.H. The head curve is of the usual form exhibited by many turbine pumps of this and larger sizes, having that peculiar rapid falling off of the head at large discharges. For discharges less than the last point plotted, it was found that any further closing of the valve, caused the discharge to drop rapidly, finally ceasing altogether although the valve was still apparently partly open. This may have been due to the type of valve employed which was simply a standard 4 inch gate valve with a certain amount of play in the disk which would render the necessary fine adjustments impossible. It may also have been due to the fact that the head in the upright discharge pipe to the elbow was greater than the head corresponding to this small discharge and the pump discharge thus simply fell off, no water passing along the horizontal portion of the discharge pipe. The conditions changed so rapidly at the small discharges that no results could be obtained.

The input power curve possesses the desirable feature of flattening off for large outputs reducing the liability of damage to the motor should the pump be overloaded.

The curve of efficiency is of good form, giving as was to be expected, maximum efficiency at normal discharge and being comparatively flat at this point so that for slight increase or decrease of discharge the change in efficiency is slight.



CURVE SHEET No. 2.

EXCEPTIONALLY HIGH EFFICIENCIES.

The efficiencies obtained are quite high for a pump of this size acting under so low a head. As will be seen from the tables and curves efficiencies slightly over 65% were obtained. Although customary in large multistage pumps to obtain efficiencies in the neighbourhood of 80%, efficiencies of 60% or over may be considered very good in pumps of this size.

In small single stage pumps the bearing and gland friction is relatively much larger than in larger multistage pumps and uses a much larger part, comparatively, of the input power reducing the mechanical and therefore the net efficiency. Thus if the shaft of a large pump is twice the diameter of that of a small pump, while the power is eight times as great the friction loss is only increased to double that of the small pump. The power consumed in overcoming friction (running pump light) is in the pump as in other machines a certain fraction of the total input, and this fraction decreases as the pump increases in size so that evidently higher mechanical efficiencies should be expected in pumps of large size, than in small ones. Such efficiencies as have been obtained from this pump are the result of very careful design, and accurate workmanship which in turbine pump work are absolute necessities when high efficiencies are sought. In addition as previously mentioned the impeller and guide ring, guides, passages and surfaces were all finely finished and polished in this pump, the construction being so modified as to permit of this being done.

Effect of Exit Guide Ring.

The results obtained from the test with the exit guide ring removed (blank ring substituted) are tabulated in Table 4 and characteristics are plotted on the same curve sheet 2 as those with guide ring, for purposes of comparison. During this test it was found that with the valve wide open the discharge as shown by the Venturi and weir fluctuated considerably and that there was considerable vibration and noise, indicating at once the presence of considerable hydraulic shock losses, whereas with the guide ring in, the discharge was steady and operation

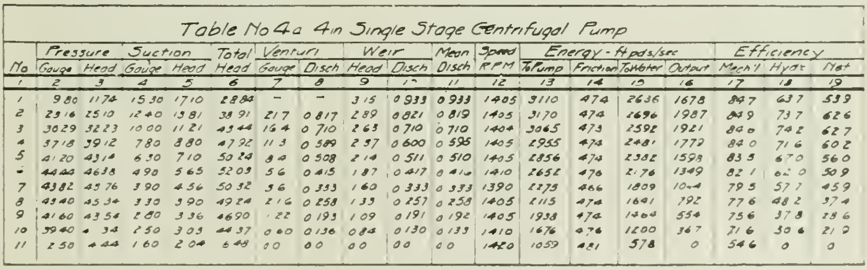


Table No 4a 4in Single Stage Centrifugal Pump

	Pressure		Suction		Total	Venturi		Weir		Mean	Speed	Energy - Ft pds/sec				Efficiency		
No	Gauge	Head	Gauge	Head	Head	Gauge	Disch	Head	Disch	Disch	RPM	To Pump	Friction	To Water	Output	Mech'l	Hydr	Net
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	9 80	11 74	15 30	17 10	28 84	–	–	3 15	0 933	0 933	1405	3110	474	2636	1678	84 7	63 7	53 9
2	23 16	25 10	12 40	13 81	38 91	21 7	0 817	2 89	0 821	0 819	1405	3170	474	2696	1987	84 9	73 7	62 6
3	30 29	32 23	10 00	11 21	43 44	16 4	0 710	2 63	0 710	0 710	1404	3065	473	2592	1921	84 0	74 2	62 7
4	37 18	39 12	7 80	8 80	47 92	11 3	0 589	2 37	0 600	0 595	1405	2955	474	2481	1773	84 0	71 6	60 2
5	41 20	43 14	6 30	7 10	50 24	8 4	0 508	2 14	0 511	0 510	1405	2856	474	2382	1598	83 5	67 0	56 0
6	44 44	46 38	4 90	5 65	52 03	5 6	0 415	1 87	0 417	0 416	1410	2652	476	2176	1349	82 1	62 0	50 9
7	43 82	45 76	3 90	4 56	50 32	3 6	0 333	1 60	0 333	0 333	1390	2275	466	1809	1044	79 5	57 7	45 9
8	43 40	45 34	3 30	3 90	49 24	2 16	0 258	1 33	0 257	0 258	1405	2115	474	1641	792	77 6	48 2	37 4
9	41 60	43 54	2 80	3 36	46 90	1 22	0 193	1 09	0 191	0 192	1405	1938	474	1464	554	75 6	37 8	28 6
10	39 40	4 34	2 50	3 03	44 37	0 60	0 136	0 84	0 130	0 133	1410	1676	476	1200	367	71 6	30 6	21 9
11	2 50	4 44	1 60	2 04	6 48	0 0	0 0	0 0	0 0	0 0	1420	1059	481	578	0	54 6	0	0

Table No. 4a.

poses of comparison. During this test it was found that with the valve wide open the discharge as shown by the Venturi and weir fluctuated considerably and that there was considerable vibration and noise, indicating at once the presence of considerable hydraulic shock losses, whereas with the guide ring in, the discharge was steady and operation vibrationless and with no rumbling. So that even before making any calculations the guide ring appeared to possess advantages.

A comparison of the characteristics with and without the guide ring shows very effectively the advantages of the exit guide ring in centrifugal pump construction.

With the guide ring in, it is seen that at discharges below about 0.8 c.f.s. there is an increase of head, decrease of power and consequent increase of efficiency compared with similar quantities without the guide ring, while above this discharge the effects are the reverse.

The addition of the guide affects only slightly the mechanical operation of the pump as is shown by the two mechanical efficiency curves which practically coincide. The effect of the guide ring has been to flatten out all the curves, the net effect, however, at the point of normal operation of the pump being comparatively small. It will be noted that the majority of the curves for the pump with the guide ring intersect those for pump without the ring, in the neighbourhood of 0.8 c.f.s. discharge. This characteristic has been noted in pumps of much larger size, a change in the guide ring causing an intersection of the two sets of curves. This bears out the claim that it is possible to design two pumps, one with and one without a guide ring to fulfil certain conditions at some certain discharge but that a change from this discharge will result in one or the other exhibiting bad characteristics.

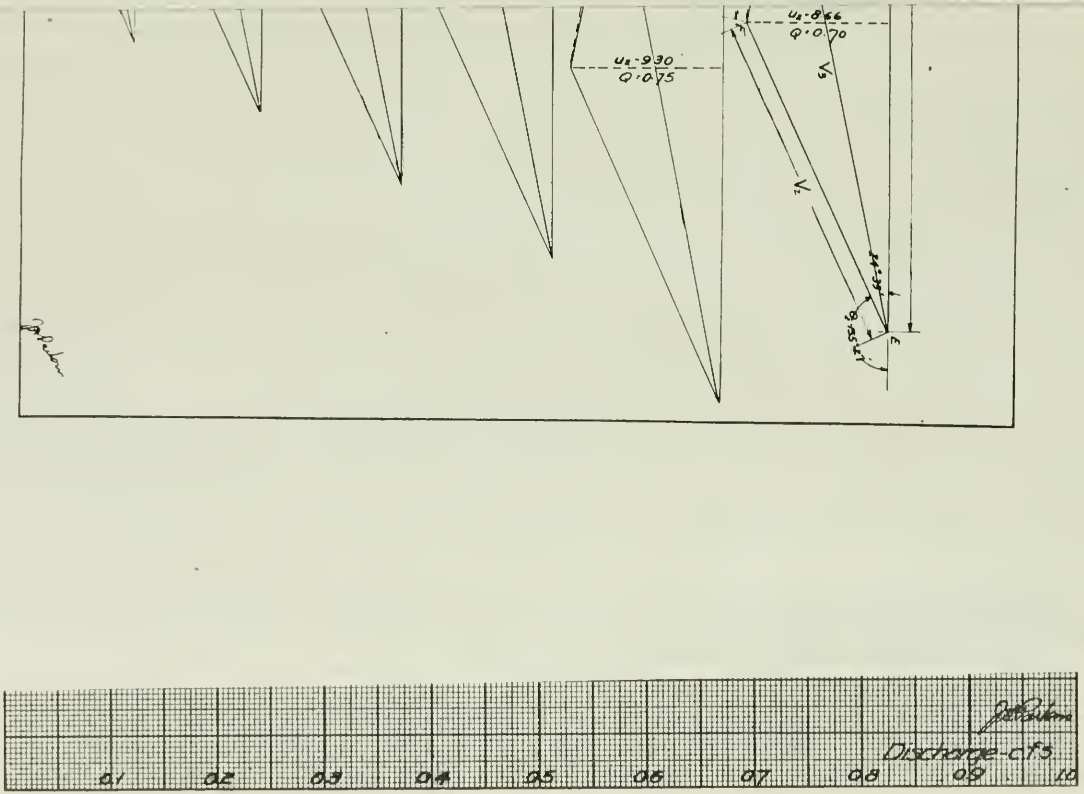
EFFECT OF SPEED VARIATION ON THE CHARACTERISTICS.

The effect of speed variation on the characteristics is clearly shown on curve sheets (3, 4 (*a*) and (*b*) and 5). These show the effect so well that comment is hardly necessary. When the speed is increased the head and power are correspondingly increased for the same discharge. The effect on the efficiencies is not so marked, the total variation in efficiency being small and the efficiency curves for the various speeds being almost coincident. However, it may be noted that the maximum efficiency obtainable increases with increase of speed the maximum efficiency occurring at a greater discharge as the speed increases. That is the peaks of the efficiency curves show a general trend upward and to the right.

The above is more effectively shown on curve sheet (6). This type of curve is very generally used now in pump, turbine and fan work and is a combination of the three sheets above. The curves show at a glance practically everything of value in connection with the operation of the pump.

LOSS OF HEAD DUE TO FOOT VALVE.

An interesting fact was brought out by the test, illustrated on curve sheet (7), namely the tremendous loss introduced by the foot valve.

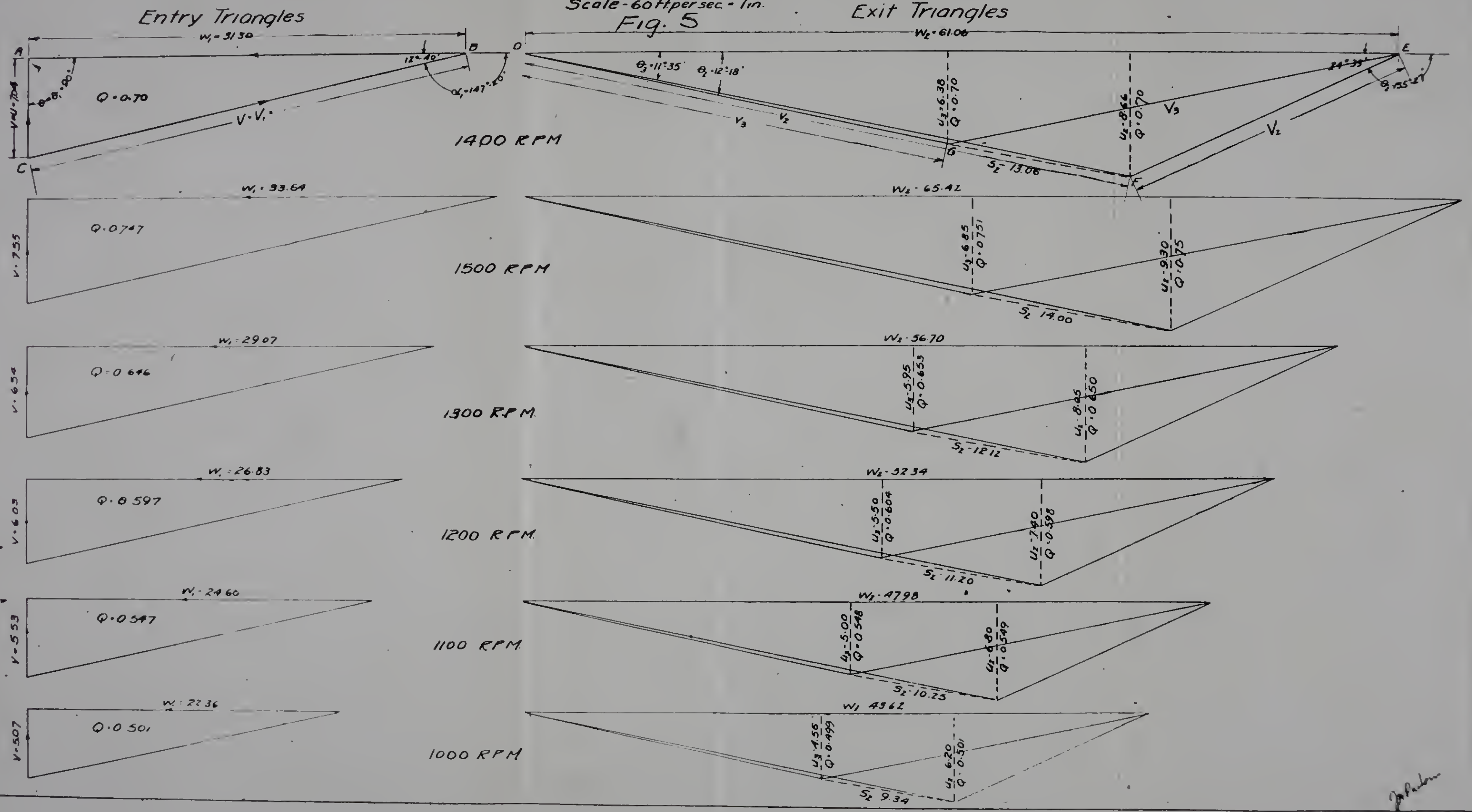

u₂·8.56
Q·0.70
V₃
V₂
24°35'
θ₂ 135°27'
E
F
u₂·9.30
Q·0.75
Discharge-cfs
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

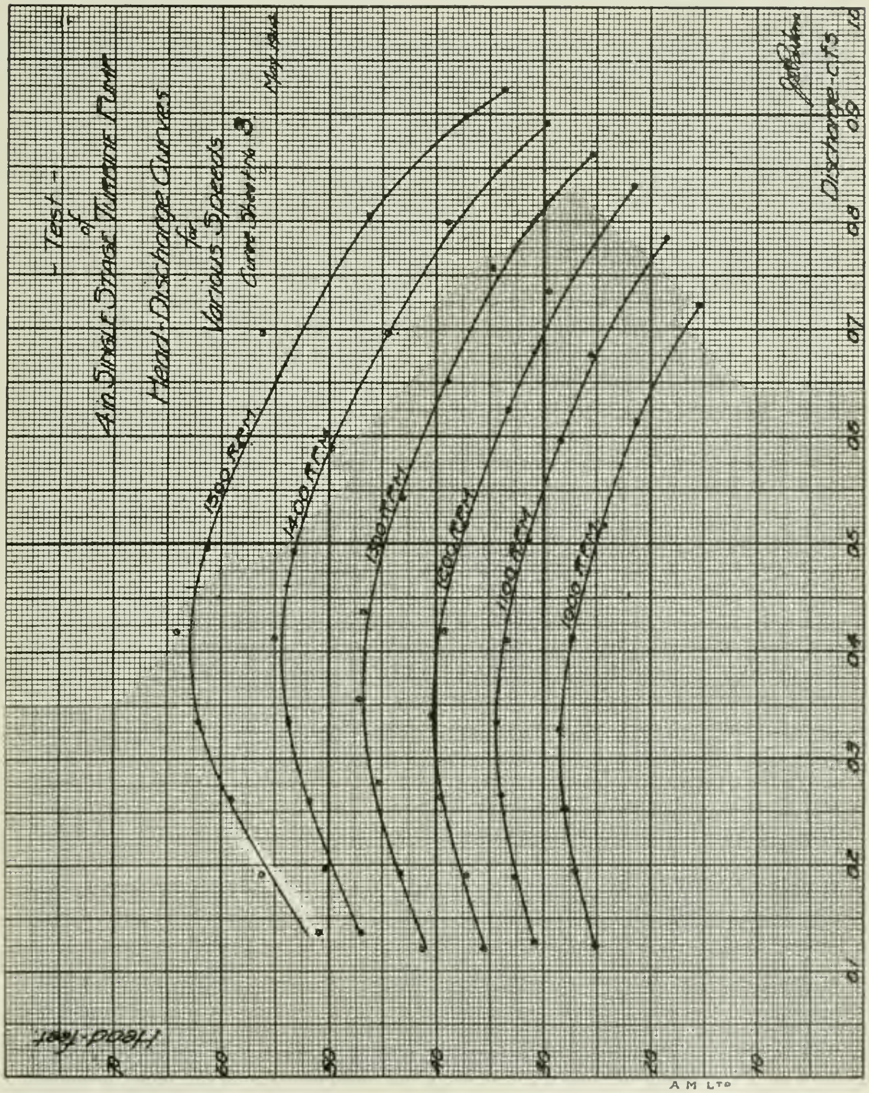
Test of a 4in Single Stage Turbine Pump

Velocity Triangles

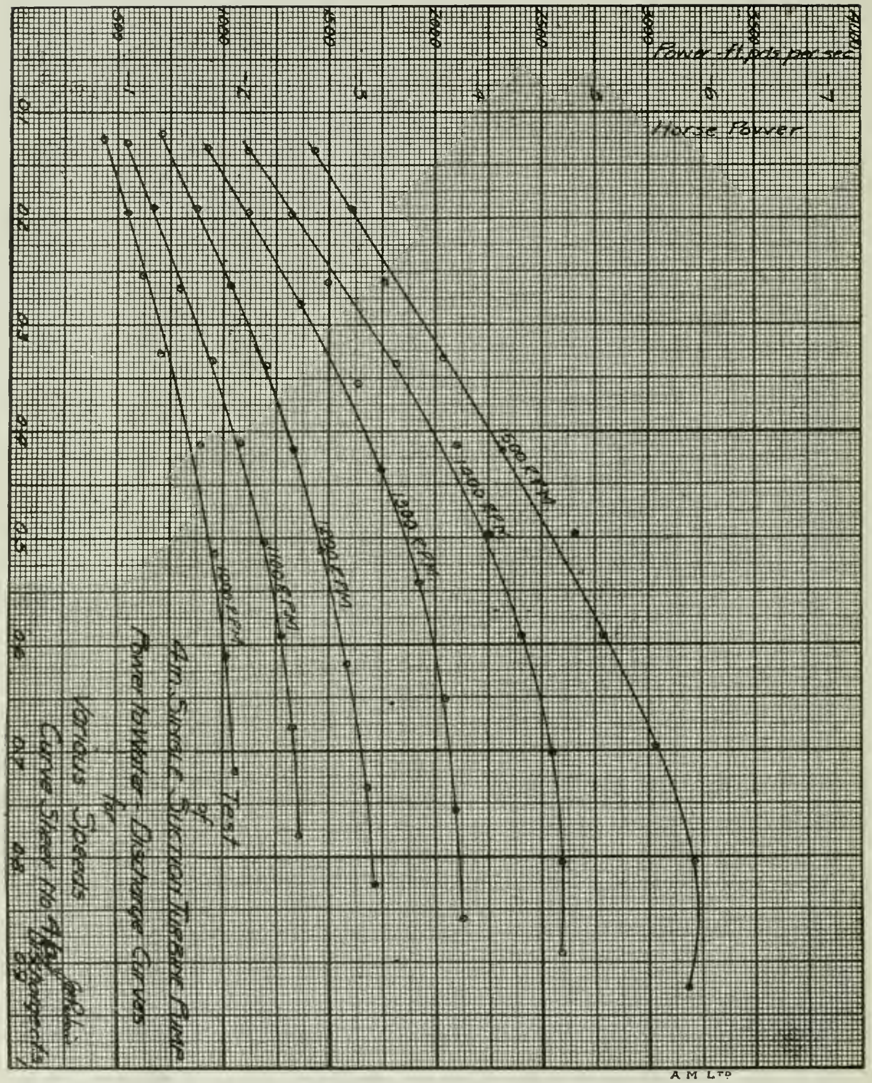
Scale - 60 ft per sec. = 1 in.

Fig. 5

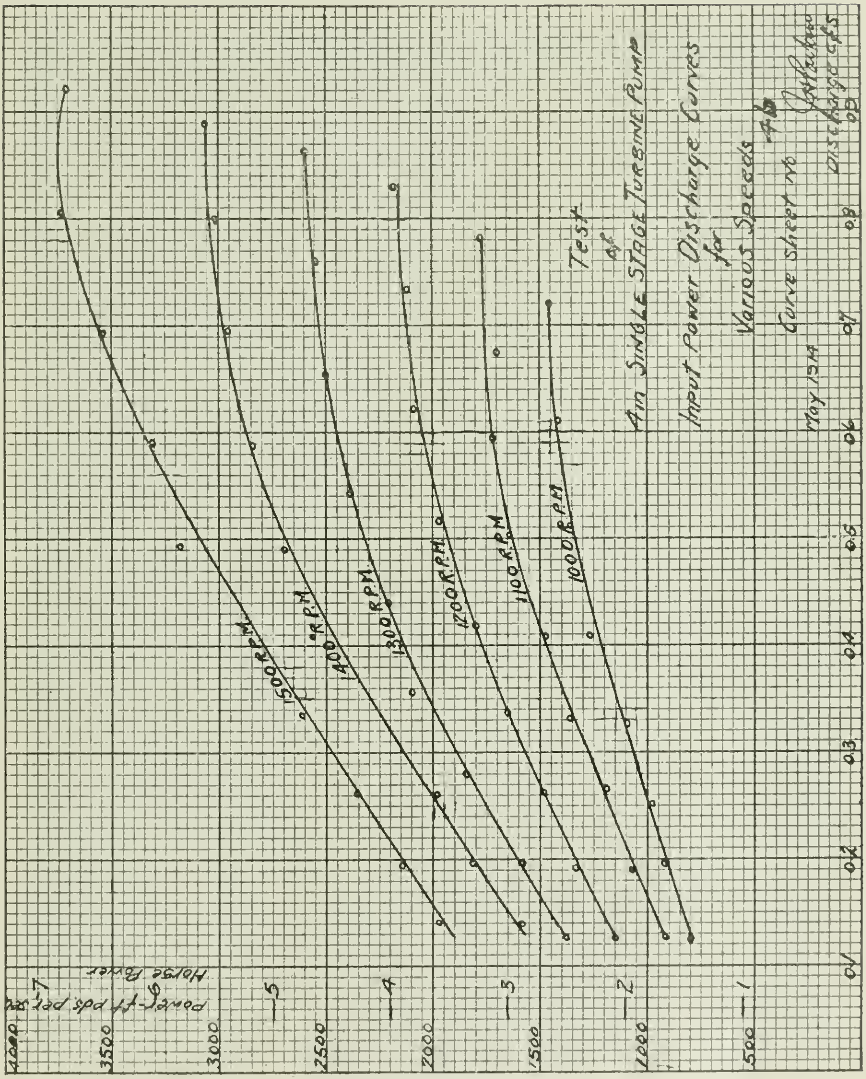




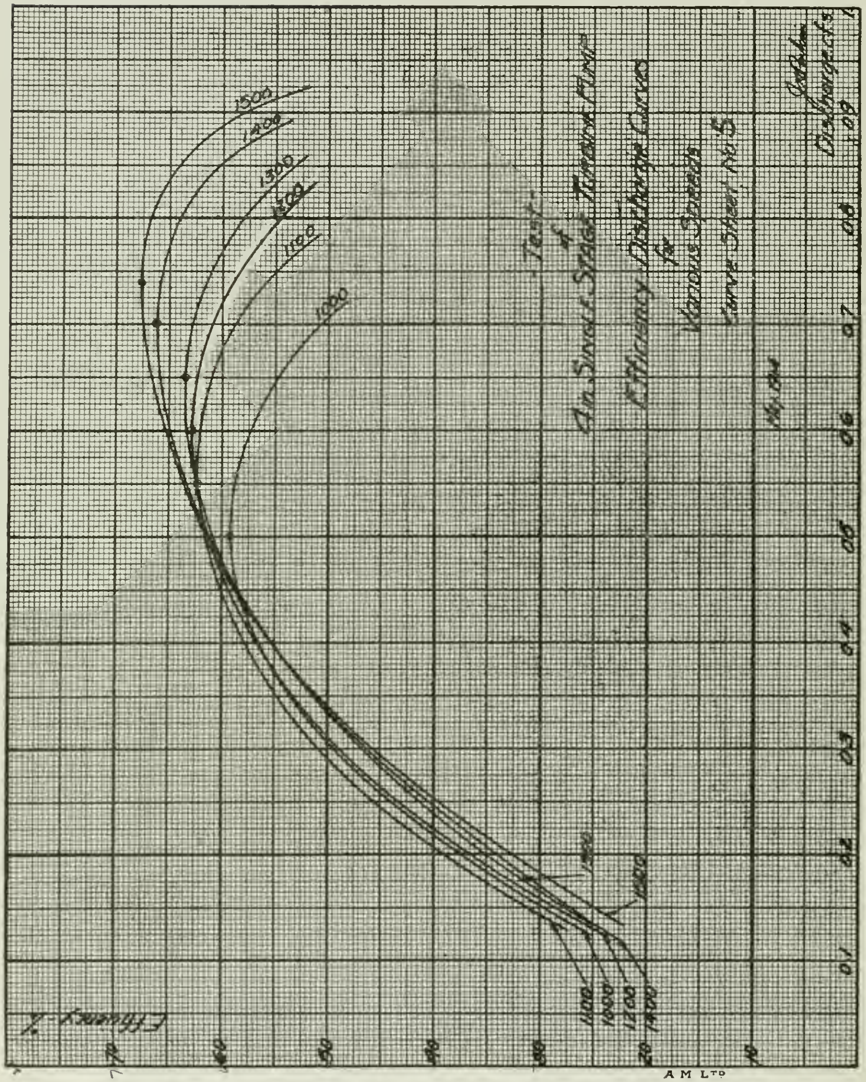
CURVE SHEET NO. 3.



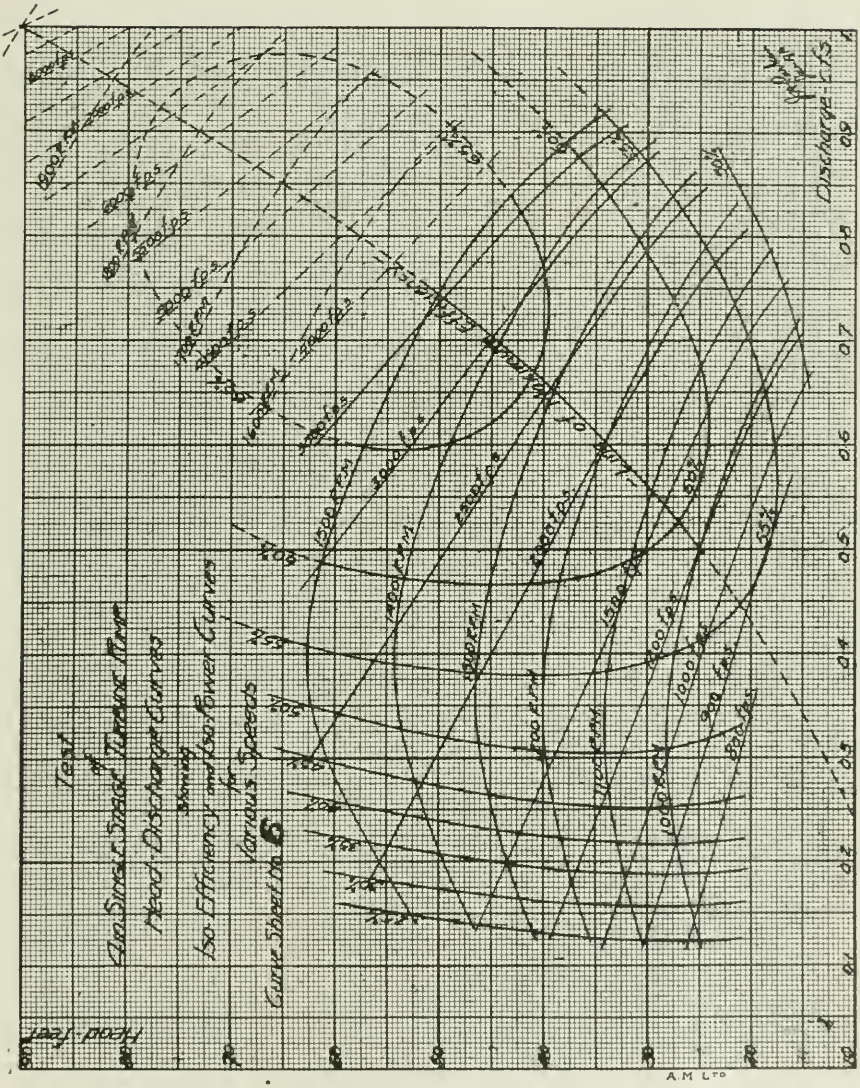
CURVE SHEET NO. 4A.



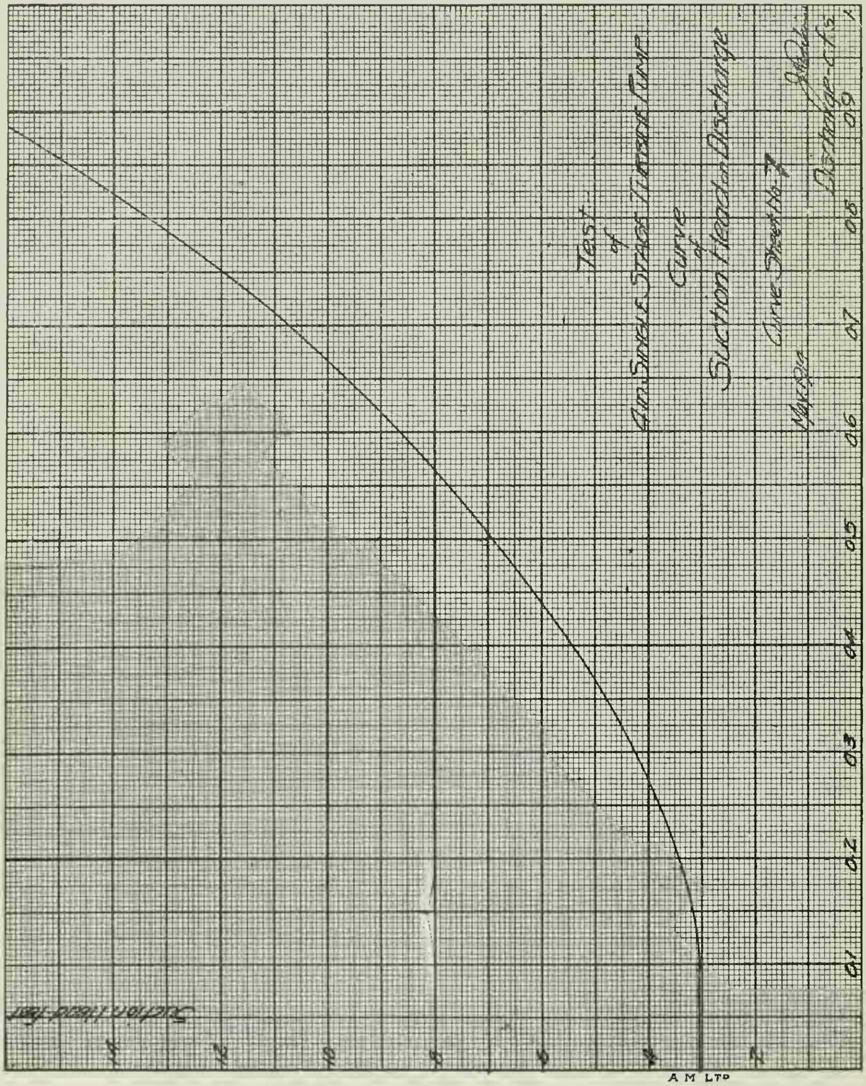
CURVE SHEET NO. 4B.



CURVE SHEET NO. 5.



Curve Sheet No. 6.



CURVE SHEET NO 7

The total suction head under ordinary circumstances, allowing for a reasonable amount of friction would have been about 3.6 feet. Here up to a discharge of 0.1 c.f.s. the suction head is 3.53 feet, but for larger discharges increases rapidly to about 15 feet at 0.9 c.f.s. Part of this increase is of course due to increased pipe friction due to increased velocity of water, but by far the greater portion is directly attributable to the foot valve. The valve employed was of the standard stock, strainer type and was in good condition. Such a large suction head means that the lifting power of the pump is thereby decreased to a great extent.

PART II. THEORETICAL CONSIDERATIONS.

Prediction of Performance at a Different Speed.

In this research, owing to the special arrangements provided, it was possible to run very full tests and obtain the complete characteristics of the pump at various speeds from which the performance at any speed covered by the range of the test could be foretold. In practice this is rarely possible. The pump is designed and supplied to the customer to give a certain discharge at a given head when running at a certain speed and using not over a given amount of power; and is tested at this one speed only, generally at the one discharge or perhaps for a small range of discharges. If it is desired to change the type of prime mover or make some other alteration necessitating changing the speed of the pump, the question arises how may the performance of the pump at the new speed be predicted. Or again if the pump capacity is to be increased by increasing its speed, how may the necessary new speed be determined. This is done in practice by use of the fundamental relations, already herein employed, that the head varies as the square of the speed, the discharge as the square root of the head and the power as the three-halves power of the head. Applying these relations to this test (using results at 1,400 r.p.m. normal speed, as a basis) the results obtained check remarkably well with those actually found by test. It should be here particularly noted that these regulations only hold good for conditions giving the highest hydraulic efficiency at each speed or least hydraulic losses. Since the mechanical efficiency is as already seen from curves, practically constant at the larger discharges, the highest hydraulic efficiency occurs at the same discharge as the highest total efficiency, hence total efficiencies may be used. Also the relations only apply to the *hydraulic* operation of the pump and do not consider mechanical losses so that the power used in these calculations is the "power to the water" as tabulated. See curve sheet 4 (*a*).

Thus assuming the conditions at normal speed 1,400 r.p.m. as a basis for the calculations, namely, discharge 0.7 c.f.s. head 44.3 feet and power to water 2,540 f.p.s. first the head corresponding to any new speed is found from:

$$H' = \left(\frac{N'}{N}\right)^2 H$$

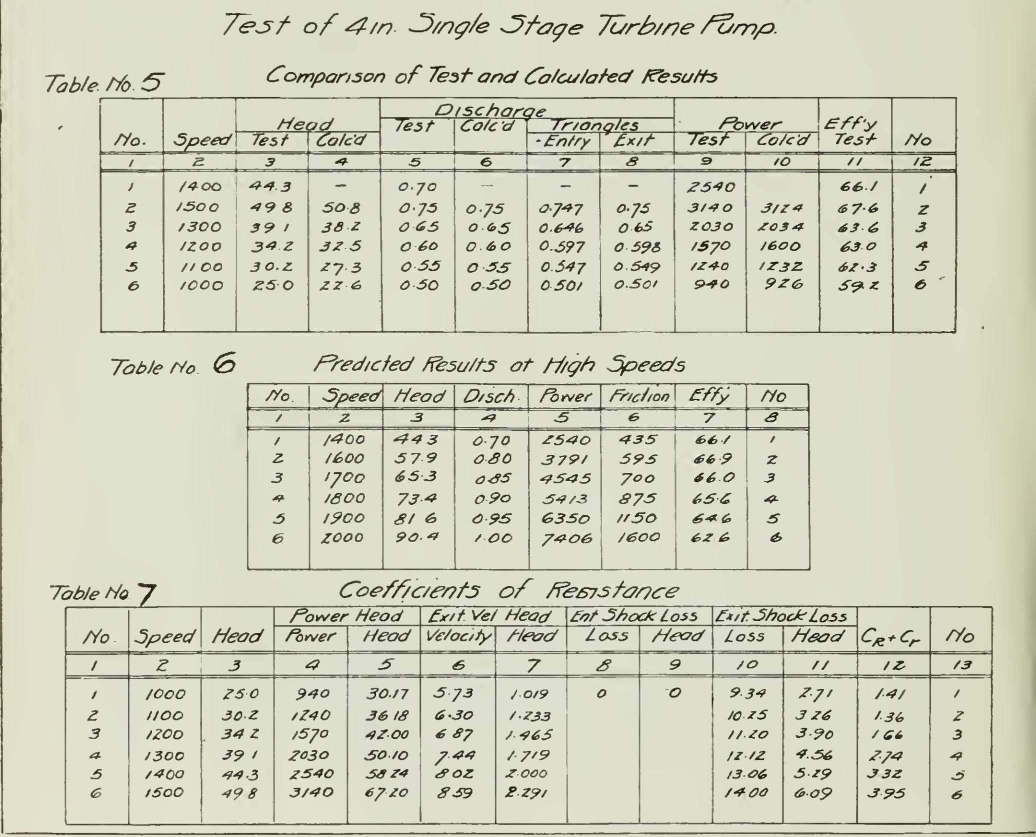
then the discharge from:

$$Q' = \sqrt{\frac{H'}{H}}\, Q$$

and finally the power from:

$$P' = \left(\frac{H'}{H}\right)^{\frac{3}{2}} P.$$

The results of these for the other five speeds at which the pump was run are tabulated in Table 5.



Test of 4 in. Single Stage Turbine Pump.

Table No. 5 Comparison of Test and Calculated Results

No.	Speed	Head		Discharge				Power		Eff'y Test	No
		Test	Calc'd	Test	Calc'd	Triangles · Entry	Triangles Exit	Test	Calc'd		
1	2	3	4	5	6	7	8	9	10	11	12
1	1400	44.3	–	0.70	–	–	–	2540		66.1	1
2	1500	49.8	50.8	0.75	0.75	0.747	0.75	3140	3124	67.6	2
3	1300	39.1	38.2	0.65	0.65	0.646	0.65	2030	2034	63.6	3
4	1200	34.2	32.5	0.60	0.60	0.597	0.598	1570	1600	63.0	4
5	1100	30.2	27.3	0.55	0.55	0.547	0.549	1240	1232	62.3	5
6	1000	25.0	22.6	0.50	0.50	0.501	0.501	940	926	59.2	6

Table No. 6 Predicted Results at High Speeds

No.	Speed	Head	Disch.	Power	Friction	Eff'y	No
1	2	3	4	5	6	7	8
1	1400	44.3	0.70	2540	435	66.1	1
2	1600	57.9	0.80	3791	595	66.9	2
3	1700	65.3	0.85	4545	700	66.0	3
4	1800	73.4	0.90	5413	875	65.6	4
5	1900	81.6	0.95	6350	1150	64.6	5
6	2000	90.4	1.00	7406	1600	62.6	6

Table No. 7 Coefficients of Resistance

No.	Speed	Head	Power Head		Exit Vel Head		Ent Shock Loss		Exit Shock Loss		$C_R + C_r$	No
			Power	Head	Velocity	Head	Loss	Head	Loss	Head		
1	2	3	4	5	6	7	8	9	10	11	12	13
1	1000	25.0	940	30.17	5.73	1.019	0	0	9.34	2.71	1.41	1
2	1100	30.2	1240	36.18	6.30	1.233			10.25	3.26	1.36	2
3	1200	34.2	1570	42.00	6.87	1.465			11.20	3.90	1.66	3
4	1300	39.1	2030	50.10	7.44	1.719			12.12	4.56	2.74	4
5	1400	44.3	2540	58.24	8.02	2.000			13.06	5.29	3.32	5
6	1500	49.8	3140	67.20	8.59	2.291			14.00	6.09	3.95	6

TABLES NOS. 5, 6, 7.

COMPARISON WITH TEST RESULTS.

Since these relations only apply to points of maximum (hydraulic or total) efficiency at each speed, which on the efficiency-discharge curves correspond to the peaks of the curves the actual discharge at maximum efficiency at each speed is readily found. From the head-discharge and power-discharge curves at these discharges the corresponding head and power are read. These actual test results are tabulated beside the equivalent results by calculation in Table 5 to facilitate comparison.

Comparing the results it is seen that the agreement is perfect in the case of the discharge and very good in case of head and power, showing very well the close agreement between actual test results and results that may be calculated from a single known set of readings.

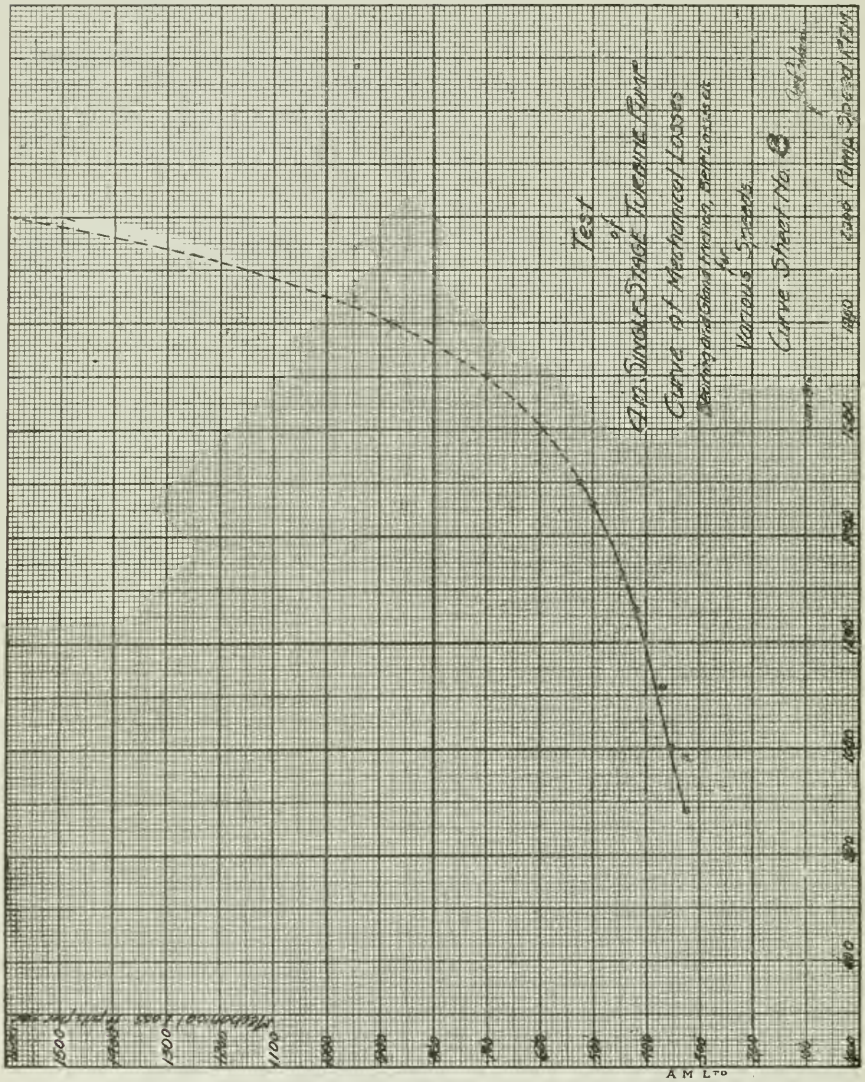
PREDICTION FOR HIGHER SPEEDS.

To further illustrate, should it be desired to determine the operation at speeds above those covered by the test, the determination may be carried out similarly to the above. Using the same figures as a basis the results at speeds from 1,600 to 2,000 r.p.m. are obtained as given in Table 6. To determine the corresponding efficiencies it is necessary to know the input power which is the sum of the "power to the water" and the power lost in mechanical losses. The latter may be estimated by continuing the mechanical loss-speed curve (sheet 8) to these higher speeds. Having the input power the efficiency is readily obtained. The results thus obtained may be used to extend the curves on sheet (6) as shown, from which the performance of the pump at these higher speeds may with a fair degree of accuracy be predicted.

VELOCITY TRIANGLES.

An examination of the velocity triangles of the pump yields some very interesting facts. Enough information is available to plot all the triangles at entry and exit from the runner at normal speed assuming the impeller full of water (full discharge) and radial entry. Thus the circumferential velocities w_1 and w_2 are readily calculated, so also the angles of the runner guides α_1 and α_2 and of the fixed guides θ_3 and θ_4. In addition the radial velocities u_1, u_2 and u_3 may be found from the circumferential water areas c_1, c_2 and c_3 and the discharge.

In calculating the above circumferential areas allowances are made for the areas occupied by the guides at exit from the runner but no allowances are made at entry since here the guides are sharpened. It should be further noted that at entry to the runner the so called circumferential area is not the true circumferential area or the surface of a cylinder of radius r_1 and length 1 but is the area of a frustrum of a cone such that the mean direction of flow of the water into the runner is at right angles to its surface. Thus the direction of flow of water varies from parallel to the shaft to a direction at an angle y to the radial, hence the mean direction of flow and the so-called radial velocity is in a direction at an angle y' to the radial where y' is the average of 90° and y and therefore the so-called radial velocity is $\frac{1}{\cos y'}$ of the true



CURVE SHEET NO. 8.

radial velocity. The corresponding circumferential area is the surface of a frustrum of a cone whose surface is at right angles to the direction of the so-called radial velocity and hence $c'_1 = c_1 \cos y'$.

ENTRY TRIANGLES.

The triangle *ABC* (see Fig. 5) at entry to the runner may therefore be drawn which gives direction of v the absolute entrance velocity at right angles to the circumferential velocity w_1 (angle $\theta = 90°$) or radial entry. Hence if the direction of flow of the water into the runner be considered as radial, there is no shock loss at entry. The assumption of radial entry is however one upon which considerable difference of opinion exists at this time.

EXIT TRIANGLES.

The velocity triangle *DEF* at exit from runner and the triangle *DEG* at entry to the guide ring may also be plotted as shown. Evidently there is some shock loss here since there is a change in direction of absolute velocity from v_2 to v_3 ($\theta_2 = 11° \; 35'$ and $\theta_3 = 12° - 18'$) and also in direction of relative velocity from v_2 to v_3. This loss is due to two causes, the first and greater being the sudden increase in the water area causing a decrease in radial velocity from $u_2 = 8.663$ to $u_3 = 6.381$ feet per second and the second being the slightly incorrect entrance angle of the distributor guide. This shock loss s_2 is represented by *GF* on the diagram. To eliminate s_2 the blade angle must be altered from $12° - 18'$ to $11° - 35$ and the areas made equal or that is the width of the water passage (see Fig. 2) in the impeller made equal to that in the distributor and the runner blades sharpened at exit as well as entry. On the other hand, if the two widths be made equal any slight axial movement of the impeller will greatly reduce the discharge and as there is always this possibility present in pumps, the width of the guide ring at entry is generally made larger than the other and some shock loss thereby necessitated, as in this pump.

TRIANGLES AT SPEEDS OTHER THAN NORMAL.

If similar triangles be drawn for the other test speeds they will be as shown in Fig. 5. The angles α_1, α_2 and θ_3 are in these triangles fixed by the blade angles while the magnitude of the remaining angle in each triangle is fixed by the radial velocity. The radial velocity depends on the discharge which varies as the root of the head and the latter in turn varies as the square of the speed; so that the radial velocity varies with the speed and the remaining angles are thus independent of the speed. These triangles are drawn then on this basis that the angles in all cases are equal and hence that the directions of the relative and absolute velocities do not change.

The entry triangles are thus a series of similar right angled triangles which give the magnitude of the absolute entry velocity in each case or what is the same thing, the radial velocity as indicated. These radial velocities multiplied by the circumferential area give the discharges, which agree exactly with those already determined by test and calculation (see Table 5).

The triangles at exit are also a series of similar triangles and give the radial velocities u_2 and u_3 as shown corresponding to discharges as indicated which check with the corresponding discharges given by the entry triangles by test and by calculation (see Table 5).

SHOCK LOSSES.

It will be noted that working on this basis the shock loss at exit from the impeller is represented by a series of parallel vectors increasing in magnitude as the speed increases. These magnitudes are indicated on the triangles. It will be evident both from geometrical reasoning as well as from the actual figures scaled from the triangles that the shock loss varies directly as the speed.

The equation showing the distribution of the input power to a pump is as follows:

$$h_p = H + h_{v_4}(1 + C_R + C_r) + h_{s_1} + h_{s_2}$$

where $h_p = \frac{P}{w}$ the power per pound of discharge (power head)

P = power input

w = weight of discharge per second

H = total pumping head

h_{v_4} = velocity head in discharge pipe

C_R = coefficient of resistance of fixed elements of pump, suction pipe, fixed guides, scroll and discharge pipe, referred to discharge velocity

C_r = coefficient of resistance of moving element or impeller

h_{s_1} – entrance shock loss head

h_{s_2} – exit shock loss head

From the results of the test and the subsequent analysis the numerical values of all these quantities are known or readily calculable except C_R and C_r and hence the sum of these latter two may be found. Table 7 shows the quantities necessary in the determination together with the values of $(C_R + C_r)$ as found. There would be no particular object in finding the individual values C_R and C_r since if it were desired to use these results to find the losses in a similar pump the sum $C_R + C_r$ would in any case be used and not the separate values. Evidently the coefficients vary with the speed, increasing with it and therefore with the discharge. This is contrary to all facts and theory which show that the coefficient of resistance increases with decrease in discharge. This curious result here obtained may be due first to the fact that the coefficients are determined as a difference thus

$$1+C_R+C_r=\frac{h_p+H-h_{s_2}-h_{s_1}}{h_{v_4}}$$

which tends to magnify any errors occurring in the determination of the individual factors, and secondly may be due to the comparative numerical smallness of the coefficients compared to the figures from which they are obtained.

If these coefficients be calculated by determining from tables friction factors for the conditions of flow in suction pipe, impeller, distributor, volute and discharge pipe the values of the sum C_R+C_r for a discharge of 0.7 c.f.s. is found as 3.26 and for 0.5 c.f.s. as 3.50 which shows a comparatively small decrease with the increase in discharge from 0.5 to 0.7 c.f.s. or that is speed. The value obtained for 0.7 c.f.s. corresponds very closely to that obtained from the test results. Apparently a fair average value of C_R+C_r for this pump would be 3.25.

THE STRENGTH OF CAST IRON IN BENDING AS AFFECTED BY VARIATIONS IN CROSS SECTION.

BY J. H. BILLINGS, B.A.Sc; S.M.; ASSOC. MEM. AM. SOC. M.E.

INTRODUCTION.

The progress of mechanical construction in the last decade has been marked by two outstanding tendencies, one towards increased speeds of moving parts, the other towards decreased weights. In the machine shop the general introduction of high speed tool steel and the spread of the gospel of efficiency, and, more recently, the pressure of war orders have ushered in an increased speed era. In the industries the development and wide introduction of alloy steels and more careful observance of the laws of balancing have made possible a large increase in engine speeds and a corresponding decrease in weights. From the stand point of design all this means greatly reduced factors of safety and the apex of such development is to be found in the aeroplane engine where the safety factors are shaved until the distribution of every ounce of material becomes of the utmost importance.

To permit of such a reduction of safety factors, that is of the working of materials close up to their elastic limits, two conditions are essential. The physical properties of the materials used must be accurately known, and reliably uniform. If these conditions are not realized allowance must be made in the factor of safety—more suitably called, in this case, "factor of ignorance". Any research then, I take it, which gives promise of extending the knowledge of the physical properties of any of the materials of construction needs no further justification.

THE PROBLEM.

The resistance of a member to an externally applied bending moment is generally expressed by the equation,

$$M_b = \frac{I}{c} f_t$$

where if all units are in lbs. and inches,

M_b is the applied bending moment;

I is the moment of inertia of the section where the bending is maximum;

c is the distance from the neutral axis to the outside fibre in tension;

f_t is the unit fibre stress in such outside fibre.

For most materials of construction this formula is near enough to the truth, when the stresses do not exceed the elastic limit, to be used in design without correction, but for cast iron parts this is not the case. For example, a gray cast iron beam of square section which, if tested in pure tension, would give an ultimate tensile strength of 24,000 lbs. per sq. inch, if broken as a beam, and the fibre stress computed according to the above beam formula, will indicate a fibre stress of about 37,000 lbs. per sq. inch. It is evident that the latter value is not a fibre stress at all. The beam formula is more properly written,

$$M_b = \frac{I}{c} f_b$$

calling f_b the "modulus of rupture".

That this modulus of rupture for cast iron was quite different from its fibre stress and that for the same material it varied for different shapes and sizes of section has been commonly known, but whenever I have had occasion to use data of this kind I have been disappointed from the fact that any data available on the subject was not only meagre but unusually conflicting. Hence when the opportunity arose it was decided to attack the problem with the apparatus available at the University.

The object then was to obtain, for those cross-sections in most common use, values of the rupture factor for gray cast iron beams and its relation to the tensile strength. Over 1,400 lbs. of iron comprising some 90 specimens was specially cast for the investigation.

THE SECTIONS USED.

The sections chosen as those for which results would be most useful are shown dimensioned in Fig. 1 and broken specimen are shown in Fig. 2. Nos. 4, 6, 7 and 8 have the same area so that comparison is simplified. All except the "I" sections have the same wall thickness, the object being to eliminate, as a factor in the results, the variations caused in cast iron by the difference in cooling rates between thick and thin sections. The $1\frac{1}{4}$ inch wall thickness used is that of the "Arbitration Bar" of the Am. Soc. for Testing Materials which has come to be adopted as standard on this continent. No. 7 is the uniform "I" section commonly used and No. 8 illustrates the section often seen where an attempt is made to utilize the fact that cast iron is four times as strong in compression as in tension. The thick side should, in all cases, be the tension side.

THE QUALITY OF THE METAL USED.

It was of essential importance that the quality of metal in all specimen should be the same in order that the results should be independent of this variable. To ensure the required uniformity a close supervision of the foundry methods was necessary. A kettle large enough to hold the total weight of iron was secured and a unit charge from the cupola discharged into it. All the moulds were poured using metal from this large kettle.

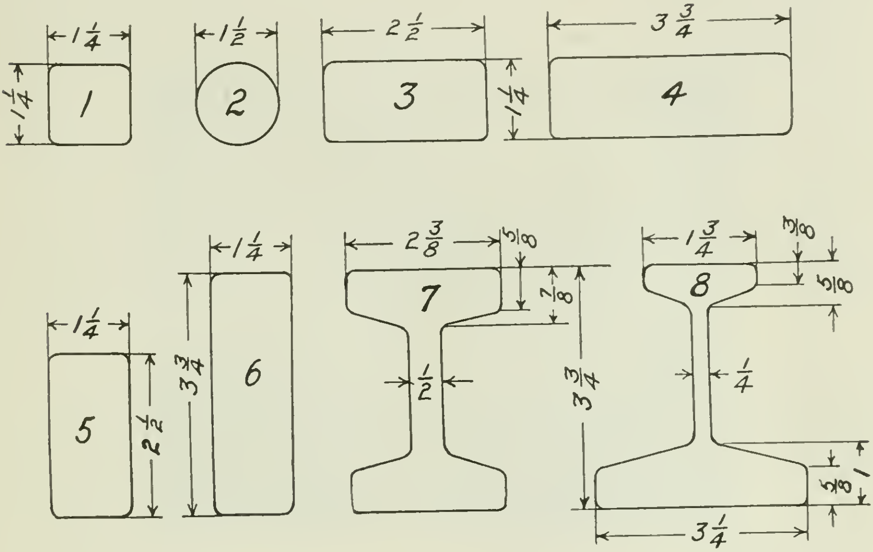


Fig. 1. Cross-Sections Investigated.

The chemical analyses of several samples are given in Table 1 and a satisfactory uniformity is indicated.

METHOD.

The human factor in the art of moulding is still quite a large variable and to obtain castings absolutely free from blowholes or sponginess is not to be expected. In order to eliminate this as a factor in results all fractures were carefully examined and the few specimens which showed faults were discarded.

The tests were made on two machines in the Strength of Materials Laboratory. The small specimen were broken in the 5,000 pound Riehle transverse machine and the larger beams on the 33,000 pound Buckton machine. The deformation was measured for about half the tests and the results used as a check on the strength results. Calibration tests for strength and deformations were initially made on both machines.

The spans used conform to standard testing practice in that no span was used which was less than nine times the depth of the section measured at right angles to the neutral axis.

One half of the rectangular beams were machined all over while all other specimen were tested in the rough. The measurement of the rough specimen was a delicate problem. Experiments were made to arrive at the average depth, on an unmachined casting, of the layer of rough sand and iron particles which adhere to the surface but which can in no way add to the strength. The thickness of this layer was found to be close to 0.005 inch and a correction based on this was made to all rough dimensions. The results for the rough and finished specimen ran so close together that it was found unnecessary to distinguish between them in the result summary.

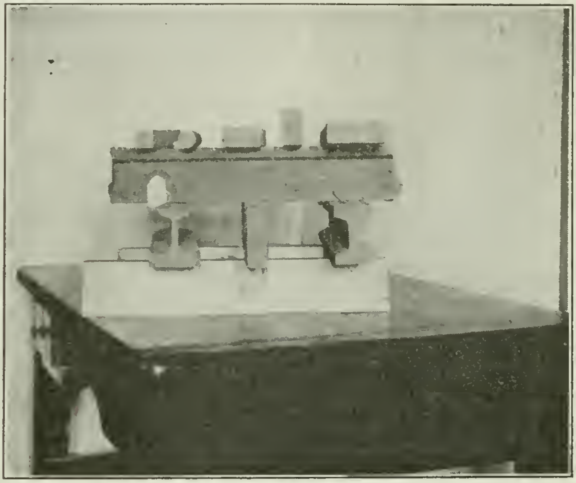


FIG. 2.

THE RUPTURE FACTOR.

A summary of results is shown in Table 2. Under the modulus of rupture for the various sections investigated is found the "rupture factor" which is a term coined to show the relation between the modulus of rupture and the pure tensile strength.

$$\text{Rupture Factor} = \frac{\text{modulus of rupture}}{\text{unit tensile strength}}$$

For a given section the modulus of rupture will vary as different qualities of iron are used while the rupture factor is independent of variations in the tensile strength of the metal and only varies with the section. Hence the rupture factor is the most useful to the designer.

THE ECONOMY FACTOR.

The values given under "economy factor" in Table 1 are got by dividing the maximum bending moment in ft. lbs. by the cross-sectional area in sq. inches. It therefore expresses the relation between carrying capacity and weight of the specimen and is a true measure of the efficiency of the beam.

It will be seen from the table that, although the *I* sections have the lowest rupture factor, they are still the most efficient beams by a large margin. Again, the special *I* has about the same efficiency as the uniform *I* although the latter has 22% larger rupture factor. This might be taken to indicate that there is little to be gained by thickening the compression side of such a section. In the writer's opinion, however, the only safe conclusion is that extremely non-uniform sections are not advisable. It would probably be found that a section somewhere between the two would be most efficient.

POSSIBLE REASONS FOR RESULTS OBTAINED.

The deviation of the rupture factor from unity is the measure of the deviation of the actual strength of a beam from the theoretical strength, as computed from the beam formula. What are the causes of this deviation?

First, the beam formula is based on the assumption that the stress increases uniformly from zero at the neutral axis to a maximum at the outside fibre. We know this is not exactly true for the stress-strain curve for cast iron is never a straight line. It is concave to the axis of strain which means that the fibres nearer the neutral axis take more than their proportional share of the stress. This fact, however, will not account for one tenth of the variation in the case of the round section.

Another factor is the presence of inferior metal at the core of a casting and denser, stronger metal at the surface. This helps very little, however, in explaining the variation between the round and square sections.

The problem is a very interesting one and it is evident both from these results, and from the conflicting opinions to be found in the literature on the subject, that much remains to be found out as to what actually takes place in a beam when it is bent. More research along this line is to be expected.

The following constants are recommended for use in design:

Shape of Section	*Rupture Factor*
Rectangular	1.5
Round	2
Uniform "*I*"	1.2
Non-uniform "*I*"	1

TABLE 1 (Showing Uniformity of Composition).

Sample No.	C	Mn.	S	P	Si.
1	3.41	.49	.090	.685	2.38
2	3.26	.49	.105	.685	2.32
3	3.32	.50	.080	.693	2.32
4	3.35	.48	.125	.697	.36
Average	3.33	.49	.100	.690	2.33

TABLE 2 (Summary of Results).

Name of Section	Square	Round	Narrow Plate	Wide Plate	Medium Rectangle	High Rectangle	Uniform I	Special I
Number in Fig. 1	1	2	3	4	5	6	7	8
Modulus of Rupture	37600	48350	35700	33600	32800	32700	28000	23800
Rupture Factor	1.64	2.10	1.55	1.46	1.43	1.42	1.22	1.03
Economy Factor ft. lbs. per sq. in.	652	629	620	583	1140	1700	2230	2300

Pure unit tensile strength of metal used = 23000 lbs. per sq. inch

SUMMARY.

This investigation deals with the practical deviation in the strength of cast iron beams from the commonly accepted theory of beam resistance. Some of the possible reasons for this deviation are discussed and from the experiments reported, constants are derived for use in the design of beams of various cross-section.

The writer wishes to express his thanks to Professors McGowan and Gillespie for their generous co-operation in having machine work done on specimens and in having testing machines set apart for this investigation, also to Professor Angus for constructive advice.

HEAT TRANSFER TESTS OF BUILDING MATERIALS.

By L. M. ARKLEY, B.A. Sc., M.E.I.C.

In 1912 the following series of tests were started at the request of the Department of the Provincial Secretary of the Ontario Government for the purpose of assisting in the selection of the proper materials to be used in buildings in course of erection at that time by the Department. The general method of making the tests was decided at a conference including Professor Angus of the University of Toronto, Capt. R. J. Durley, Consulting Engineer to the Ontario Government, Mr. James Govan, Architect of the Department of the Provincial Government and the writer.

In 1913 another set of tests was authorized to determine the effect on the transfer of heat through a 12 inch hollow tile wall of laying it up, first with the hollow spaces horizontal, and second with the hollow spaces vertical and directly over each other.

And in 1914 a third set of tests was required to investigate the heat insulating qualities of a number of materials which might be used in the walls of refrigerating rooms.

The apparatus shown in Figs. 1 and 3 was installed in the Thermodynamic Laboratory of the University of Toronto in order to carry out these tests.

OBJECT OF TESTS MADE IN 1912.

To compare the insulating values of walls of approximately equal thickness but built of different materials, viz.:

8″ hollow concrete block
9″ brick wall
$7\frac{3}{4}$″ hollow tile

and further to determine the effect in insulating value of plastering the walls and of stopping air circulation in the hollow walls by filling the spaces with cheap materials, or by using paper on the inside of the walls.

The most common methods that have been employed in making such tests are as follows:

(1) A box is made of the material to be tested, inside this box is placed a vessel containing a weighed quantity of hot water at a known temperature (the water must be agitated in order to get the true average)

by taking the temperature after a certain time the B.T.U. loss may be approximately computed.

(2) The same arrangement as in method No. 1 .is employed but a weighed quantity of ice is used instead of the hot water, and the B.T.U.'s transmitted computed from the weight of ice melted in a certain time.

(3*e* An electric heater made of fine resistance wire wound on an electrically insulated sheet is embedded in the material to be tested and the B.T.U.'s computed from the electrical energy supplied to the heater in a given time.

(4) A box is built of the material to be tested and the heat is supplied to the inside of this box by means of an electric heater. This box is placed in a room which is kept at a low temperature by means of a refrigerating machine. By measuring the current and temperature inside and outside the box the B.T.U. loss may be computed.

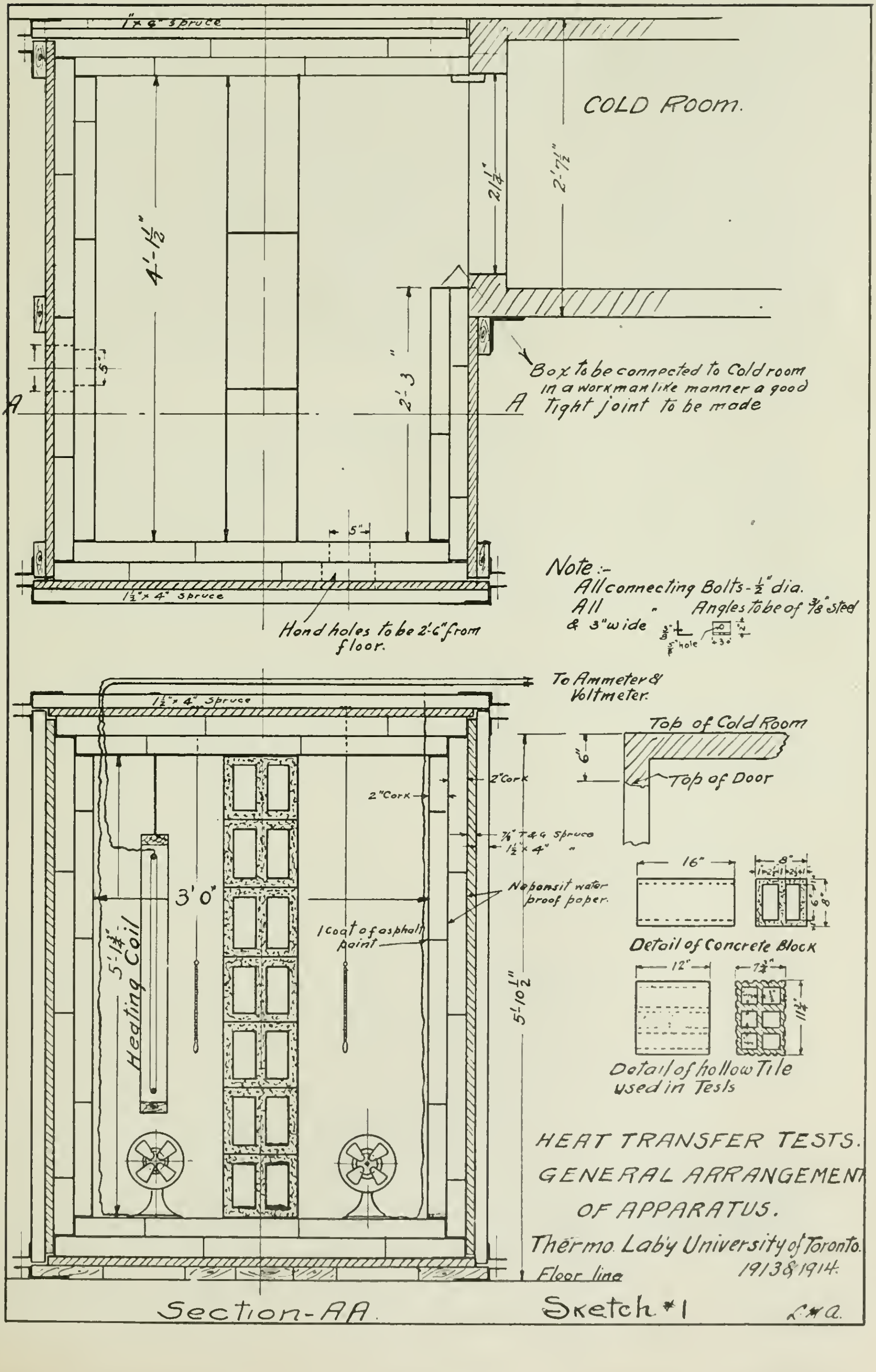
The main objection to the first three methods mentioned is that they do not represent conditions found in practice and from the nature of methods No. 1 and No. 2 it is almost impossible to get accurate results.

In most cases in practice either in dealing with heating problems or problems in refrigeration the medium of transfer of heat is air, and air usually in motion, so that in choosing a method for making these tests a modification of method No. 4 was decided upon, but instead of using a box it was decided to use a wall whose exact area could be measured directly. See Sketch No. 1.

Arrangement of Wall and Instruments in Making Tests.

By referring to sketch No. 1 the arrangement of the testing apparatus will be seen. A box was made of 7/8″ T. and G. sheeting supported by 2″×4″ scantling and held together by steel angle braces and bolts. This box was lined with two 2 inch layers of pressed cork held together by wooden pegs, all joints were carefully broken and a sheet of water proof paper was placed between the layers of cork, the inside of the box was finished with a coat of asphalt paint.

Into this box, which is a very good heat insulator, the wall or material to be tested was built in such a manner as to divide the box into two compartments. When the wall to be tested was in place the cover ends and sides of the box were firmly drawn together by means of bolts. One of these compartments communicated directly with the cold room of a three ton refrigerating machine and in this way a temperature of 20° F., or lower if desired, was easily maintained. While on the opposite side of the wall heat was supplied by means of an electric heater made of resistance wire strung on a 3′×3′ frame, a bank of lamps and a rheostat


1"× 4" Spruce
COLD Room.
4'-1½"
2'1½"
2'7½"
2'-3"
5"
5"
A
A
Box to be connected to Cold room in a workman like manner a good tight joint to be made
1½"× 4" Spruce
Hand holes to be 2'-6" from floor.
Note:-
All connecting Bolts-½" dia.
All " Angles to be of ⅜" steel & 3" wide
⅜"
⅝" hole
3"
To Ammeter & Voltmeter.
1½"× 4" Spruce
Top of Cold Room
6"
Top of Door
2" Cork
2" Cork
⅞" T & G Spruce
1½"× 4" "
Neponsit water proof paper.
1 Coat of asphalt paint
3'0"
5'-1¼"
Heating Coil
5'-10½"
16"
8"
6"
8"
Detail of Concrete Block
12"
7¾"
11¼"
Detail of hollow Tile used in Tests
HEAT TRANSFER TESTS.
GENERAL ARRANGEMENT OF APPARATUS.
Thermo. Lab'y University of Toronto.
1913 & 1914.
Floor line
Section-AA.
Sketch #1
C.H.A.

placed in the circuit supplying the heater allowed any desired quantity of heat to be supplied.

To get the average temperature on each side of the wall four thermometers were suspended from the roof of the box, two on the cold side and two on the hot. When readings were required these thermometers were partially withdrawn through small openings in the covers, but the bulb of the thermometer was never exposed to the room temperature. These thermometers indicated the necessity of circulating the air on each side of the wall in order to get a uniform temperature, and two small electric fans were placed one on each side of the wall inside the box to give the required circulation. The electric current being supplied to the heater and fan was measured by means of an accurately calibrated voltmeter and ammeter.

Method of Conducting Test.

The refrigerating machine and fans were operated until the temperatures on each side of the wall remained constant. This indicated that all the heat being put into the high temperature side of the box was being transferred through the wall. After this state had been reached a run of about an hour's duration was made while readings of temperature amperes and volts were taken every ten minutes.

Method of Making Calculations.

As an example we will work out the value found in test No. 1 on log. sheet page 7.

1 watt = .0009477 B.T.U's. per sec.
1 " = 3.41172 " " hr.
Watts = Amperes × volts
" = .647 × 227.7 = 147.32 used by heater
" = .240 × 113.8 = 27.31 used by fan

Total watts used 174.63

$$\text{B.T.U.'s per sq. ft. per hour} = \frac{174.63 \times 3.41172}{19.65 \times 48.1} = 0.63$$

per degree diff. of temperature.

Possible Errors in the Above Method.

(1) Leakage of heat from the high temperature side of box to the surrounding air or *vice-versa*.

From the dimensions of the box it will be seen that the surface exposed to such a loss is about 26 square feet.

A liberal value for the heat loss in B.T.U.'s per square foot per hour per degree difference of temperature for the box is .0875.

Assuming a difference of temperature between the room and high temperature side in box of $1\frac{1}{2}°$ the heat loss per hour would be

$$26 \times .0875 \times 1.5 = \text{B.T.U. per hr.} \times 3.4.$$

LOG OF RESULTS—TESTS MADE IN 1912.

Date	No. of test	Conditions	Average Temperature			Total watts	No. of sq. ft. of wall surface	B.T.U.'s transferred per 1 degree diff. of temp. per hr. per sq. ft.
			Room	High side	Low side			
1912 Dec. 23	1	Hollow concrete block wall. Not plastered. Air spaces empty	F 67.9	F 68.1	F 20°	174.63	19.65	0.630
Dec. 28	2	Hollow concrete block wall. Not plastered. Air space filled with sawdust	70.7	72°	22°	126.21	19.65	0.438
1913 Jan. 8	3	Hollow concrete block wall. Plastered both sides. Air space filled with sawdust	72.43	73.5	22.75	99.83	19.65	0.342
Jan. 9	4	Hollow concrete block wall. Plastered both sides. Air spaces empty	74.5	76°	23.75	152.76	19.65	0.506
Jan. 13	5	Hollow concrete block wall. Plastered both sides. Air spaces filled with gravel	71.75	71.15	21.90	109.52	19.65	0.385
Jan. 18	6	Hollow concrete block wall. Plastered both sides. Air spaces empty. One layer of flooring felt on high temperature side of wall	72.1	73.45	22.75	75.70	19.34	0.263

LOG OF RESULTS—TESTS MADE IN 1912. (*Continued*).

Date	No. of test	Conditions	Average Temperature			Total watts	No. of sq. ft. of wall surface	B.T.U.'s transferred per 1 degree diff. of temp. per hr. per sq. ft.
			Room	High side	Low side			
Jan. 21	7	Hollow concrete block wall. Plastered both sides. Air spaces empty. One layer of tarred bldg. paper on high temp. side of wall	74°	74.75	22.82	75.92	19.34	0.258
Jan. 22	8	Hollow concrete block wall. Plastered both sides. Air spaces empty. One layer of asphalt paper on high temp. side	74	73	23	74.46	19.34	0.263
Feb. 4	9	Brick wall without plaster, 9″	73	72.45	22.5	118.92	20.56	0.392
Feb. 6	10	Brick wall without plaster, 9″	69	72.25	21.6	121.04	20.56	0.397
Feb. 17	11	Hollow tile without plaster. Air spaces empty	68.8	69.16	20.33	119.43	20.05	0.416
Feb. 19	12	Hollow tile without plaster. Air spaces filled with gravel	71.3	71.82	22.75	102.32	20.05	0.355

This value is so small as to be negligible for practical purposes.

In order that there might be little loss from this source the temperature of the room was kept as near 70° F. as possible. In most cases the difference between the room temperature and that inside the box on the high temperature side of wall was much less than 1½°. The range of temperature decided on for the tests was from 70° F. on hot side to 20° F. on cold side giving a temperature difference of 50° F.

(2) Possible error due to length of run.

The accuracy of the method employed in making these tests depends on the length of run being sufficient to allow the heat to permeate the wall and the heat gradient to be established This is indicated by the temperature on both sides of the wall becoming constant; until this point is reached it would not be correct to assume that all heat being supplied was passing through the wall.

In testing the hollow walls with spaces vertical a good check on this point was obtained by inserting a thermometer through the box cover into the centre of the wall. When the thermometers on the high and low temperature sides of the wall showed a stationary condition the thermometer in the centre indicated very nearly the mean of the other two readings.

As stated in the beginning of this report, three series of tests were made, in 1912-13 and 1914 and the report is therefore divided into three parts. Descriptions of the walls tested. Specifications for plastering and log sheets of tests follow each part.

TESTS MADE IN 1912.

Description of materials used in walls tested.

CONCRETE BLOCKS.

For dimensions see sketch No. 1

Composition. One bag cement to four cubic feet of sand and one cubic ft. of $\frac{3}{4}$ inch crushed stone with $\frac{1}{4}$ inch and all below screened out.

Age—about two months. Weight $38\frac{1}{2}$ lbs. Weight of water absorbed per block after being submerged 24 hours = $3\frac{3}{4}$ lbs.

TILE.

For dimensions see sketch No. 1.

Dry weight of tile $30\frac{3}{4}$ lbs. Weight of water absorbed per block after being submerged for 24 hours $3\frac{1}{2}$ lbs.

BRICK.

Hard burned common brick, dimensions $2\frac{1}{2}'' \times 4\frac{1}{4}'' \times 8\frac{1}{2}''$. Weight dry = 4.67 lbs. Weight of water absorbed per brick after being submerged 24 hours, 1.125 lbs.

Flooring felt used in test No. 6, actual weight per 100 square feet = 11.5 lbs.

Tarred felt used in test No. 7 known as 16 oz. felt, actual weight per 100 square feet = 25 lbs.

Asphalt felt used in test No. 8 known as Neponsit, Asphalt felt.

Specification of the construction and finish of walls to be built at Thermodynamics Building, University of Toronto, for thermal-conductivity tests,

BY JAMES GOVAN, ARCHITECT.

TEST NO. 1—TILE WALL.

TILES TO BE USED.

Wall to be built of 8″×8″×16″ concrete hollow wall tiles.

MORTAR.

Mortar for building tile to be composed one part lime to three parts of good clean sharp sand.

JOINTS.

Joints to be $\frac{3}{8}$″ wide.

Tiles to be wetted before laying to prevent absorption of water from mortar.

FURRING STRIPS.

Provide and build into side of wall away from refrigerator at every second joint, wood furring strips $\frac{3}{8}$″×1$\frac{1}{4}$″ for wood strapping.

STRAPPING.

Strap this side of wall with wood straps at 12″ centres.

METAL LATH.

Metal lath for plastering to be expanded metal No. 24 gauge.

PLASTERING.

Plastering on this side of wall to be three coat work composed as follows:

1st Coat—One part alca and lime mixture as supplied.
Three parts sand.
Fibering material or hair as required.
This coat to be thoroughly scratched.
2nd Coat—One part alca and lime mixture.
Four parts sand.
Fibering material as required.
Finishing Coat—One part alca and lime mixture.
One part fine white-sand, trowelled to a smooth polished surface.

STUCCO ON SIDE OF WALL NEXT REFRIGERATOR.

Base coat shall be applied direct to the tile wall and shall consist of one part alca and plaster mixture to four parts sand. Surface of tile wall must be thoroughly wetted before this base coat is applied and the mortar scratched in preparation for the second coat.

Second coat to be composed of one part alca and plaster mixture to four and one-half parts sand, trowelled smooth.

This second coat must be applied before the first coat is allowed to dry out.

TEST NO. 2—BRICK WALL.

BRICKS.

Bricks to be good, common stock bricks, approved of by the architect.

MORTAR.

Mortar for building bricks to be composed of one part lime to three parts clean sharp sand.

Joints to be $\frac{3}{8}$ in. thick and struck on both sides of wall. Bricks to be thoroughly wetted before laying.

NO PLASTERING INSIDE OR OUTSIDE OF BRICK WALL.

NOTE—There will be no plastering on either inside or outside of brick wall for this test.

CONCLUSIONS.

The most important conclusion to be drawn from the above results is that the commonly held assumption that a hollow block wall is more efficient from a heat loss standpoint than a solid brick wall of approximately equal commercial standard thickness is not founded on fact, this will be clearly seen by comparing tests No. 1, 9, 10 and 11. On the other hand tests Nos. 2, 3, 5 and 12 show that if the spaces in the hollow block wall are filled with some material which separates the air into small pockets and prevents circulation that the rate of transfer of heat is considerably diminished.

An interesting result is shown by comparing test No. 4 with No. 7. The conditions in these tests are exactly the same with the exception that one layer of tarred building paper was placed on the high temperature side of the wall in test No. 7. The effect of this layer of paper was to reduce the heat transfer by nearly 50%.

Tests Nos. 9 and 10 serve as a check on the work as many tests have been made on 9 inch brick walls and the commonly accepted value as the coefficient of conductivity is 0.4 B.T.U.

It may be mentioned that in having the brick wall built, the architect took special pains to see that the laying of the bricks was no better than the ordinary standard of bricklayers practice, no attempt was made to have all points in the interior of the wall filled and the face jointing was struck with the trowel as the wall was laid up. If the bricklayer had been allowed to fill carefully every joint with mortar and to have the wall so well flushed as to reduce air circulation through the wall to a minimum, better results would no doubt have been obtained. On the other hand in building the $7\frac{3}{4}$ inch hollow tile wall the practice adopted was decidedly better than would obtain in ordinary contract work.

Where hollow tile are laid with webs vertical it is practically impossible to get the bricklayers to spread mortar on all webs of the tile, as a rule the two outside and the two end webs receive mortar while the interior webs are left uncovered. In the wall tested every web was carefully covered and every precaution taken to keep the air spaces free.

When the wall was built the outside and inside face joints were carefully pointed up so as to reduce air circulation through the joints to a minimum. Under these conditions the results obtained are probably

LOG SHEET OF RESULTS—TESTS MADE IN 1913.

Date	No. of test	Conditions	Average Temperature			Total watts	Sq. ft. of wall surface	B.T.U.'s transfer per degree diff. of temp. per hr. per sq. ft.
			Room	High side	Low side			
1913 Dec. 16	1	12″ tile wall laid with hollow spaces horizontal	70.7	71.75	20.9	91.78	20.91	0.295
Dec. 1 & 17	1 2	12″ tile wall as above but with 1 layer of paper on high temp. side of wall	70.6	69.55	19.00	6.5	20.91	0.210
Dec. 18	3	12″ tile wall as above with paper removed and 1 coat of dehydratine painted on high temp. side of wall	70.00	70.13	19.9	63.75	20.91	0.206
Dec. 25	4	12″ wall as above but plastered both sides, plaster $\frac{5}{8}''$ thick	65.00	67.00	18.25	53.75	20.91	0.179
Feb. 27	5	New 12″ tile wall built with hollow spaces vertical and directly over each other	72.00	70.00	20.4	99.7	21.28	0.322
Mar. 2	6	12″ wall as above with one coat of dehydratine on high temp. side	69.5	69.25	20.45	76.4	21.28	0.250
Mar. 9	7	12″ wall as above but plastered both sides plaster $\frac{5}{8}''$ thick	73.3	70.00	20.8	69.97	21.28	0.228

more favourable to the hollow tile wall than they would be in ordinary construction practice.

Note—Saw dust was not used as filling with the idea that it would be a suitable material for building construction but simply to show the effect of breaking up the circulation of the air in a hollow block wall.

Tests Made in 1913.

Object of Tests.

(*a*) To determine the rate of transfer of heat through a 12 inch hollow tile wall, first when the tile was laid with the hollow spaces horizontal; second when the tile was laid with the hollow spaces vertical and directly over each other.

(*b*) To determine the effect on the rate of transfer of heat of placing a layer of ordinary building paper on the high temperature side of wall.

(*c*) To determine the effect on the rate of transfer of heat of painting the high temperature side of the wall with one coat of "Dehydratine".

Materials Used in Walls.

Tile.

The hollow tile used was a shale tile made at the Ontario Government's Clay Plant at Mimico, Ont., for dimensions, see sketch No. 1.

"Dehydratine".

Described by the manufacturers as a "Foundation compound for waterproofing substructures".

It has about the consistency of the ordinary ready mixed paint, is black in colour and is made up principally of asphalt.

These tests were made in the same manner as those previously described in this report and the specifications for plastering are the same for both series of tests.

Conclusions.

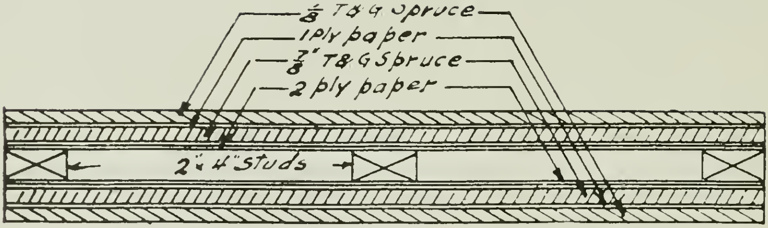
Comparing tests No. 1 and No. 2 the marked improvement in the insulating qualities of a layer of paper is again demonstrated.

From test No. 3 it would appear that one coat of "dehydratine" accomplishes practically the same result as a layer of paper, this is what might be expected as both the paper and "dehydratine" serve to close the pores in the wall.

Comparing test No. 1 with No. 6, we see that the wall is a better heat insulator when the hollow spaces in the tile run horizontally than when they are vertical. In the second case the circulation of air would be much more pronounced due to the chimney effect of the vertical hollow spaces.

Comparing tests Nos. 1 and 3 with Nos. 6 and 7, approximately the same saving is shown due to one coat of "dehydratine".

Comparison of the results obtained with walls plastered and without plaster show that plaster is not a good heat insulator.



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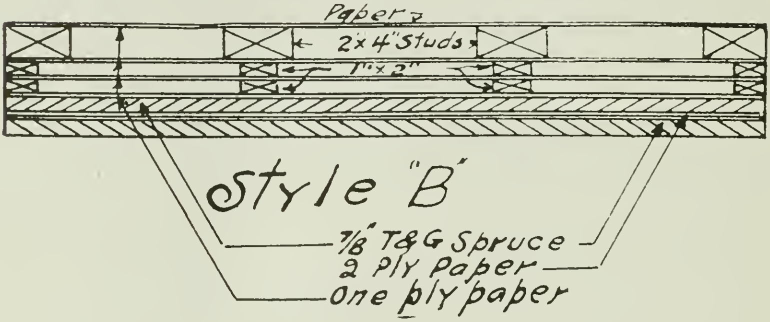

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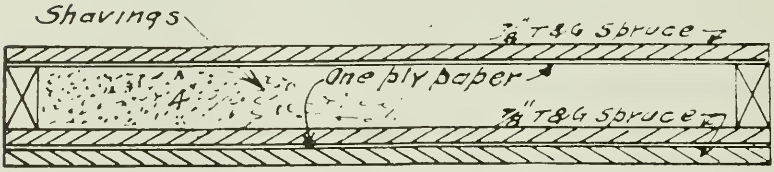
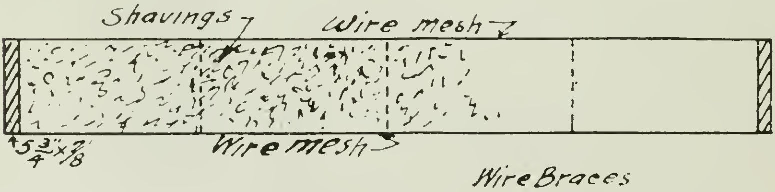


Style "C"



Style "D"

LOG SHEET OF RESULTS OF TESTS MADE IN 1914.

Date	No. of test	Description of material tested	Average temperature			Total watts	No. of sq. ft. of wall surface	B.T.U.'s transferred per sq. ft. per degree diff. of temp. per hr.
			Room	High side	Low side			
1914 Nov. 6	1	1 ply oiled paper	70	71	31.5	320.25	20	1.38
Nov. 9	2	1 ply climasat water proof building paper	68	68	26	297.6	20	1.21
Nov. 12	3	Wall shown on sketch No. 2 marked D	72	69	20	79.3	20	0.276
Nov. 13	4	Wall shown on sketch No. 2 marked A	70	70.25	20.8	41.49	21.335	0.134
Nov. 17	5	Wall shown on sketch No. 2 marked B	69.75	68.7	20.2	63.84	21.335	0.205
Nov. 19	6	Wall shown on sketch Style C, filled with sawdust	69	70	18.75	47.0	21.335	0.147
Nov. 20	7	1 layer of hair felt held together between two sheets of thin paper	72	69.5	26.9	144.17	20	0.578
Nov. 23	8	Style C, hollow space filled with granulated cork	68.0	70.0	18.1	46.94	21.335	0.144
Nov. 23	9	3″ cork wall made up of 2 layers 1½″ thick with 1 layer of paper between the 1½″ layers of cork	72.5	70.3	19	41.47	21.11	0.130
Nov. 23	10	The preceding test repeated	72.0	69.8	18.33	42.0	21.11	0.131

For details of walls marked A, B, C, D see sketch No. 3.

MISCELLANEOUS MATERIALS TESTED SINCE 1914.

Date	No. of test	Description of material tested	Average temperature			Total watts	No. of sq. ft. of wall surface	B.T.U.'s transferred per sq. ft. per degree diff. of temp. per hr.
			Room	High side	Low side			
1917 Nov. 22	1	Corrugated galvanized iron wall supported on 2×4 frame	67	69.6	19.7	368	21	1.2
Dec. 4	2	Wall made up of corrugated galvanized iron on one side of 2×4 frame and $\frac{7}{8}$ T. & G. spruce sheathing on other side	69	68.4	18.1	168.8	21	0.55
1918 Jan. 24	3	Beaver Board 3/16″ thick	65	68.9	22.5	230	20.3	0.83
Feb. 18	4	Limabestos wall board 3/16″ thick, dark red in colour made up of asbestos and concrete	67	71	20.6	389.5	20.3	1.305
Feb. 22	5	Asbestos lumber 3/16″ thick	67	69.7	21	345.5	21.2	1.14
Feb. 28	6	Asbestos corrugated sheathing 2/16″ thick	68	70.3	22.3	340.3	21.2	1.13
Mar. 11	7	Asbestos shingles laid on 1 layer paper and $\frac{7}{8}$ T. & G. sheathing, laid 7″ to the weather, size of shingles 16×16×3/16	67	71.1	18.9	137.75	21.2	0.442

TESTS MADE IN 1914.

OBJECT OF TESTS.

To determine the rate of transfer of heat through walls suitable for refrigerating rooms including built up walls (for details see sketch No. 3), cork walls, and ordinary building papers.

MATERIALS USED.

Paper used in test No. 1—oiled paper—thickness 10/1000 of an inch weight per square foot = $5\frac{1}{4}$ pounds.

Paper used in test No. 2—trade name Clima—sat-building paper, advertised an acid, salt, alkali and waterproof, thickness 18/1000 of an inch, weight per 100 sq. feet = 10.4 lbs.

Test No. 3 was made to test a wall of planer shavings 6 inches thick. A frame 6 inches deep shown on sketch No. 3 as style D was made of $\frac{7}{8}$ inch boards. A wire screen, 12 mesh to the inch was fastened to each side of the frame and shavings packed between the screens giving a wall of shavings 5 inches thick.

For details of walls used in tests No. 4, No. 5 and No. 6, see sketch No. 2.

Cork walls used in tests No. 8, No. 9 and No. 10 were made of slabs of pressed cork $1\frac{1}{2}$ inches thick by 12 inches wide by 3 feet long.

The composition of the asbestos products is approximately 15% asbestos and 85% Portland cement.

We wish to acknowledge assistance given by Professor Angus of the Mechanical Engineering Department, University of Toronto, Mr. James Govan, Architect, Department of the Provincial Secretary of the Ontario Government and the Asbestos Manufacturing Co. of Montreal who furnished the asbestos products tested.

THE FUSION OF SODIUM HYDROXIDE WITH SOME INORGANIC SALTS.

Reprinted from the Journal of the American Chemical Society, Vol. XL.

By Maitland C. Boswell, M.A.; Ph.D. and J. V. Dickson, B.A.Sc.

In the course of an investigation initiated for quite another purpose, it became desirable to determine the nature of the action of fused sodium hydroxide upon sodium arsenite and other salts. The working hypothesis which had been devised by the senior author to assist in the main investigation, indicated the probability that in such fusions the elements of water are active, and that oxidations might be possible whose mechanism would consist of a decomposition of water, the oxygen being taken up by the sodium arsenite or other oxygen acceptor present, and the hydrogen evolved in the free state. Our experiments have confirmed this expectation. Sodium arsenite is oxidized to arsenate with the evolution of an equivalent amount of hydrogen, according to the equation $Na_3AsO_3 + H_2O \longrightarrow Na_3AsO_4 + H_2$. It seemed desirable to expand this investigation to other oxygen acceptors, with the object of determining to what extent this oxidizing action induced by sodium hydroxide is applicable. We have accordingly studied the action in the case of a number of inorganic salts, which might conceivably pass, under such conditions, from a lower to a higher stage of oxidation, and thus act as oxygen acceptors.

A search of the literature showed a few isolated cases recorded of such oxidations. Most of these are qualitative observations that oxidation has occurred, but without any quantitative observation regarding the change. To this class belong the observations of Peligot,[1] that chromous hydroxide mixed with aqueous potassium hydroxide is oxidized with the evolution of hydrogen, of Bloxam,[2] who states in his text book that hypophosphites when boiled with caustic alkalis are converted into phosphates with the evolution of hydrogen, of Baugé[3] that the compound $2Cr_3O_4.3H_2O$ has been obtained by the decomposition of potassium chromous carbonate by boiling water, with the evolution of hydrogen and carbon dioxide. Quantitative observations were made by Muthmann and Nagel[4] on the transformation of molybdenum dichloride into

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[1] *Ann. chim. phys.*, [3] 12, 538 (1844).

[2] 10th Ed., p. 222.

[3] *Compt. rend.*, 127, 551 (1898).

[4] *Ber.*, 31, 2009 (1898).

TABLE I.—Gas Analytical Results.

No.	Compounds	Oxygen. At start Cc.	Oxygen. At end Cc.	Oxygen. Used up Cc.	Hydrogen produced Cc.	Nitrogen present Cc.	Time of heating Mins.	Temperature Degrees
1	0.4000 g. sod. arsenite + 1 g. sodium hydroxide	10.4	7.4	3.0	2.1	39.0	...	Free flame
2	0.4000 g. sod. arsenite + 1 g. sodium hydroxide	10.8	7.6	3.2	24.8	41.1	...	Free flame
3	0.4000 g. sod. arsenite + 1 g. sodium hydroxide	10.7	7.0	3.7	29.9	40.5	...	Free flame
4	0.4000 g. sod. arsenite + 1 g. sodium hydroxide	11.5	4.4	7.1	11.5	43.8	30	325-340
5	1.0 g. sod. sulphite (38% sulphite) + 2 g. sodium hydroxide + 1 drop of water.......	10.1	9.6	0.5	0.4	38.3	...	Free flame
6	1.0 g. sod. sulphite (38% sulphite) + 2 g. sodium hydroxide + 1 drop of water.......	10.4	10.0	0.4	0.3	39.3	10	Free flame
7	0.2000 g. sod. nitrite + 1 g. sodium hydroxide + 1 drop of water......................	9.3	9.3	0	0	35.5	25-30	320-355
8	1 g. ferrous sulphate (crystals) + 2 g. sodium hydroxide............................	8.2	2.9	5.3	20.4	31.3	25	410
9	0.553 g. ferrous sulphate (crystals) + 2 g. sodium hydroxide......................	10.7	7.0	3.7	17.1	40.6	15-20	410
10	0.7 g. cuprous chloride + 2 g. sodium hydroxide + 3 drops of water....................	10	2	8	0	40	40-50	Free flame
11	1 g. nickel sulphate (crystals) + 2 g. sodium hydroxide...........................	10.8	10.8	0	0	41.0	25-30	400-450
12	1 g. manganous sulphate (crystals) + 2 g. sodium hydroxide......................	10	3	7	0	40	30	400-450

TABLE I.—*continued.*

No.	Compounds	Oxygen At start Cc.	Oxygen At end Cc.	Oxygen Used up Cc.	Hydrogen produced Cc.	Nitrogen present Cc.	Time of heating Mins.	Temperature Degrees
13	1 g. cobalt sulphate (crystals)+2 g. sodium hydroxide	10	7	3	0	40	30	400-450
14	0.5 g. bismuth subnitrate + 1 g. sodium hydroxide	8	8	0	0	32	10-20	Free flame
15	1 g. stannous chloride (crystals)+2 g. sodium hydroxide	..	..	..	40	..	10	300-350
16	0.5 g. lead sulphate+2 g. sodium hydroxide+0.5 c.c. of water	12	12	0	0	46	5	400-450
17	0.7 g. cupric chloride (crystals)+2 g. sodium hydroxide	13	13	0	0	51	..	400-450
18	0.7 g. magnesium sulphate (crystals)+2 g. sodium hydroxide	14.5	14.5	0	0	55	10	400-450
19	0.5 g. bismuth oxide+2 g. sodium hydroxide +2 c.c. of water	13	13	0	0	50	..	300-350
20	Sodium antimonite + sodium hydroxide (amounts unknown)	7.4	6.3	1.1	0.2	28.4	..	..
21	Titanous sulphate + sodium hydroxide (amounts unknown)	9	9	0	1	34	..	..
22	Titanous sulphate+sodium hydroxide solution	3.6	3.6	0	0	13.7	..	Boiled
23	Cerous sulphate+sodium hydroxide	15	14	1	6	57	..	Free flame
24	0.5 g. mercurous chloride+2 g. sodium hydroxide	14	14	0	0	52.5	..	Free flame
25	0.5 g. vanadium sulphate+2 g. sodium hydroxide	13.5	12	1.5	20	52	..	300-350
26	Uranous sulphate+sodium hydroxide solution	8.5	8.5	0	0.8	32	..	Boiled

the trihydroxide with evolution of hydrogen, by boiling with potassium hydroxide solution.

None of these papers make any reference to the general nature of the reaction, or offer any explanation of its mechanism.

DESCRIPTION OF EXPERIMENTS.

The fusions were carried out in a hard glass tube (Pyrex glass was used in most of the experiments), of approximately 50 c.c. capacity, bent in such a form as to catch the water which distilled off from the fusion mixture and prevent it from running back into the hot portion of the tube. This tube (see figure) was provided with a rubber stopper carrying a small dropping funnel and a small bore tube leading to a water-jacketed gas buret containing mercury. This buret was provided with a two-way tap. Heating was effected by means of a bath of fused sodium nitrite which melts at 271° and can be used for temperatures up to 350°. For temperatures above 350° a bath of fused potassium dichromate was used. Below 350° the temperatures were recorded by an ordinary thermometer and above that temperature by a mercury thermometer under 60 atmospheres pressure of nitrogen in quartz.

The procedure consisted in placing a carefully weighed amount of substance in the bottom of the fusion tube, and adding quickly a known weight of sodium hydroxide in coarse powder, inserting the stopper and connecting with the gas buret. As gas was evolved in the reaction the levelling tube was lowered so that the pressure in the apparatu was always approximately atmospheric. After the fusion, water was run into the tube through the tap funnel and all the gas driven into the buret. The gas was then analyzed for hydrogen and oxygen and, where necessary, for other constituents, and in some cases the contents of the fusion tube was also analyzed in order to determine the amount of oxidation which had occurred. Oxygen was determined by absorption by phosphorus in a Hempel pipet and hydrogen by combustion with hot copper oxide contained in a small quartz tube heated to redness by a Bunsen burner the accuracy of this method for determining hydrogen having first been proved by the analysis of gas mixtures of known hydrogen content. For convenience and accuracy the gas buret was directly connected with the small quartz tube containing copper oxide, and this to the phosphorus pipet. The gas for analysis was first passed from the buret into the pipet, the quartz tube being at room temperature. This was repeated until all oxygen which remained in the quartz tube after the first passage had been absorbed. The copper oxide was then heated and the gas again passed over and back. The oxygen present in the apparatus at the start was calculated from the nitrogen remaining after the fusion.

The accuracy of the method for determining hydrogen was tested by preparing mixtures of hydrogen and nitrogen of known composition and analyzing them. The nitrogen was prepared from air by removing carbon dioxide and oxygen. The hydrogen was prepared electrolytically and freed from traces of oxygen by passing over hot copper.

	I.	II.
	Cc.	Cc.
Nitrogen used	37.4	26.4
Hydrogen used	47.3	59.0
Hydrogen found	47.2	58.9

In order to determine whether any error exists in the application of this method to mixtures of hydrogen and oxygen, due to a union of hydrogen and oxygen induced by a union of phosphorus and oxygen, the following gas mixtures were analyzed:

	I.	II.
	Cc.	Cc.
Air used	50.0	50.0
Oxygen in air	10.4	10.4
Hydrogen used	21.6	30.2
Oxygen found	10.4	10.4
Hydrogen found	..	30.2

Table I gives the details of the experiments.

The following table shows the results of the determinations of the extent of oxidation in the case of the above experiments with arsenious acid and ferrous sulphate. Col. I gives the volume of oxygen equivalent to the hydrogen evolved plus the free oxygen absorbed; Col. II the oxygen taken up by arsenite; Col. III that taken up by ferrous iron.

TABLE II.

Expt. No.	I.	II.	III.
	Cc.	Cc.	Cc.
1	4.1	6.6	..
2	15.6	19.0	..
3	18.7	21.2	..
9	12.3	..	12.5

The above values of the extent of oxidation of arsenite were determined by titration against standard iodine in the presence of sodium hydrogen carbonate. Oxidation of ferrous iron was determined by titration against standard permanganate in sulphuric acid solution. The values in Cols. I and II and I and III are substantially equal, showing that an equivalent oxidation has occurred.

DISCUSSION.

In the experiments with sodium arsenite and ferrous sulphate the extent of the oxidation, as well as the amount of hydrogen evolved were determined. These were found to be equivalent.

In the case of stannous chloride, as well as of vanadium sulphate, much hydrogen was evolved, with cerous sulphate a less amount and with uranous sulphate only a small amount. The extent of the oxidation effected in these 4 cases was not measured.

It is interesting to observe that neither sodium nitrite nor sodium sulphite was oxidized. In view of the fact that sodium sulphite in water solution is oxidized by free oxygen at ordinary temperature, and that both of these salts are rather easily oxidized in water solution by several oxidizing agents, this failure to undergo oxidation when fused with sodium hydroxide is noteworthy.

Upon examining the bases whose salts were oxidized it is observed that they belong, with the exception of iron, to Groups 4, 5 and 6 of the periodic table. Tin and cerium fall in Group 4; phosphorus, vanadium and arsenic fall in Group 5, while chromium and molybdenum fall in Group 6. If this has any bearing at all on the matter it is the more surprising that nitrogen and sulphur, which fall in Groups 5 and 6, respectively, are not oxidized from nitrites and sulphites to nitrates and sulphates. In this connection it would also be of interest, and especially in view of the industrial importance of the reaction in the manufacture of formates, to study the action of carbon monoxide with sodium hydroxide with a view to determine whether oxidation to carbon dioxide according to this general reaction occurs or not.

The mechanism of these reactions consists ultimately in the decomposition of water, the oxygen carrying the oxygen acceptor to a higher stage of oxidation and the hydrogen being evolved in the gaseous state. Reactions with fused or dissolved alkali hydroxides are described in the literature in which the other reacting compound acts as both oxygen and hydrogen acceptor, one mol. being oxidized and one mol. simultaneously reduced. In such cases the hydrogen, instead of being evolved in the free state, goes to reduce a second mol. These are in all probability special cases of the general oxidizing action of alkali hydroxides referred to in this paper, with probably the same mechanism involving the decomposition of water. The following are instances of this action: Ditte[1] observed that stannous hydroxide reacts with very strong solutions of potassium hydroxide at ordinary temperatures with the formation of tin and potassium stannate; also Rammelsberg[2] observed that lower phosphorus salts such as $NaH_2PO_2.H_2O$, when heated pass into phosphates and phosphine.

The evolution of hydrogen when many metals as aluminium, zinc, and lead are boiled with sodium hydroxide solution is no doubt initiated by this general oxidizing action of water catalyzed by sodium hydroxide.

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[1] *Ann. chim. phys.*, [5] 27, 145 (1882).

[2] *Ber.*, 5, 494 (1872).

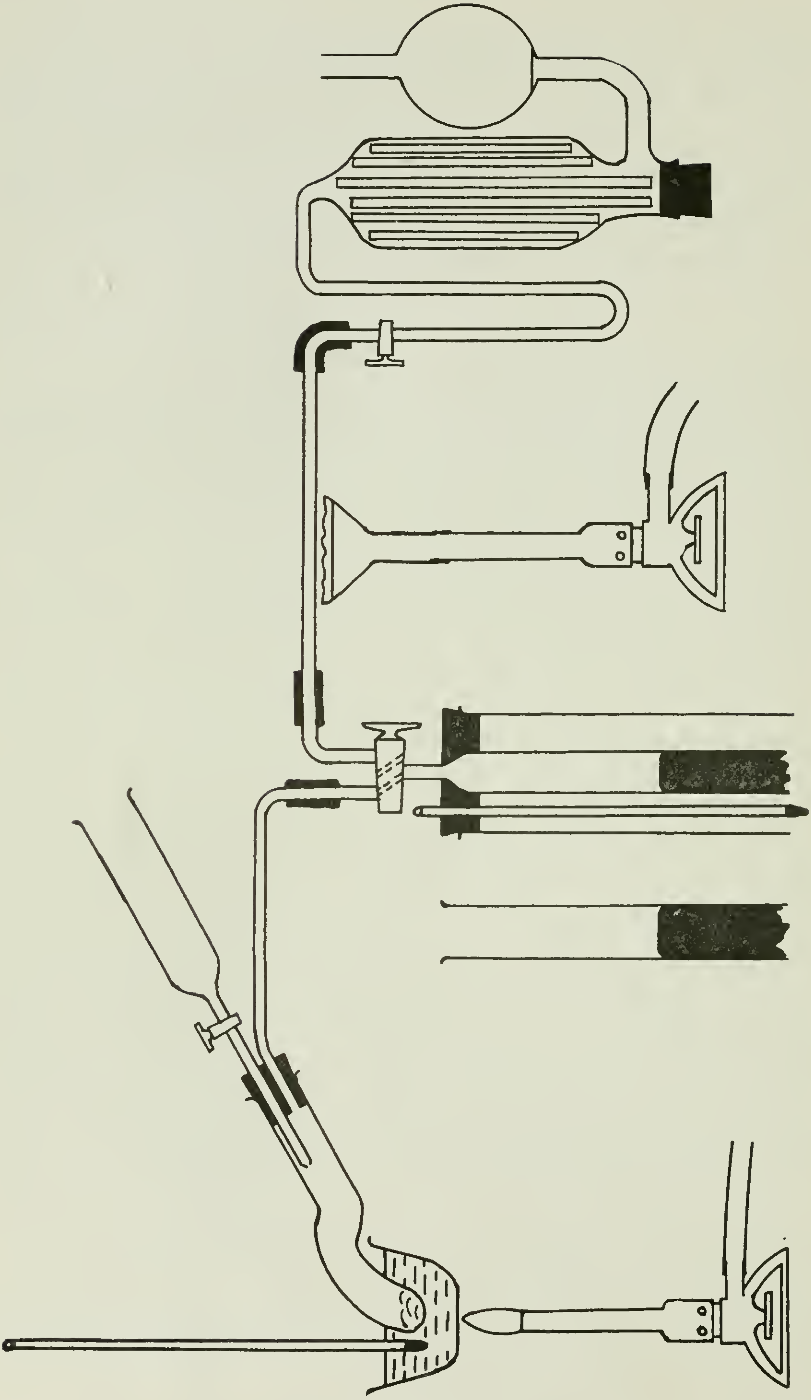
$$Zn + 2H_2O = Zn(OH)_2 + H_2.$$

The inner mechanism of the catalysis of this general oxidizing reaction induced by alkali hydroxides, constitutes the working hypothesis which, as stated at the outset, led to this investigation. It has to do with the relationship between water and alkali hydroxide in the dissolved and fused states. A statement of this theory with experimental evidence and argument to support it will be presented in the near future.

SUMMARY.

The general nature of the action of fused sodium hydroxide in effecting oxidations has been pointed out.

The reaction has been shown to involve ultimately a decomposition of water, one mol. of oxygen going to effect oxidation and two mols. of hydrogen being either evolved or fixed by a hydrogen acceptor.



THE ACTION OF SODIUM HYDROXIDE ON CARBON MONOXIDE, SODIUM FORMATE AND SODIUM OXALATE.

Reprinted from the Journal of the American Chemical Society, Vol. XL.

By Maitland C. Boswell, M.A.; Ph.D. and J. V. Dickson, B.A.Sc.

In a previous paper[1] we have shown that fused sodium hydroxide is very active in effecting oxidations. As pointed out there, it seemed desirable in the light of this fact to investigate more fully the action of sodium hydroxide upon carbon monoxide, sodium formate and sodium oxalate. The necessity for this became further apparent upon examining the literature with its confusing statements regarding the reactions involved.

It has long been known that carbon monoxide acts on sodium hydroxide to form sodium formate at moderately high temperatures. Merz Tibirica[2] recommended the use of moist carbon monoxide and loose soda lime at temperatures not above 220°. A commercial method described by Ullmann[3] consists in the use of producer gas at 6 to 8 atmospheres pressure acting on finely ground sodium hydroxide at 120-130°. Merz and Weith[4] have stated that a residue consisting of at least 70% oxalate and the remainder carbonate can be obtained by heating sodium formate rapidly to above 400° but that at about 360° the formation of carbonate is much greater. Levi and Piva[5] state that formate begins to decompose at 300°, that the decomposition is violent at 400° and complete at 550°. They give two analyses of the products which include small quantities of carbon dioxide, large amounts of carbon monoxide and hydrogen, carbonate and oxalate. The admixture of sodium carbonate is stated to bring about a quantitative yield of sodium oxalate from formate, without the necessity of particularly rapid heating or of heating above 360°. The investigators who took out the patent[1] based upon this observation, believed that it gave support to the following mechanism:

$$2HCOONa = Na_2CO_3 + H_2 + CO$$

$$CO + Na_2CO_3 = Na_2C_2O_4$$

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[1] *J. Am. Chem. Soc.*, 40, 1773 (1918).

[2] *Ber.*, 13, 23 (1880).

[3] Ullmann, "Enzyklopädie der Technischen Chemie."

[4] *Ber.*, 15, 1507 (1882).

[5] *Ann. chim. applicata*, 1, pp. 1-24.

However, the efficacy of the addition of carbonates in increasing the yield was called in question shortly afterwards,[2] it being stated that the yield claimed had not been reached in a great number of experiments that were tried. The addition of oxalate, however, was stated to be a marked improvement. Later[3] the addition of small amounts of sodium hydroxide (under 5%) to the formate was again claimed to have a good effect, technically pure oxalate being produced at a temperature below the melting point of the formate. With regard to the action of carbon monoxide on sodium hydroxide at high temperatures, Bain and Tyrrell[4] showed that no oxalate (or at most only traces) is formed at temperatures between 355° and 490°, carbon monoxide being used at atmospheric pressure. Dumas and Stas[5] found that sodium formate heated with barium hydroxide gives oxalate and hydrogen, and that oxalate heated with barium hydroxide gives carbonate and hydrogen. However, neither the temperatures nor the amounts of substances used were recorded.

The above statements show the unsatisfactory condition of the whole question from a scientific point of view, much of the work evidently having been done with a view solely to the commercial method of preparing oxalate.

The present investigation was undertaken with a view of ascertaining the action, if any, of carbon monoxide on sodium hydroxide at about the temperature of decomposition of sodium formate, with which it was thought the mechanism of the decomposition of the formate might be closely connected. In view of the oxidizing action of fused sodium hydroxide on various substances,[6] it was thought that the non-formation of oxalate from sodium hydroxide and carbon monoxide (at temperatures at which formate gives oxalate) might be due to the oxidation of the monoxide to dioxide according to the equation $CO+H_2O->CO_2+H_2$ (carbonate, of course, then being formed). Our experiments have shown this conjecture to be correct.

The oxidizing action of sodium hydroxide on carbon monoxide having been shown, it was thought desirable to determine the action of sodium hydroxide on formate and oxalate, particularly in view of the statement quoted above that small amounts of sodium hydroxide added to formate increase the yield of oxalate. The experiments described below show that at temperatures far below those at which formate or oxalate alone is decomposed with any degree of rapidity, fused sodium hydroxide carries them over rapidly to carbonate with the evolution of hydrogen.

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[1] D. R. P. 111,078, *Z. angew. Chem.*, 1900, p. 448.

[2] D. R. P. 144,150, *Ibid.*, 1903, p. 924.

[3] D. R. P. 161,512, *Ibid.*, 1905, p. 1793.

[4] E. J. Tyrrell, *Thesis*, Univ. of Toronto, 1917.

[5] *Ann. Chem.*, 35, 129 (1840).

[6] This Journal, 40, 1773 (1918).

In view of the oxidizing action of sodium hydroxide it was thought that the decomposition of pure sodium formate by heat might be initiated by dissociation of some of the formate into carbon monoxide and sodium hydroxide, which latter, reacting then with more formate and also, at higher temperature with the carbon monoxide would form oxalate, carbon dioxide and carbonate, all the sodium hydroxide being carried over to carbonate before oxidizing all the oxalate to carbonate. One experiment was performed in order to detect, if possible, such a dissociation, but the results were negative.

EXPERIMENTAL WORK.

The apparatus used in all these experiments, with the exception of the last one, was similar to that used in previous fusions.[1] The carbon dioxide, oxygen and hydrogen were determined as already described. The carbon monoxide was determined by absorption by ammoniacal cuprous chloride solution. After removal of the hydrogen by hot copper oxide the gas was again passed into the potash pipet, the final residue being called nitrogen. This last treatment with potash would show the presence of any hydrocarbons or any carbon monoxide which might have escaped the cuprous chloride and which would have burned to carbon dioxide. The carbon dioxide found at this stage was never more than 0.2 c.c. so that it did not affect the results appreciably.

ACTION OF SODIUM HYDROXIDE ON CARBON MONOXIDE.

Two experiments were performed, the procedure being as follows: The sodium hydroxide was weighed out of a weighing bottle into a small glass tube about the size of a thimble which was quickly slipped down to the bottom of the fusion tube. The latter was stoppered quickly and connected with the gas buret. The end of the fusion tube was immersed in a bath of fused potassium dichromate which was kept at 410-430°. When the volume of air in the buret had come approximately to equilibrium, the delivery tube was closed with a pinchcock and disconnected from the buret. The air was expelled from the buret and a measured quantity of analyzed carbon monoxide was admitted in its place. The buret was again connected with the delivery tube, the pinchcock, however, closed. A water suction pump was attached to the tap funnel and the fusion tube evacuated. The carbon monoxide was then given access to the fusion tube. After heating for some time the tube was allowed to cool and an excess of hot, boiled, dil. hydrochloric acid was drawn in through the tap funnel, and the gas driven over into the buret for analysis. The solution remaining in the tube was tested for oxalate. The results of these two experiments are as follows:

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[1] This Journal, 40, 1776 (1918).

TABLE I.—ACTION OF SODIUM HYDROXIDE ON CARBON MONOXIDE.

NaOH used, g.	CO taken, c.c.	CO used up, c.c.	CO^2 produced, c.c.	H^2 produced, c.c.	O^2 used up, c.c.	Time of heating
1.000	47.0	25.8	26.5	23.1	0.5	30 min.
0.441	55.0	31.2	29.0	27.9	0.8	25 min.

No oxalate remained. The oxygen used up was found by subtracting the amount left after the fusion from the amount corresponding (in the composition of air) with the nitrogen present after the fusion, allowance being made for the nitrogen and oxygen content of the carbon monoxide. In determining the amount of carbon dioxide produced allowance had to be made for the carbonate in the alkali used, which was determined from time to time.

DECOMPOSITION OF SODIUM FORMATE BY HEAT.

The formate used was prepared by neutralizing sodium carbonate (Baker's C. P.) with excess of formic acid (Merck's), evaporating nearly to dryness on the water bath and drying at 110-130° in an oven. 0.2072 g. of formate was heated in a fusion tube connected with the gas buret at about 325° for 10 minutes. The decomposition being very slow the temperature was raised to about 345° where it was kept for 15 minutes. The tube was allowed to cool when it was seen that the volume of gas had increased only 3 c.c. The tube was again heated for 30 minutes at 390-400° when the volume of gas increased considerably. The temperature was raised to 420° where it was kept for 10 minutes without much increase in the volume of gas. The gas, including the carbon dioxide from the carbonate, was displaced by the addition of dil. sulphuric acid and analyzed with the following results:

	Cc.
Hydrogen produced	17.0
Carbon monoxide produced	22.0
Carbon dioxide produced	21.7
Oxygen present at the start	10.4
Oxygen used up	4.4

The solution in the fusion tube was titrated with standard permanganate, giving a titer corresponding to 0.046 g. oxalate.

ACTION OF SODIUM HYDROXIDE ON SODIUM FORMATE.

This experiment was carried out in a manner similar to the previous one, 0.157 g. formate and 0.352 g. hydroxide being used. The temperature was 275°. The mixture began quickly to froth and within 5 minutes the reaction appeared to be complete. The heating was continued for 10 minutes. Analysis of the gas resulted as follows:

	Cc.
Hydrogen produced	47.9
Carbon dioxide produced	50.2
Carbon monoxide produced	0.3
Oxygen used up	0.1

No oxalate was found in the solution.

Action of Sodium Hydroxide on Sodium Oxalate.

The experiment was similar to the previous one, the temperature, however, being 290°. 0.1982 g. oxalate and 0.52 g. hydroxide were used. Here again the action appeared to be over in 5 minutes. The gas analysis gave the following results:

	Cc.
Hydrogen produced	27.9
Carbon dioxide produced	63.2
Carbon monoxide produced	0.1
Oxygen used up	0.0

The solution remaining required permanganate corresponding to 0.0025 g. oxalate.

Behaviour of Formate at Lower Temperatures.

The following experiment was performed in order to detect, if possible, any dissociation of formate that might take place below the temperature of its ordinary decomposition.

About 0.2 g. of sodium formate was heated in a U-tube of hard glass about $\frac{1}{4}$ inch in diameter, having a small bulb at the bottom holding the formate. One end of the tube was closed by means of rubber tubing and a pinchcock, and the other end attached to the gas buret. The bulb was heated in an oil bath to 150° and then gradually during more than an hour to about 315° where it was kept for 20 minutes. While still hot, air was drawn through the tube and the gas drawn into the buret. The gas in the buret was analyzed with the following results:

	Cc.
Oxygen used up	0.1
Hydrogen produced	0.8
Carbon monoxide produced	0.7
Carbon dioxide produced	0.2

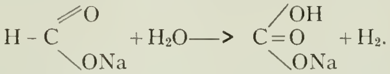
Discussion of Results.

The main reaction of sodium hydroxide upon carbon monoxide is evidently the oxidation of carbon monoxide to carbon dioxide with the liberation of a corresponding amount of hydrogen

$$CO+H_2O=CO_2+H_2.$$

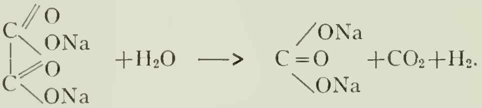
If this were the sole reaction, the volumes of carbon monoxide used up, dioxide produced, and hydrogen produced would be equal. The oxygen used up may be considered as having carried twice its volume of monoxide to dioxide so that the 3 volumes, carbon monoxide, carbon dioxide and hydrogen concerned in the main reaction in Expt. 1 when corrected for this will be, respectively, 24.8 25.5 and 23.1 c.c.; and in Expt. 2, 29.6, 27.4 and 27.9 c.c. While these volumes are not quite equal in either case they are near enough to indicate that the main reaction is an oxidation with the liberation of an equivalent amount of hydrogen.

The fusions of sodium hydroxide with formate and oxalate show that the latter are carried rapidly and almost completely to carbonate at temperatures far below their ordinary decomposition temperatures,



$$H-C\begin{matrix}/\!\!/O\\ \diagdown ONa\end{matrix} + H_2O \longrightarrow C\begin{matrix}/OH\\ =O\\ \diagdown ONa\end{matrix} + H_2.$$

With excess of sodium hydroxide this sodium hydrogen carbonate is carried to carbonate.



$$\begin{matrix}C\begin{matrix}/\!\!/O\\ \diagdown ONa\end{matrix}\\ |\\ C\begin{matrix}/\!\!/O\\ \diagdown ONa\end{matrix}\end{matrix} + H_2O \longrightarrow C\begin{matrix}/ONa\\ =O\\ \diagdown ONa\end{matrix} + CO_2 + H_2.$$

With excess of sodium hydroxide the carbon dioxide is fixed as carbonate. Tyrrell[1] and Bain found that sodium oxalate decomposed very slowly at 465° and lower. According to these equations the oxidation of formate to carbonate is accompanied by the formation of carbon dioxide and hydrogen in equal volumes. In the experiment the hydrogen is found to be 2.3 c.c. less than the carbon dioxide. The latter (50.2 c.c. accounts for 0.1525 g. formate although 0.1570 g. was used. In the oxidation of oxalate to carbonate the hydrogen formed is by the equation $\frac{1}{2}$ of the carbon dioxide and we find in the experiment it is 3.7 c.c. short of this. Carbon dioxide formed (63.2 c.c.) accounts for 0.1890 g. oxalate, while the amount remaining undecomposed according to the permanganate titration is 0.0025 g., 0.1915 g. out of 0.1982 g. thus being accounted for.

The practice of mixing sodium hydroxide with sodium formate to the extent of 5% in the manufacture of oxalate from formate according to D. R. P. 165,512 can now be given an interpretation. Pure sodium formate, when heated alone, undergoes decomposition at a very slow rate at temperatures below about 400°. At this temperature the chief reaction is the formation of carbonate and carbon monoxide with very little oxalate. At temperatures up to 480° there is considerable carbonate formed with a corresponding poor yield of oxalate. If, however, a small amount of sodium hydroxide is present the temperature of fusion is low, and at this temperature the oxidation from formate to oxalate occurs with the evolution of an equivalent amount of hydrogen, the temperature however, being so low as to keep the velocity of the oxidation of formate and oxalate to carbonate small. That is, the transformation from formate to oxalate in the presence of a small amount of sodium hydroxide as carried out in practice in the manufacture of oxalate is really an oxidation of formate by the water present in the fusion, the hydrogen being evolved in the free state. That is, the direct source of the hydrogen evolved is not the sodium formate as represented in the usual reaction equation

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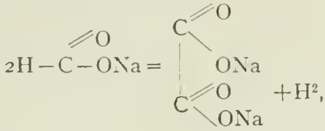
[1] Bain and Tyrrell, *Loc cit.*

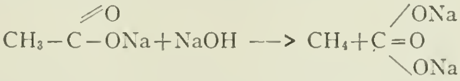


$$2H-\overset{\diagup\!\!\diagup O}{C}-ONa=\begin{array}{l}C\begin{array}{l}\diagup\!\!\diagup O\\ \diagdown ONa\end{array}\\ |\\ C\begin{array}{l}\diagup\!\!\diagup O\\ \diagdown ONa\end{array}\end{array}+H^2,$$

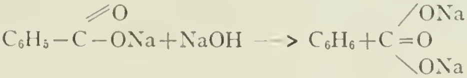
but the water present. This may be considered as an oxidation of an acid, whose anhydride is carbon monoxide to an acid whose anhydride is the hypothetical oxide $C_2\,O_3$, with the evolution of an equivalent amount of hydrogen.

$$2CO+H_2O=C_2O_3+H_2.$$

This immediately raises the question of the mechanism of the fusion of carboxylic acid salts in general with sodium hydroxide. This is one of the general reactions of organic chemistry and is always represented without any regard to the mechanism in the following way in the case of sodium acetate and of sodium benzoate:

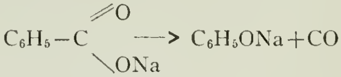


$$CH_3-\overset{\diagup\!\!\diagup O}{C}-ONa+NaOH\longrightarrow CH_4+\overset{\diagup ONa}{\underset{\diagdown ONa}{C=O}}$$



$$C_6H_5-\overset{\diagup\!\!\diagup O}{C}-ONa+NaOH\longrightarrow C_6H_6+\overset{\diagup ONa}{\underset{\diagdown ONa}{C=O}}$$

The carboxyl group has been replaced by hydrogen. It seems highly probable that these actions as far as they involve the replacement of carboxyl by hydrogen (and apart altogether from those secondary reactions recognized as oxidations such as the formation of *p*-oxybenzoic acid in the case of the fusion of sodium benzoate and sodium hydroxide), are all oxidations catalyzed by sodium hydroxide and involving the elements of water. In the production of methane and benzene from sodium acetate and sodium benzoate the oxidation product, sodium carbonate, is formed. The hydrogen, however, instead of being evolved as such is fixed by the methyl and phenyl groups, respectively, and appears, not as free hydrogen, but as methane and benzene. However, some of the hydrogen escapes this action and appears as free hydrogen, for it is a well-known fact that the methane prepared by this process may contain as high as 8% of free hydrogen. The mechanism in the case of the fusion of sodium benzoate and sodium hydroxide (as far as it concerns the production of benzene) would accordingly be



$$C_6H_5-C\begin{matrix}\nearrow O\\ \searrow ONa\end{matrix} \longrightarrow C_6H_5ONa+CO$$

$$CO+H_2O \longrightarrow CO_2+H_2$$

$$C_6H_5ONa+H_2O+H_2 \longrightarrow C_6H_6+NaOH+H_2O.$$

It seems that in general the primary and chief action of sodium hydroxide in fusion actions is an oxidizing one with the mechanism referred to. Whether this action is accompanied by the evolution of free hydrogen or not depends on whether any compound is present which can act as hydrogen acceptor and thus prevent, completely or in part, its evolution in the free state.

SUMMARY.

When carbon monoxide is heated with excess of sodium hydroxide at temperatures at which formate is transformed into oxalate, oxidation almost quantitatively to carbon dioxide occurs with the evolution of approximately an equivalent amount of hydrogen.

At temperatures far below those at which formate and oxalate alone are decomposed, sodium hydroxide carries both of them almost quantitatively to carbonate with the evolution of equivalent amount of hydrogen.

The general reaction involving the replacement of the carboxyl group by hydrogen in alkali fusions is probably a special case of the general reaction under discussion involving a decomposition of water, both the hydrogen and oxygen being fixed.

THE FUSION OF SODIUM HYDROXIDE WITH SEVERAL PHENOLS AND SUPLHONIC ACIDS.

Reprinted from the Journal of the American Chemical Society, Vol. XL.

BY MAITLAND C. BOSWELL, M.A.; Ph.D. AND J. V. DICKSON, B.A.Sc.

Sodium hydroxide fusions of organic compounds particularly of salts of sulphonic acids are common both in the laboratory and in the factory. It is only necessary to mention such well known examples as the manufacture of carbolic acid from benzene mono sulphonate;

$$C_6H_5SO_3Na+NaOH \longrightarrow C_6H_5ONa+NaHSO_3$$

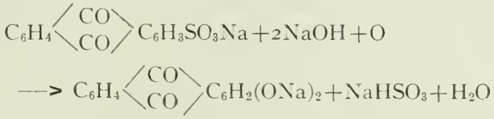
of α and β naphthols from α and β naphthalene sulphonates

$$C_{10}H_7SO_3Na+NaOH \longrightarrow C_{10}H_7ONa+NaHSO_3$$

of resorcin from benzene meta di sulphonate

$$C_6H_4(SO_3Na)_2+2NaOH \longrightarrow C_6H_4(ONa)_2+2NaHSO_3$$

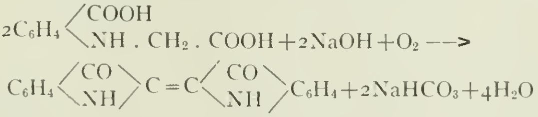
of the dye alizarin from anthraquinone βmono sulphonate



$$C_6H_4\left\langle\begin{matrix}CO\\CO\end{matrix}\right\rangle C_6H_3SO_3Na+2NaOH+O$$

$$\longrightarrow C_6H_4\left\langle\begin{matrix}CO\\CO\end{matrix}\right\rangle C_6H_2(ONa)_2+NaHSO_3+H_2O$$

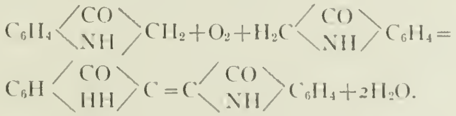
and of indigo from phenyl glycine ortho carboxalate



$$2C_6H_4\left\langle\begin{matrix}COOH\\NH\,.\,CH_2\,.\,COOH\end{matrix}\right. +2NaOH+O_2 \longrightarrow$$

$$C_6H_4\left\langle\begin{matrix}CO\\NH\end{matrix}\right\rangle C=C\left\langle\begin{matrix}CO\\NH\end{matrix}\right\rangle C_6H_4+2NaHCO_3+4H_2O$$

In the production of alizarin by the above reaction a mono sulphonate yields a diphenol requiring the introduction of one atom of oxygen per mol. of alizarin formed. Potassium chlorate is added to the fusion mixture to furnish this oxygen. In view of the fact that in the early days of alizarin manufacture by this reaction, no commonly recognized oxidizing agent was used, it became of importance to determine whether any oxidation induced by the fused sodium hydroxide and involving the decomposition of water, occurs in this process as originally conducted. Similarly in the production of Indigo from phenyl glycine orthocarboxylic acid the compound indoxyl formed as intermediate product is carried to indigo by an oxidation process as follows:



$$C_6H_4\left\langle\begin{matrix}CO\\NH\end{matrix}\right\rangle CH_2+O_2+H_2C\left\langle\begin{matrix}CO\\NH\end{matrix}\right\rangle C_6H_4=$$

$$C_6H\left\langle\begin{matrix}CO\\HH\end{matrix}\right\rangle C=C\left\langle\begin{matrix}CO\\NH\end{matrix}\right\rangle C_6H_4+2H_2O.$$

Is any of this oxidation produced by this general reaction under discussion involving the decomposition of water?

In view of the commercial importance of these industries as well as of the interest from a scientific standpoint of the problems involved we decided to investigate some of them in the light of the oxidizing action already treated of in two previous papers[1],[2] dealing with this subject. The present paper gives the results of our investigation of this reaction in the case of sodium benzene sulphonate, β-naphthalene sulphonate, anthraquinone β-monosulphonate, sodium salt of phenylglycine-*o*-carboxylic acid and of the 7 phenols, carbolic acid, hydroquinone, pyrocatechine, resorcin, pyrogallol, oxyhydroquinone and phloroglucine.

Considerable work done in this laboratory had already shown that fusion of sodium benzene sulphonate with sodium hydroxide under widely varying conditions failed to give a yield of phenol in excess of ·90% of the theory. The discovery of the widespread applicability of sodium hydroxide in effecting oxidations, made it appear that in this property of sodium hydroxide might be found the secret of the failure to obtain a greater yield. This involved the study of the gas changes occurring in the fusion with sodium hydroxide of sodium benzene sulphonate and of sodium phenate, in the presence and absence of free oxygen. It soon became evident that the evolution of hydrogen in both of these reactions is conditional upon this reaction being preceded by an oxidation by free oxygen. In the case of several separate fusions of sodium hydroxide with these two compounds, the ratio of hydrogen evolved to free oxygen absorbed was approximately one-half. The inference is that free oxygen carried the sodium phenate over to some polyhydroxy phenol, which was then oxidized in the sodium hydroxide fusion according to the general reaction under discussion, with the evolution of free hydrogen. This naturally led to a study of the fusion reaction of the 6 dihydroxy and trihydroxy benzenes with sodium hydroxide. It also led to an investigation of the effect upon the yield of phenol, of conducting the fusion of sodium benzene sulphonate out of contact with free oxygen, with the result that it was found possible to increase the yield of carbolic acid in this way by about 8% and thus make the yield approximately theoretical.

In view of this marked increase in yield it seems that manufacturers of phenol should consider the feasibility of conducting the fusion out of contact with atmospheric oxygen.

Although the yields of β-naphthol in the presence and absence of free oxygen were not compared, yet the fact that oxidation (as indicated by evolution of hydrogen) occurs only when free oxygen is present, makes it appear highly probable that an increased yield would be obtained by carrying out the reaction in the absence of free oxygen.

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[1] J. Am. Chem. Soc., 40, 1773 (1918).

[2] *Ibid.*, 40, 1779 (1918).

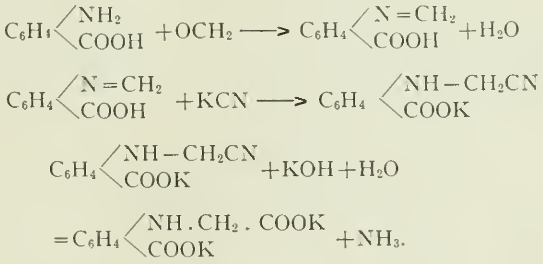
EXPERIMENTAL.

PREPARATION OF MATERIALS.

Sodium benzene sulphonate was made by the sulphonation of benzene and isolation of the sodium salt by the "lime method" and recrystallization. The sulphonate content was determined by extraction of a weighed amount under a reflux condenser with boiling 95% alcohol, filtering, evaporating in a dish and weighing.

β-sodium naphthalene monosulphonate and β-sodium anthraquinone monosulphonate were made from naphthalene and anthraquinone by similar processes.

Sodium phenylglycine orthocarboxylate was made according to the method of D. R. P. No. 120105 by the action of anthranilic acid, potassium cyanide and formaldehyde and the hydrolysis of the resulting cyanmethyl anthranilic acid according to the equations



$$C_6H_4\left\langle\begin{matrix}NH_2\\COOH\end{matrix}\right. + OCH_2 \longrightarrow C_6H_4\left\langle\begin{matrix}N=CH_2\\COOH\end{matrix}\right. + H_2O$$

$$C_6H_4\left\langle\begin{matrix}N=CH_2\\COOH\end{matrix}\right. + KCN \longrightarrow C_6H_4\left\langle\begin{matrix}NH-CH_2CN\\COOK\end{matrix}\right.$$

$$C_6H_4\left\langle\begin{matrix}NH-CH_2CN\\COOK\end{matrix}\right. + KOH + H_2O$$

$$= C_6H_4\left\langle\begin{matrix}NH.CH_2.COOK\\COOK\end{matrix}\right. + NH_3.$$

The pyrocatechine, resorcin, hydroquinone, oxyhydroquinone, pyrozallol, phloroglucin, used were "Merck" products.

The fusions were performed in the same manner and with the same apparatus as already described in a previous paper,[1] with the additional precaution that the residual gas after combustion with hot copper oxide, and measurement, was passed into a potassium hydroxide pipet to determine whether any carbonaceous gas had been present in the original gas. In almost all cases the carbon dioxide found was quite small and thus the error introduced may in consequence be neglected.

In Expts. 7 and 8 oxygen was admitted to the fusion tube only after the main reaction was over (quiet fusion) 1.0 and 18 minutes, respectively, from the outset.

In order to determine the effect of excluding free oxygen upon the yield of phenol in the fusion of sodium benzene sulphonate and sodium hydroxide, two experiments were conducted, one in an open crucible with access to the air, and the other in a closed crucible in an atmosphere of nitrogen.

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[1] This Journal, 40, 1773 (1918).

TABLE I.—GAS ANALYTICAL RESULTS.

No.	Wt. of substance G.	Wt. of sodium hydroxide G.	Temperature Degrees	Oxygen			Hydrogen produced Cc.	"Methane" produced Cc.	Time of heating Mins.	Ratio oxygen used to hydrogen produced	Nitrogen present Cc.
				at the start CC.	at the end Cc.	used up Cc.					
Sodium Benzene Sulphonate.											
1	1	1	300-330	9.8	2.6	7.1	3.1	0.5	40	2.3	37.2
2	1	2.5	300	9.9	3.4	6.5	3.2	0.5	35	2.0	37.6
3	1	2.5	290	9.4	4.6	4.8	2.6	0.2	40	1.9	35.5
4	1	2.5	350	9.5	2.5	7.0	3.9	0.3	40	1.8	36.1
5	1	2.5	325	35.3	10.9	24.4	12.6	0.4	30	1.9	37.5
6	1	2.5	325	75.7	7.1	68.6	37.4	0.7	60	1.8	16.2
7	1	2.5	325	42.0	12.0	30.0	16.7	0.6	40	1.8	40.8
8	1	2.5	325	20.4	1.5	18.9	9.5	0.9	75	2.0	77.3
9	1	2.5	325	1.0	0.5	0.5	0.6	0.0	30	...	37.6
10	1	2.5	325-355	1.0	0.5	0.5	0.3	0.2	..	...	39.6
Sodium Phenolate.											
11	1	2	330	10.0	1.9	8.1	5.7	0.2	40	1.4	37.9
12	0.64	2.5	335	11.2	2.1	9.1	6.1	0.4	45	1.5	42.3
13	1	2	315-340	10.6	0.3	10.3	6.1	0.2	40	1.7	40.1
14	1	2	315-335	1.0	0.4	0.6	0.6	0.3	..	...	34.3
Sodium Sulphite.											
15	1	2	Flame	10.4	10.0	0.4	0.3	..	10	...	39.3
16	1	2	300	10.1	9.6	0.5	0.4	..	..	...	38.3
β-Sodium Naphthalene Sulphonate.											
17	1.28	4.5	300	9.6	2.8	6.8	2.9	0.1	40	2.3	36.4
18	1.28	4.5	275-315	0	0	0	0.4	0.1	30	...	45.4
β-Sodium Anthraquinone Monosulphonate.											
19	1	4.5	275-295	12.4	1.9	10.5	6.5	0.1	35	1.6	47.1
20	1	4.5	280-310	1.0	0.5	0.5	15.2	1.0	30	...	52.7
Sodium Phenylglycine-*o*-carboxylate.											
21	0.65	*	230-270	11.3	3.6	7.7	13.8	0.2	30	0.6	42.7
22	0.65	*	265	0.5	0.1	0.4	2.5	0.1	35	...	58.0
Pyrocatechine.											
23	0.5	2.0	315	10.2	4.0	6.2	4.5	0.4	30	1.4	38.8
24	0.5	2.0	340	2.5	1.2	1.3	1.7	0.7	..	...	39.4
Resorcin.											
25	1	2	315-330	10.3	3.5	6.8	4.2	0.5	..	1.6	39.1
26	0.5	2	350	1.5	0.6	0.9	0.4	0.1	40	...	41.4
Hydroquinone.											
27	0.5	2	325	10.9	1.9	9.0	6.7	1.0	60	1.3	41.3
28	0.5	2	340	1.0	0.2	0.8	0.5	0.1	40	...	35.7
Oxyhydroquinone.											
29	0.5	2	350	10.3	1.3	9.0	25.2	8.7	55	0.4	40.6
30	0.5	2	...	0	0	0	16.0	3.6	60	...	53.0
Pyrogallol.											
31	0.5	2	350	11.0	1.5	9.5	6.6	1.7	55	1.4	41.6
32	0.5	2	330	0	0	0	1.7	0.2	55	...	64.1
Phloroglucin											
33	0.5	2	340	9.8	6.5	3.3	6.0	24.0	45	0.5	37
34	0.5	2	280-340	1.0	0.4	0.6	2.0	27	50	...	67

*1.5 g. NaOH+1.5 g. KOH.

The method of fusion and isolation of the phenol was as follows: 70 g. of sodium hydroxide was heated in a small iron crucible to above 400° for 15 minutes. It was then allowed to cool to 330°. 100 g. of dry sodium benzene sulphonate of known sulphonate content was added with constant stirring, during a period of 30 minutes, the temperature being kept at 330° for 10 minutes after the addition of all the salt. The fusion mixture was poured on an iron plate, powdered in a mortar and dissolved in 250 c.c. of water. This solution was treated with sulphuric acid (1 : 1 by weight), until distinctly acid. Care was taken to stir the phenol layer well into the acid as previous experiments had shown that considerable loss may arise due to the solution of phenolate in the separated phenol layer. The phenol was separated by means of a separating funnel and distilled. The first runnings were caught in a small separating funnel, the phenol layer separated and returned to the distilling flask. The fraction boiling between 175-185° was caught in a small weighed beaker.

Extraction with ether of the water layer after separation of phenol gave 5 to 7 g. of phenol in several determinations. In the calculations below, 6 g. is added to the phenol actually obtained in each case, thus giving a close approximation to the actual yield of phenol produced in the fusion.

The following are the results obtained by fusion in air:

Wt. of phenol	36.3 g.	35.9 g
Phenol dissolved in the water layer	6.0	6.0
	42.3	41.9
Sulphonate content	89%	89%
Theoretical yield of phenol	46.48 g.	46.48 g.
Actual yield	91.1%	90.1%
Average yield		90.6%

The fusion in an atmosphere of nitrogen was conducted according to the same procedure with the exception that oxygen was excluded. This was accomplished by covering the iron crucible with an asbestos cover which was provided with a hole sufficiently large to add the salt and to stir. A stream of nitrogen was led in through a glass tube projecting through the cover, during the fusion process and while the product was cooling. The following are the results obtained:

Weight of phenol	40.0 g.
Phenol dissolved in water layer	6.0
	46.0
Theoretical yield	46.5
Actual yield	99.0%

DISCUSSION.

In the benzene sulphonate fusion Expts. 1 to 8 inclusive in which free oxygen was present, oxygen was used up and hydrogen produced in approximately the proportion 1 : 2, notwithstanding that very varying amounts of oxygen disappeared in the several experiments. In these experiments very small amounts of methane were formed. In Expts. 9 and 10, where no free oxygen was present, practically no hydrogen or methane was produced. Apparently under the conditions of the experiments sodium hydroxide is almost without action on phenol in the absence of free oxygen.

Expts. 11 to 14 inclusive with sodium phenolate are parallel in their results with those of the fusion of the sulphonate, that is, hydrogen is formed only when free oxygen is used up, and practically no methane is produced. That is, under the conditions of the experiment sodium hydroxide is almost without action on sodium phenolate in the absence of free oxygen. This fact together with the results of Expts. 7 and 8, where oxygen was admitted to the sulphonate fusion only after the main reaction had occurred, makes it appear very probable that the oxidation of phenolate is the chief, if not the entire cause, of the secondary reaction in the case of the sulphonate fusion.

Expts. 15 and 16 show that sodium sulphite (also a product in the sulphonate fusion) is practically unacted on by sodium hydroxide in the presence of air under the conditions of the experiment.

It follows that in the fusion of the sulphonate in the presence of air the first action is the taking up of free oxygen by the phenolate formed to produce some compound (or compounds) which is then oxidized by the water present in the fusion with the evolution of hydrogen according to the general reaction under discussion.

The fact that the ratio of oxygen used up to hydrogen produced is in all cases so nearly constant and approximately equal to 2 in the sulphonate fusions, indicates that the compound sought may have been formed from phenol by the introduction of two oxygens, which compound is then oxidized by the "water reaction" with the evolution of one volume of hydrogen. The experiments with the 6 dihydroxy and trihydroxy phenols were performed in order to determine, if possible, whether any of these react with sodium hydroxide in such a way as to point to one or more of them taking part in the mechanism of this reaction. An inspection of results of Expts. 24, 26, 28, 30, 32 and 34 shows that in the absence of free oxygen, with the exception of oxyhydroquinone, practically no hydrogen is evolved. This eliminates 5 of the phenols as possible intermediate products. In the case of oxhydroquinone (Expt. 30) although much hydrogen is evolved, there is also considerable methane, which eliminates this compound as a possibility. Hence none of the dihydroxy or trihydroxy phenols are involved in this mechanism.

It is of interest to observe that the oxidation of the 3 dihydroxy benzenes as well as pyrogallol by fused sodium hydroxide occurs only when free oxygen is present. The reaction involving the elements of water is not involved in these fusions under the conditions employed.

In the case of sodium β-naphthalene monosulphonate the results recorded in Expts. 17 and 18 are quite similar to those of sodium benzene sulphonate; that is, hydrogen is evolved only when free oxygen has been taken up.

In the case of sodium anthraquinone sulphonate recorded in Expts. 19 and 20 the results are different from the preceding cases in that the hydrogen produced does not depend on the previous absorption of free oxygen.

In the case of the sodium salt of phenylglycine *o*-carboxylic acid the absorption of oxygen is necessary before an evolution of hydrogen occurs.

These two latter results show that in the production of alizarin from sodium anthraquinone sulphonate some of the oxygen necessary may come from the decomposition of water and, where no oxidizing agent is added to the fusion, undoubtedly does come from this source, also that in the production of indigo from the sodium salt of phenyl glycine orthocarboxylic acid this "water reaction" plays no part, the oxygen coming solely from the air.

In a recent paper[1] on the oxidation of ethyl alcohol by soda lime Carroll has shown that the oxidation is due in the main to the sodium hydroxide in the soda lime, and that fused sodium hydroxide is more active in oxidizing alcohol than soda lime. In discussing the mechanism of the conversion of ethyl alcohol to sodium acetate Carroll expresses the belief that the reaction consists in splitting off the hydrogen from the ethyl alcohol and acetaldehyde, the sodium hydroxide acting as catalyzer for these dissociations. In the oxidation of acetaldehyde to acetic acid Carroll assumes the intermediate formation of keten and the subsequent addition reaction of the keten and sodium hydroxide to form sodium acetate. In view of the fact there is no evidence of the formation of keten in this reaction, and of the fact that alkali hydroxide can act simultaneously as both oxidizing and reducing agent (as for instance with the compounds benzaldehyde, *o*-chlorophenol, phenol at high temperatures, stannous hydroxide, etc.) it seems simpler and more in harmony with the facts to represent the reaction as involving the elements of water, the oxygen being used to oxidize the compound, and the hydrogen being fixed or evolved in the free state, depending on whether a hydrogen acceptor is present or not.

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[1] *J. Phys. Chem.*, 22, 128 (1918).

In a recent paper[1] Witzemann has studied the action of potassium permanganate in effecting the oxidation of acetone in the presence of alkali hydroxide, and finds that the extent of the oxidation is largely dependent on the concentration of the latter. Witzemann moreover observes that the permanganate used is somewhat less than corresponds to the oxidation produced, and in discussing the mechanism of reaction comes to the conclusion that one possible explanation is the reduction of the manganese dioxide (presumably by the organic compounds present) to manganous oxide which latter is then reoxidized by the oxygen of the air, and superposed on this an oxidation of the acetone, catalyzed by the alkali. Witzemann however says nothing regarding the nature of this latter action. The fact that numerous oxidations occur (which form the subject of this series of papers) in which sodium hydroxide acts as catalyzer to a general reaction involving the elements of water, the hydrogen being either evolved in the free state or fixed by some hydrogen acceptor, may afford an explanation of this observation of Witzemann. From this point of view acetone or one or more products of its oxidation may be further oxidized by this reaction involving the decomposition of water, either the manganese dioxide or permanganate acting as hydrogen acceptor. The fact, as Witzemann points out, that sodium hydroxide alone or permanganate alone is without action, but in conjunction effect the oxidation, makes it appear at least a possibility that a large part of the total oxidation reaction is due to this reaction of water, the permanganate and (after the reaction has set in), the manganese dioxide, acting as hydrogen acceptors.

Further support of this view regarding the mechanism of oxidations in general in alkaline solution will be presented by one of us in the near future.

SUMMARY.

It has been determined that the yield of carbolic acid by the fusion of sodium benzene sulphonate with sodium hydroxide is increased by about 8% and made approximately theoretical by carrying out the entire fusion and cooling of the melt in an atmosphere free from gaseous oxygen.

When the fusion is performed in the absence of free oxygen no hydrogen is evolved and practically the only action occurring is that represented by the equation

$$C_6H_5SO_3Na+NaOH=C_6H_5ONa+NaHSO_3.$$

In the presence of free oxygen secondary reactions occur involving first an absorption of free oxygen followed by an oxidation involving the elements of water, free hydrogen being evolved.

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[1] This Journal, 39, 2657 (1917).

A study of the oxidations of the 6 di- and tri-hydroxy benzenes by sodium hydroxide showed that none of these could be the direct cause of this hydrogen evolution.

The oxidation of the 3 di-hydroxy benzenes and of pyrogallol by sodium hydroxide occurred only in the presence of free oxygen. Oxidation by reaction with water catalyzed by sodium hydroxide was not observed.

This last observation is also true in the case of the fusion of sodium β-naphthalene sulphonate as well as of phenylglycine *o*-carboxylic acid with sodium hydroxide.

In the fusion of sodium anthraquinone β-monosulphonate with sodium hydroxide, oxidation occurs by means of the "water reaction" without the necessity of a previous oxidation by free oxygen.

In the production of indigo from phenylglycine orthocarboxylic acid the "water reaction" plays no part, the oxygen coming solely from the air.

It seems highly probable that the yield of β-naphthol would be increased by carrying out the fusion of the corresponding sulphonate with sodium hydroxide in the absence of free oxygen.

THE ADSORPTION OF ARSENIOUS ACID BY FERRIC HYDROXIDE.

Reprinted from the Journal of the American Chemical Society, Vol. XL.

BY MAITLAND C. BOSWELL, M.A.; Ph.D. AND J. V. DICKSON, B.A.Sc.

In the course of the investigation of the catalysis of certain reactions by silicic acid and ferric hydroxide gels, the large adsorption of arsenious acid from solutions of sodium arsenite by ferric hydroxide was encountered.

Bunsen and Berthold[1] observed the great adsorption power of ferric hydroxide for arsenious acid, and considered the combination to be a basic ferric arsenite. Biltz[2] studied the reaction quantitatively and concluded that it was a case of adsorption and not chemical combination, since it followed the "adsorption law" expressed by the empirical and very flexible equation $E=\beta A^p$, where E is the concentration of adsorbed substance in the absorbing phase, A is the concentration of absorbed substance in solution at equilibrium, and β and p are constants. Lockmann and Paucke[3], and Lockmann and Lucius[4] studied the conditions under which the adsorption of arsenious acid from solution is complete. A comparison made in the latter paper shows that a given amount of arsenious acid is removed from solution by a considerably smaller amount of precipitate, when an exactly stoichiometric amount of ammonia is used than when twice that amount is used. They also observed that the adsorption of arsenious acid is considerably reduced by the use of potassium hydroxide or sodium hydroxide instead of ammonia. Mecklenburg[5] worked with ferric hydroxide and arsenious acid in demonstrating his principle of "affine Adsorptionskurven", *i.e.*, adsorption curves so related, each by a particular factor, to a unit curve, that the ordinate corresponding to any given abscissa on one of the curves may be obtained by multiplying, by the factor, the ordinate on the unit curve corresponding to the given abscissa. He showed that the adsorption curves obtained from differently prepared samples of hydrated ferric oxide were actually related in such a way to a unit curve, which he at first determined for one particular preparation.

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[1] "Das Eisenhydroxyd," 1834.

[2] *Ber.*, 37, 3138 (1904).

[3] *Z. Chem. Ind. Kolloide*, 8, 273 (1911).

[4] *Z. Physik. Chem.*, 83, 735 (1913).

[5] *Ibid.*, 83, 609 (1913).

We have studied (as far as seemed profitable for the purposes of our main investigation) the effects of sodium hydroxide and arsenious acid on the adsorption of each other by ferric hydroxide.

DESCRIPTION OF EXPERIMENTS.

The ferric hydroxide was prepared several months before the experiments by precipitation of ferric chloride solution with ammonium hydroxide. It was washed a great many times with tap water by decantation. Ammonium hydroxide was again added, and the gel again washed until the liquid gave only a very slight reaction for ammonia with Nessler's solution. The gel covered with water was kept so that it could be shaken up and poured.

The arsenious acid solutions were made by dissolving pure arsenious oxide in water. The normalities were determined by titration against standard iodine solution in the presence of sodium hydrogen carbonate.

The required amounts of arsenious acid and sodium hydroxide solution were mixed and made up to about 150 c.c. in a 200 c.c. standard flask. 31.5 c.c. of the ferric hydroxide suspension carefully measured in a cylinder was added and the flask filled with water to the mark. Several determinations showed that this amount of ferric hydroxide contained 2.13 g. Fe_2O_3. To each flask of Series A was added one c.c. of concn. sodium chloride solution and of Series B, 5 c.c. of sodium chloride solution. These two series were allowed to stand for about two weeks with occasional shaking. When equilibrium had been attained (determined by titration of small portions withdrawn) the arsenious acid remaining was determined. The difference between the amount originally put in and the amount remaining in solution gave the amount adsorbed.

Series C and D were prepared similarly, but were allowed to stand a month with shaking every day or two. They were then allowed to stand for about 10 days to settle. In these two series although no sodium chloride was used, there was not very much difference in turbidity from A and B. In a few of the more concentrated solutions the turbidity was great enough to make the methyl orange end-point rather uncertain but the arsenite-iodine titration could be carried out without any serious difficulty. In those experiments where both sodium hydroxide and arsenious acid were used they were determined in the same sample, the alkali being titrated first.

Series A gives the adsorption of arsenious oxide in the absence of alkali. Series B gives the adsorption of arsenious oxide and of sodium hydroxide in the presence of varying amounts of sodium hydroxide and a constant amount of arsenious oxide. These results are plotted in Fig. 1. Series C gives the adsorption of sodium hydroxide in the absence of arsenious oxide. Series D gives the adsorption of arsenious oxide and

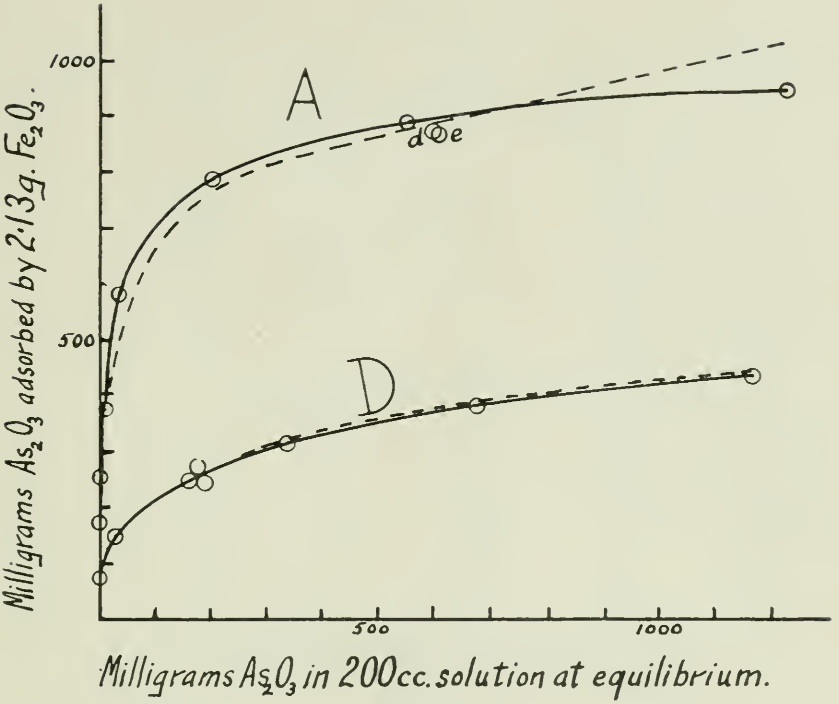
Milligrams As_2O_3 adsorbed by 2·13g. Fe_2O_3.
1000
500
A
D
d
e
500
1000
Milligrams As_2O_3 in 200cc. solution at equilibrium.

FIG. 1.

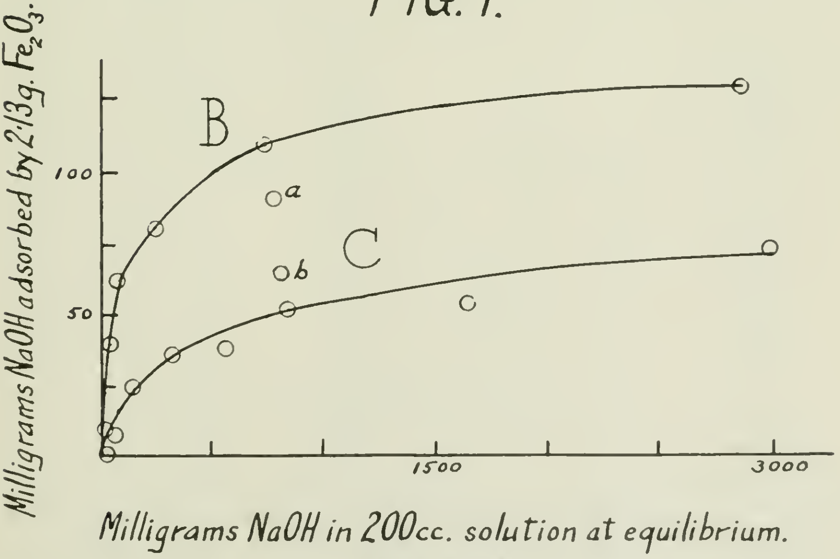
Milligrams NaOH adsorbed by 2·13g. Fe_2O_3.
100
50
B
C
a
b
1500
3000
Milligrams NaOH in 200cc. solution at equilibrium.

FIG. 2.

TABLE I.

Adsorption by Ferric Hydroxide.

Total As_2O_3	As_2O_3 adsorbed	As_2O_3 remaining	Total NaOH	NaOH adsorbed	NaOH remaining
Series A.—Pure Arsenious Oxide.					
73.5	73.5	0	..	..	..
172.1	172.1	0	..	..	..
246.4	244	2.38	..	..	..
384.7	378	6.73	..	..	..
393.1	384	9.1	..	..	..
614.1	578	36.1	..	..	..
987.5	784	203.5	..	..	..
1424.0	878	546	..	..	..
2175.0	955	1220	..	..	..
Series B.—Arsenious Oxide Constant.					
443.5	431	12.5	4	..	13
443.2	436	7.2	25.6	9.6	16
435.5	423	12.5	72	40	32
423.5	368	55.5	138	62	76
413	314	99	307	80	227
414	279	135	848	110	738
364	261	103	868	130	738
362	259	103	865	105	760
418	236	182	3010	130	2880
Series C.—Pure Sodium Hydroxide.					
..	..	..	25.24	0.24	25
..	..	..	75	8.0	67
..	..	..	149.8	24.8	125
..	..	..	349.4	36.4	313
..	..	..	568.8	38.8	530
..	..	..	874.4	52.4	822
..	..	..	1749	56.1	1693
..	..	..	3065	72.1	2993
Series D.—Sodium Hydroxide Constant.					
79.5	75	4.5	876	69	806
173.2	149	24.2	876	73	804
396	250	146	876	76	800
643	309	334	876	92	784
1049	381	668	876	88	789
1599	435	1164	876	104	773
449	250	199	1749	116	1633
450	233	217	3064	124	2940
Series E.—Effect of Sodium Chloride andAge.					
(*a*) 424.5	266.5	158	878	83	795
(*b*) 418	251.3	166.7	874	64	810
(*c*) 434.4	416	18.4	76.8	37.2	39.6
(*d*) 1456	846	610	..	..	..
(*e*) 1454	851	604	..	..	..

All values are expressed in milligrams and refer to 200 c.c. solution and 2.13 g. Fe_2O_3.

5 c.c. saturated sodium chloride in (*a*) and (*c*).

1 c.c. saturated sodium chloride in (*d*).

TABLE II.

Comparison of Observed with Calculated Values of E and Log E.

A	$x=\log A$	E obs.	E calc.	$y=\log E$ obs.	$y=\log E$ calc.
		Series A—($\beta=127.7$, $p=0.2102$).			
0	..	34.5	..	1.5378	..
0	..	80.8	..	1.9074	..
1.19	0.0755	114.6	132	2.0582	2.122
3.37	0.5276	177.5	165	2.2492	2.217
4.6	0.6628	180	176	2.2553	2.2455
18.05	1.2565	271.4	235	2.4436	2.370
101.8	2.0076	368	359	2.5658	2.555
273	2.4362	412	415	2.6149	2.6184
610	2.7853	448	492	2.6513	2.6917
		Series D—($\beta=33.26$, $p=0.2840$).			
2.25	0.3522	35.2	43.6	1.5465	1.6399
12.1	1.0828	70	70.4	1.8451	1.8474
73	1.8633	117.3	117.1	2.0693	2.0689
167	2.2227	145	148.2	2.1614	2.1709
334	2.5237	179	180.5	2.2529	2.2565
582	2.7649	204	211.3	2.3096	2.3251

TABLE III.

Values of the "Mecklenburg Factor" for Series A and D.

100 c.c. of solution contains x mg. of As_2O_3	1 g. Fe_2O_3 holds y mg. As_2O_3	Corresponding y value of M's unit curve	"Factor"
	Series A.		
0	34.5	..	..
0	80.8	..	..
1.19	114.6	165	0.695
3.37	177.5	237	0.746
4.6	180	264	0.682
18.05	271.4	356	0.752
101.8	368	474	0.776
273	412	548	0.752
610	448	600	0.746
	Series D.		
2.25	35.2	199.9	0.170
12.1	70.0	325.6	0.215
73	117.3	461.1	0.254
167	145	521.2	0.278
334	179	556.9	0.322
582	204	597	0.342

of sodium hydroxide in the presence of varying amounts of arsenious oxide and a constant amount of sodium hydroxide. These results are plotted in Fig. 2. Series C and D were performed about a month later than A and B. Following these, Series E was performed in order to determine whether the addition of sodium chloride had any effect upon the adsorption of arsenious acid or sodium hydroxide and also to determine whether these adsorptions were influenced at all by the somewhat greater age of the precipitate.

DISCUSSION.

It is seen from Fig. 1, and Table I, Series A and D, that the adsorption of arsenious acid by ferric hydroxide is very considerably diminished by sodium hydroxide. Figs. 2 and 4 and Table I, Series B and C, show that the adsorption of sodium hydroxide by ferric hydroxide is somewhat increased by the simultaneous presence of arsenious acid.

Expts. *a*, *b*, *d*, *e*, in series E (points *a*, *b*, *d*, *e*, Fig. 1) were performed for the purpose of determining whether the wide differences in amounts of arsenious oxide adsorbed in the presence and in the absence of sodium hydroxide is due to the influence of sodium hydroxide or to the fact that the experiments were performed at an interval of a month, and that slightly different amounts of sodium chloride were used to diminish turbidity. The results show that the amounts of arsenious oxide adsorbed by ferric hydroxide differed only slightly from those obtained in similar experiments performed with the same preparation of ferric hydroxide some two months previously. It seems then that conditions of precipitation are of very much greater influence on the adsorption than is the age of the precipitate. It is also evident that the effect of the salt used in these experiments upon the adsorption is small.

Expts. *a*, *b*, *c*, Series E (points *a*, *b*, *c*, Fig 2), performed for the similar purpose of verifying the results of Series B and C, Table I, on the adsorption of sodium hydroxide, do not definitely eliminate the somewhat greater age of the precipitate as a factor in the greater adsorption of sodium hydroxide in the presence of arsenious acid, points *a* and *b*, Fig. 2, falling midway between the two curves. However, Fig. 4, which refers to Series B and represents the variation of sodium hydroxide adsorbed with gradually increasing amounts of arsenious oxide present, makes it fairly certain that the general statement is correct that the adsorption of sodium hydroxide is increased by the presence of arsenious oxide.

Consider the general adsorption equation $E=\beta A^p$. In the present case, E is the concentration of arsenious oxide on the precipitate, A is the concentration of arsenious oxide in solution, and β and p are constants. The corresponding logarithmic equation

$$\log E=p \log A+B \text{ or } y=px+B$$

represents a straight line. If the adsorption follows the equation, the experimental values when plotted will give a straight line, the tangent of whose angle with the x axis is the constant p, and whose intercept on the y axis is the value of B. Fig. 2 shows that the plotted values

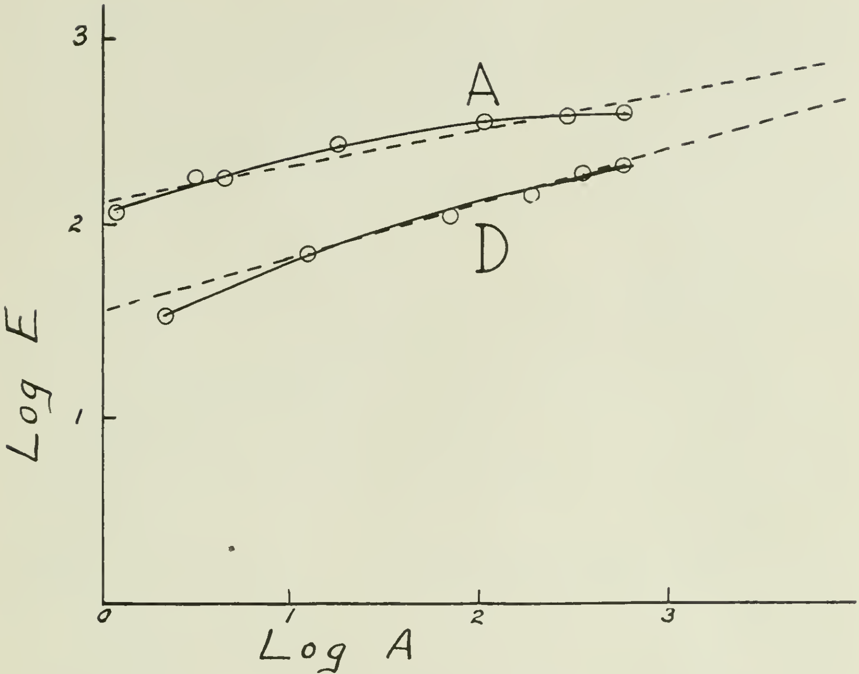
3
2
1
Log E
A
D
0
1
2
3
Log A

FIG. 3.

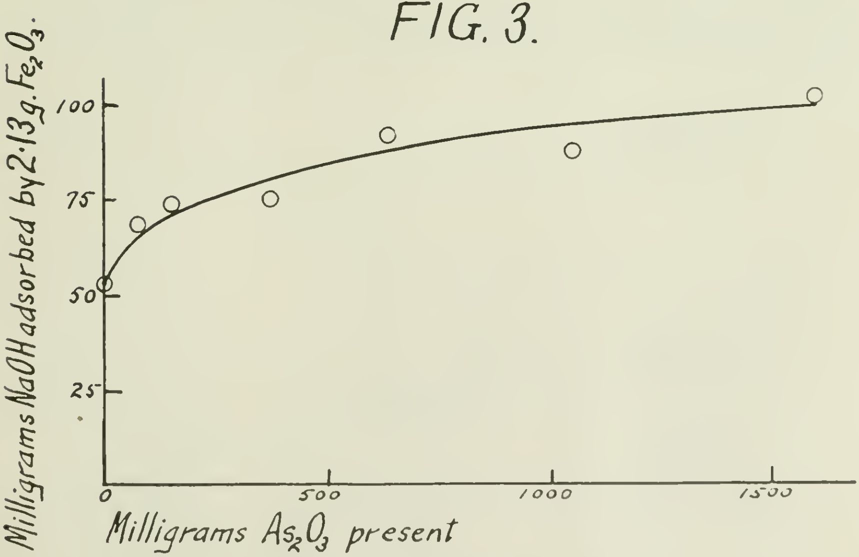
Milligrams NaOH adsorbed by 2·13 g. Fe_2O_3.
100
75
50
25
0
500
1000
1500
Milligrams As_2O_3 present

FIG. 4.

approximate to straight lines. The constants p and β were calculated, as above described, and the corresponding values of A and E (for 1 gram ferric oxide and 100 c.c. solution) were calculated for the several experiments. These calculated values along with the actual measurements are recorded in Table II. The curves obtained from these calculated values (E being multiplied by 2.13 and A multiplied by 2, corresponding with 2.13 g. and 200 c.c. solution) are compared in Fig. 1 with the experimental curves.

A comparison of the curve for Series A with Mecklenburg's "affine Adsorptionskurven" shows a close agreement, the Mecklenburg "factor" for our preparation being about 0.75 or midway between the factors obtained by Mecklenburg for preparations precipitated at 60° and 98°. That our factor is not higher than 0.75 is probably due to the fact that the gel was comparatively old and probably had as a consequence become partially dehydrated, and partially lost its original amorphous condition. However it is seen from Table III that the curve for series D (adsorption of arsenious oxide in presence of sodium hydroxide) does not correspond at all with Mecklenburg's curve but that the "factor" increases steadily to twice its initial value. This observation corresponds to the fact that the exponent constant p is different for the two curves. Had D alone been different, p remaining the same, the curves would have been "affine". This may be seen from the equation $E_1=\beta_1 A_1{}^p$ and $E_2=\beta_2 A_2{}^p$. If $A_1=A_2$, then $E_1 : E_2=\beta_1 : \beta_2$ and the fraction β_1/β_2 will be the Mecklenburg "factor" of one curve with respect to the other.

TABLE IV.

Constants of the Equation $E=\beta A^p$ for Ferric Hydroxide.

Series	Precipitation temperature	p	β
A	..	0.210	128
D	..	0.284	33.3
Mecklenburg's	1°	0.183	195
Mecklenburg's	11-12°	0.186	184
Mecklenburg's	23°	0.197	184
Mecklenburg's	45°	0.186	180
Mecklenburg's	51°	0.195	170
Mecklenburg's	60°	0.187	170
Mecklenburg's	98°	0.195	120
(Mecklenburg) unit curve	..	0.188	200
(Mecklenburg) air dried	98°	0.195	43
Biltz	..	0.198	170

From this it is seen that the only curve whose p value deviates very much from the average is that of Series D, which is the series where a constant amount of alkali was used.

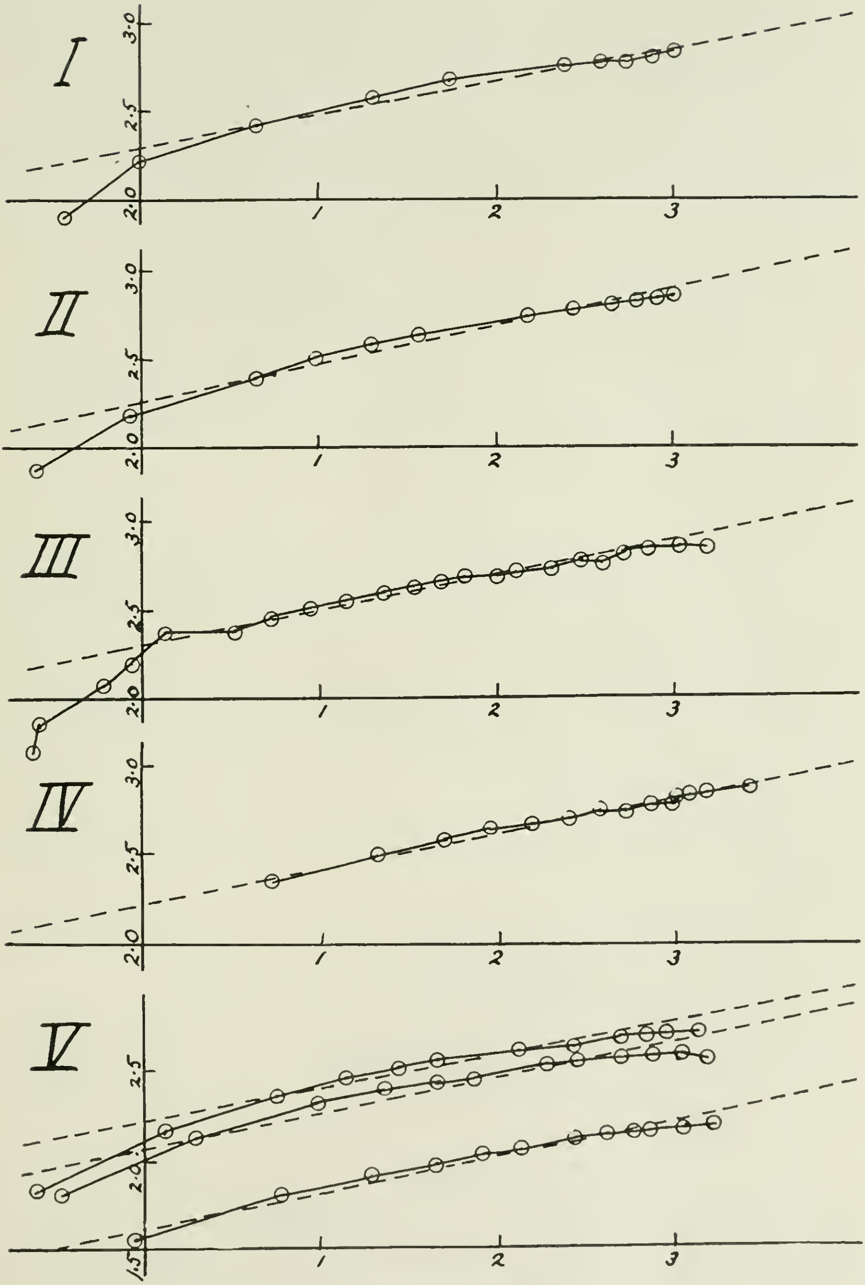
I
3.0
2.5
2.0
1
2
3
II
3.0
2.5
2.0
1
2
3
III
3.0
2.5
2.0
1
2
3
IV
3.0
2.5
2.0
1
2
3
V
2.5
2.0
1.5
1
2
3

FIG. 5.

In each of the 5 diagrams the vertical axis represents log E, *i.e.*, the logarithm of the concentration (milligrams per 1 g. Fe_2O_3) of the arsenious acid in the solid phase at equilibrium; the horizontal axis represents log A, *i.e.*, the logarithm of the concentration (milligrams per 100 c.c.) of arsenious acid in the liquid phase at equilibrium. The dotted line in each case indicates the straight line to which the curve approximates and from which the constants β and p were determined as already described.

Diagram I is for Mecklenburg's gel precipitated at 1°.
" II " " " " " " 23°.
" III " " " "unit curve".
" IV " " Biltz curve.

In " V the two upper curves are for Mecklenburg's gels precipitated at 60° and 98°, respectively, and the lowest curve is for the gel precipitated at 98° and heated subsequently.

SUMMARY.

The curves of the adsorption of arsenious acid by ferric hydroxide in the presence and absence of sodium hydroxide have been constructed. The adsorption of arsenious acid has been found to be diminished by the presence of sodium hydroxide.

The curves of adsorption of sodium hydroxide by ferric hydroxide in the presence and absence of arsenious acid have been constructed. The adsorption of sodium hydroxide has been found to be increased by the presence of arsenious acid.

Our measurements as well as those of Mecklenburg and Biltz indicate a deviation of the adsorption phenomenon from the simple adsorption equation.

An interpretation of the constants p and β in the adsorption equation $E=\beta A^p$ is given and the connection between these constants and Mecklenburg's "affine Adsorptionskurven" is pointed out.

Mecklenburg's generalization is thus equivalent to saying that where the adsorption of a compound from solution follows the equation $E=\beta A^p$, p is constant for the same adsorbing material, and that an adsorbent prepared under various conditions, or of various ages will vary only in the values of β.

The logarithmic curves plotted from our measurements although approximating to straight lines yet show unmistakably a concavity to the x axis, thus indicating a deviation of the adsorption phenomenon from the adsorption equation. In view of this fact it became desirable, especially since Mecklenburg makes no mention of the adsorption equation, to plot the corresponding logarithmic curves for Mecklenburg's results. Some of these as well as the curve for Biltz results[1] are shown in Fig. 5. In every case this same deviation from a straight line, and concavity to the x axis are found. There seems to be no doubt of the existence of a definite deviation from exact conformation with the equation. It would seem that the phenomenon of adsorption by gels from aqueous solutions has not the simplicity implied in the equation $E=\beta A^p$. What the theoretical significance of this deviation may be is not apparent. The equation however still serves as a simple approximation to the course of the adsorption.

That the exponent p remains constant for different preparations of ferric hydroxide where arsenious oxide alone is used, but changes when alkali is present, is shown in Table IV where the values of p and β have been calculated as already described.

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[1] *Ber.*, 37, 3138 (1904).

THE CALCULATION OF TRANSMISSION LINE NETWORKS.

BY T. R. ROSEBRUGH, M.A.

On account of the growing size and complexity of transmission line networks it appears desirable to develop a systematic method of calculation for such systems.

An illustration of present day conditions is the network of the Hydro Electric Power Commission of Ontario as described by A. H. Hull[1].

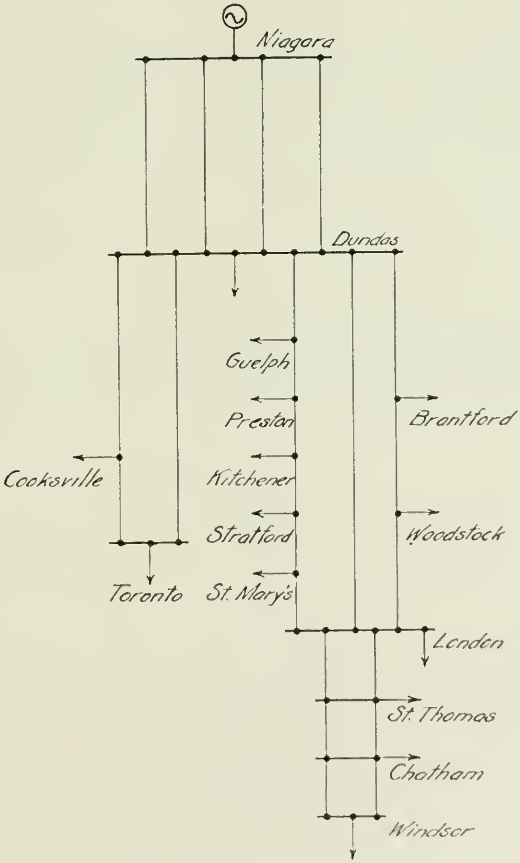

FIG. 1.

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[1] Arthur H. Hull. Electric Power Generation in Ontario on Systems of Hydro-electric Power Commission. Proc. A.I.E.E. Jan. 1919.

Fig. 1 is a diagram of the 110,000 volt lines of the Niagara system. An examination shows some of the elements which go to make up such a network of transmission lines. We find (*a*) branches with terminal load only; (*b*) branches with terminal and intermediate loads; (*c*) parallel transmission lines without intermediate loads; (*d*) looped transmission lines with several load-points. Combinations of looped transmission lines with extended branches also occur.

It is not the intention to repeat in this paper the well known methods used for calculating the simple transmission line, limited to the steady state, and to sine-wave voltages and currents, but rather to start from the results in the simplest case, and build up a general method applicable to this and also to larger and more complex networks with the same limitations.

For convenience we may assume throughout that the problem is that of three-phase transmission reduced to single-phase by any conventional method that may be preferred. This done, the calculations are to be carried out by means of complex quantities. As this involves a great amount of arithmetical work a system of checks on its accuracy is desirable and such a system will be described.

I. UNIFORM LINE.

This case comprises the usual transmission line but here regarded rather as an element of a network whether as a terminal branch or an internal element. Suppose we have a branch as in Fig. 2 extending from M where it leaves the main line to N where the load is located and the branch terminates. The circuit is represented by one instead of three conductors for simplicity. This is just the ordinary transmission line problem and, as is well known,

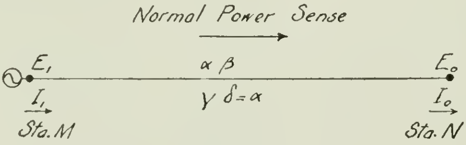

FIG. 2.

$$E_1=\alpha E_o+\beta I_o \tag{1}$$
$$I_1=\gamma E_o+\delta I_o \tag{2}$$

where[1]

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[1] The introduction of the fourth constant δ which is unnecessary here is for the purpose of making the formulae general enough to cover the case of lines in tandem.

$$\alpha=\delta=\cosh\sqrt{YZ} \tag{3}$$

$$\beta=\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ} \tag{4}$$

$$\gamma=\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} \tag{5}$$

or if preferred

$$\alpha=\delta=1+\frac{YZ}{\underline{\lvert 2}}+\frac{Y^2Z^2}{\underline{\lvert 4}}+ \tag{6}$$

$$\beta=Z+\frac{Z^2Y}{\underline{\lvert 3}}+\frac{Z^3Y^2}{\underline{\lvert 5}}+ \tag{7}$$

$$\gamma=Y+\frac{Y^2Z}{\underline{\lvert 3}}+\frac{Y^3Z^2}{\underline{\lvert 5}}+ \tag{8}$$

For every branch and every section of line between loads there will be a set α, β, γ, δ of complex quantities which should be calculated and checked by the relation

$$\alpha\delta-\beta\gamma=1 \tag{9}$$

where it may be noticed that $\alpha\delta-\beta\gamma$ is just the determinant

$$\begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$$

in which α, β, γ and δ occupy the same relative positions as in (1) and (2).

Equations (1) and (2) being two relations between four quantities E_o, I_o, E_1 and I_1 it is only necessary to know two to fix all four. Thus from (1), (2) and (9)

$$E_o=\delta E_1-\beta I_1 \tag{10}$$

$$I_o=-\gamma E_1+\alpha I_1 \tag{11}$$

which express conditions at the load-end of a section in terms of those at the end towards the source.

Also

$$I_o=\frac{1}{\beta}E_1-\frac{\alpha}{\beta}E_o=\epsilon E_1-\alpha\epsilon E_o \tag{12}$$

$$I_1=\frac{\delta}{\beta}E_1-\frac{1}{\beta}E_o=\delta\epsilon E_1-\epsilon E_o \tag{13}$$

give the currents at the two ends of the section in terms of the voltages at the two ends; in which as an abbreviation

$$\epsilon=\frac{1}{\beta} \tag{14}$$

Similarly

$$E_o=\frac{1}{\gamma}I_1-\frac{\delta}{\gamma}I_o \tag{15}$$

$$E_1 = \frac{\alpha}{\gamma} I_1 - \frac{1}{\gamma} I_o \tag{16}$$

$$I_1 = \frac{\gamma}{\alpha} E_1 + \frac{1}{\alpha} I_o \tag{17}$$

$$E_o = \frac{1}{\alpha} E_1 - \frac{\beta}{\alpha} I_o \tag{18}$$

$$E_1 = \frac{1}{\delta} E_o + \frac{\beta}{\delta} I_1 \tag{19}$$

$$I_o = \frac{1}{\delta} I_1 - \frac{\gamma}{\delta} E_o \tag{20}$$

When the load at the end of the line is zero

$$E_1 = \alpha E_o \tag{21}$$

$$I_1 = \gamma E_o \tag{22}$$

$$I_1 = \frac{\gamma}{\alpha} E_1 \tag{23}$$

or the effective admittance of the unloaded line is

$$y = \frac{\gamma}{\alpha} \tag{24}$$

As Y and Z depend on conductor size, material and spacing as well as length, it follows that α, β, γ and δ will not usually be the same even for the same length in different parts of the network.

II. PARALLEL TRANSMISSION LINES WITHOUT INTERMEDIATE LOADS.

Suppose that between two stations B and A, there are n simple uniform transmission lines, connected in parallel at both stations and none of

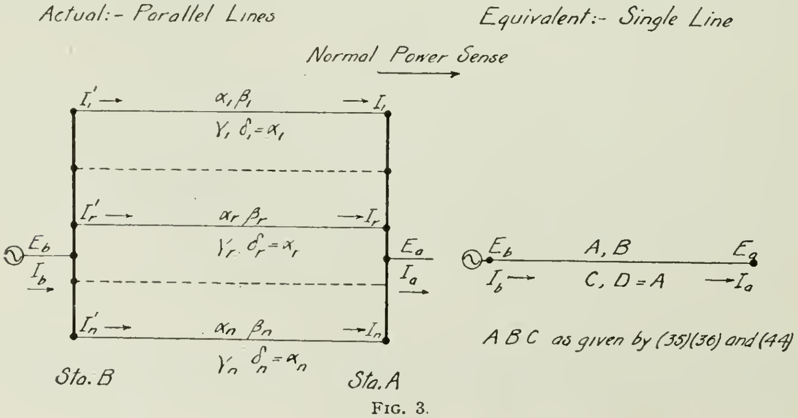


FIG. 3.

which have intermediate loads, and that their constants may be different. Such a condition is found, for instance, between Niagara Falls and Dundas on the H.E.P. Commission's transmission network.

Referred as usual to one set of conductors Fig. 3 shows the arrangement and the quantities involved. This may be reduced to the equivalent single transmission line as follows.

We have for any line say the rth

$$\alpha_r E_a + \beta_r I_r = E_b \tag{25}$$

and

$$I'_r = \gamma_r E_a + \delta_r I_r \tag{26}$$

For all together

$$I_1 + + I_r + + I_n \text{ or say } \Sigma I = I_a \tag{27}$$

and

$$I'_1 + + I'_r + + I'_n \text{ or say } \Sigma I' = I_b \tag{28}$$

Now from (25) and as in (12)

$$I_r = \frac{1}{\beta_r} E_b - \frac{\alpha_r}{\beta_r} E_a \tag{29}$$

thence from (27)

$$I_a = \Sigma I = E_b \Sigma \frac{1}{\beta} - E_a \Sigma \frac{\alpha}{\beta} \tag{30}$$

so that

$$E_b = E_a \Sigma \frac{\alpha}{\beta} \div \Sigma \frac{1}{\beta} + I_a \div \Sigma \frac{1}{\beta} \tag{31}$$

For convenience let

$$\epsilon_r = \frac{1}{\beta_r} \tag{32}$$

then

$$E_b = E_a \Sigma \alpha\epsilon \div \Sigma\epsilon + I_a \div \Sigma\epsilon \tag{33}$$

that is

$$E_b = AE_a + BI_a \tag{34}$$

where

$$A = \Sigma\alpha\epsilon \div \Sigma\epsilon \tag{35}$$

and $\quad B = 1 \div \Sigma\epsilon \quad$ (36)

From (26), (28) and (29) $\quad I_b = E_a \Sigma\gamma + \Sigma\delta_r I_r \quad$ (37)

thus from (25) $\quad I_b = E_a \Sigma\gamma + E_b \Sigma\delta\epsilon - E_a \Sigma\alpha\delta\epsilon$

$$= E_a(\Sigma\gamma - \Sigma\alpha\delta\epsilon) + \Sigma\delta\epsilon(E_a \Sigma\alpha\epsilon \div \Sigma\epsilon + I_a \div \Sigma\epsilon)$$

$$= E_a(\Sigma\gamma - \Sigma\alpha\delta\epsilon + \Sigma\alpha\epsilon\Sigma\delta\epsilon \div \Sigma\epsilon) + I_a \Sigma\delta\epsilon \div \Sigma\epsilon \tag{38}$$

That is $\quad I_b = CE_a + DI_a \quad$ (39)

where $\quad C = \Sigma\gamma - \Sigma\alpha\delta\epsilon + \Sigma\alpha\epsilon\Sigma\delta\epsilon \div \Sigma\epsilon \quad$ (40)

and $\quad D = \Sigma\delta\epsilon \div \Sigma\epsilon \quad$ (41)

Now since $\quad \alpha_r \delta_r \epsilon_r = (1 + \beta_r \gamma_r)\epsilon_r = \epsilon_r + \gamma_r \quad$ (42)

on substituting in (39) $\quad C = \Sigma\alpha\epsilon\Sigma\delta\epsilon \div \Sigma\epsilon - \Sigma\epsilon \quad$ (43)

The last result is more convenient for analytical purposes but for arithmetical work it is preferable to write

$$C=\Sigma\gamma-\Sigma(\alpha-1)\,(\delta-1)\epsilon+\Sigma(\alpha-1)\epsilon\Sigma(\delta-1)\epsilon\div\Sigma\epsilon \qquad (44)$$

This is easily deduced from (40).

The comparison of (34) *and* (39) *with* (1) *and* (2) *respectively shows that* A, B, C *and* D *play the same part for this parallel group of transmission lines as* α, β, γ *and* δ *do for a single line.* Also under the conditions of this Section, though not of Section IV (for which in general $\delta_r \neq \alpha_r$) $D=A$ for $\delta_r=\alpha_r$. This is seen by comparing (35) with (41).

We have besides $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD-BC=1 \qquad (45)$

as may be easily verified from the given expressions for A, B, C and D. This will serve as a very useful check on the arithmetic in any particular case and may be compared with the analogous equation (9) for the single transmission line.

It follows that for the parallel group of transmission lines, not subject to mutual induction, a set of relations hold analogous to the equations respectively (10) to (24) inclusive, and by which when any two of the four quantities: input and output voltage, total input current and total output current, are given, the other two may be found. That is, in addition to (34) and (39) and deduced from them and (45)

$$E_a=DE_b-BI_b \qquad (46)$$

$$I_a=-CE_b+AI_b \qquad (47)$$

which express conditions at the load-end of a parallel group of transmission lines, in terms of those at the end towards the source. Also

$$I_a=\frac{1}{B}E_b-\frac{A}{B}E_a \qquad (48)$$

$$I_b=\frac{D}{B}E_b-\frac{1}{B}E_a \qquad (49)$$

give the total currents at the two ends in terms of the voltages at the ends. Similarly

$$E_a=\frac{1}{C}I_b-\frac{D}{C}I_a \qquad (50)$$

$$E_b=\frac{A}{C}I_b-\frac{1}{C}I_a \qquad (51)$$

$$I_b=\frac{C}{A}E_b+\frac{1}{A}I_a \qquad (52)$$

$$E_a=\frac{1}{A}E_b-\frac{B}{A}I_a \qquad (53)$$

$$E_b=\frac{1}{D}E_a+\frac{B}{D}I_b \qquad (54)$$

$$I_a = \frac{1}{D} I_b - \frac{C}{D} E_a \tag{55}$$

and when the load at the end of the parallel lines is zero

$$E_b = AE_a \tag{56}$$

$$I_b = CE_a \tag{57}$$

$$I_b = \frac{C}{A} E_b \tag{58}$$

or the effective admittance of the group is

$$y = \frac{C}{A} \tag{59}$$

As to the individual transmission lines, the share of the current at the end towards the load is given by (29) and (34), viz.,

$$I_r = \frac{1}{\beta_r} E_b - \frac{\alpha_r}{\beta_r} E_a = (A - \alpha_r)\,\epsilon_r \,.\, E_a + B\epsilon_r \,.\, I_a \tag{60}$$

while that at the other end as found from (9), (26), (29) and (34) or from (13) is

$$I'_r = \frac{\delta_r}{\beta_r} E_b - \frac{1}{\beta_r} E_a = (A\delta_r - 1)\epsilon_r E_a + B\delta_r \epsilon_r I_a \tag{61}$$

Thus there will be local currents entering and leaving each individual line of a parallel group when unloaded, of the values respectively

$$I_r = (A - \alpha_r)\epsilon_r E_a = \frac{(A - \alpha_r)\epsilon_r}{A} E_b \tag{62}$$

and

$$I'_r = (A\delta_r - 1)\epsilon_r E_a = \frac{(A\delta_r - 1)\epsilon_r}{A} E_b \tag{63}$$

It seems from the results thus obtained that A is the weighted mean of the α's using their ϵ's as weights, D is the weighted mean of the δ's using their ϵ's as weights, while B is the impedance obtained by regarding the β's as impedances connected in parallel. There does not appear to be any obvious interpretation of C.

Taking the simplest instance of this case where all n lines have identical properties, it follows from (35), (36), (40) and (41) that $A = \alpha$,

$$B = \frac{\beta}{n}\,, \quad C = n\gamma, \quad D = \delta. \tag{64}$$

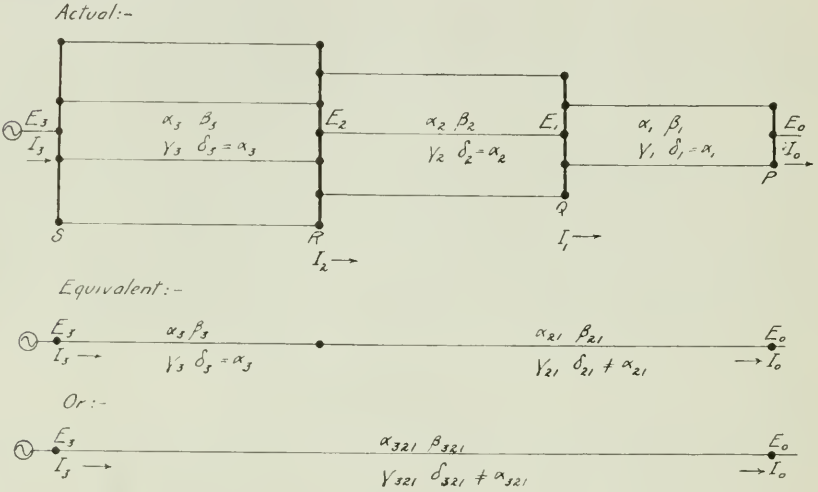
These results check with those obtained by substituting nY and $\frac{Z}{n}$ respectively in (3), (4) and (5). It should of course be understood that these transmission lines are sufficiently far apart, or so transposed, that there is no interaction throughout their length.

Conclusion: In the steady state under sine-wave conditions parallel, mutually non-inductive, uniform transmission lines without intermediate loads are structurally equivalent, as part of a network, to a single uniform line whose constants may be calculated and checked as explained above.

The subdivision, between lines, of any common terminal load, zero or other, is determined for each end of the parallel group of lines.

III. TRANSMISSION LINES IN TANDEM.

Suppose *SRQP* as in Fig. 4 to be a transmission line without intermediate loads made up of several sections in tandem, of different character, whether due to change of spacing, conductor size or material, etc., or to some or all of the sections being parallel groups whose constants as a single line have already been calculated by the method of Section II, and are here denoted by α, β, γ, δ instead of A, B, C and D.



Actual:-
E_3
I_3
$\alpha_3\ \beta_3$
$\gamma_3\ \delta_3=\alpha_3$
E_2
$\alpha_2\ \beta_2$
$\gamma_2\ \delta_2=\alpha_2$
E_1
$\alpha_1\ \beta_1$
$\gamma_1\ \delta_1=\alpha_1$
E_o
I_o
P
Q
S
R
I_2
I_1
Equivalent:-
E_3
I_3
$\alpha_3\ \beta_3$
$\gamma_3\ \delta_3=\alpha_3$
$\alpha_{21}\ \beta_{21}$
$\gamma_{21}\ \delta_{21}\neq\alpha_{21}$
E_o
I_o
Or:-
E_3
I_3
$\alpha_{321}\ \beta_{321}$
$\gamma_{321}\ \delta_{321}\neq\alpha_{321}$
E_o
I_o

FIG. 4.

The constants of the complete line *SP* may then be readily found by simple, arithmetical processes as follows:

We have
$$\left.\begin{aligned} E_2&=\alpha_2E_1+\beta_2I_1\\ I_2&=\gamma_2E_1+\delta_2I_1\end{aligned}\right\}$$

as in (1) and (2)

also
$$\left.\begin{aligned} E_1&=\alpha_1E_o+\beta_1I_o\\ I_1&=\gamma_1E_o+\delta_1I_o\end{aligned}\right\}$$

(Note that in the cases here considered $\delta_2=\alpha_2$, $\delta_1=\alpha_1$)

Hence
$$E_2=(\alpha_2\alpha_1+\beta_2\gamma_1)E_o+(\alpha_2\beta_1+\beta_2\delta_1)I_o \tag{65}$$
$$I_2=(\gamma_2\alpha_1+\delta_2\gamma_1)E_o+(\gamma_2\beta_1+\delta_2\delta_1)I_o \tag{66}$$

or
$$E_2=\alpha_{21}E_o+\beta_{21}I_o \tag{67}$$
$$I_2=\gamma_{21}E_o+\delta_{21}I_o \tag{68}$$

where
$$\alpha_{21}=\alpha_2\alpha_1+\beta_2\gamma_1 \tag{69}$$
$$\beta_{21}=\alpha_2\beta_1+\beta_2\delta_1 \tag{70}$$
$$\gamma_{21}=\gamma_2\alpha_1+\delta_2\gamma_1 \tag{71}$$
$$\delta_{21}=\gamma_2\beta_1+\delta_2\delta_1 \tag{72}$$

Since the determinant $\alpha_{21}\delta_{21}-\beta_{21}\gamma_{21}=1$ (73)
we have available the usual check on the accuracy of the arithmetic.

As the set of equations (67), (68) and (73) are identical in form with (1), (2) and (9) above there must be a further set of equations here analogous to (10) to (24) inclusive, and also to the equations (46) to (59) inclusive for parallel groups which arose in a similar manner. These are omitted for brevity as they can easily be written out by substituting α_{21} for α, β_{21} for β, etc. So that if a composite line consists of two uniform or "smooth lines" in tandem without intermediate load its behaviour is fully described by these equations.

By continuing the process for the next section we have

$$E_3=(\alpha_3\alpha_{21}+\beta_3\gamma_{21})E_o+(\alpha_3\beta_{21}+\beta_3\delta_{21})I_o \tag{74}$$

$$I_3=(\gamma_3\alpha_{21}+\alpha_3\gamma_{21})E_o+(\gamma_3\beta_{21}+\delta_3\delta_{21})I_o \tag{75}$$

or

$$E_3=\alpha_{321}E_o+\beta_{321}I_o \tag{76}$$

$$I_3=\gamma_{321}E_o+\delta_{321}I_o \tag{77}$$

where again the check is $\alpha_{321}\delta_{321}-\beta_{321}\gamma_{321}=1$ (78)

The situation is then exactly the same for three or more lines or groups of paralleled lines in tandem, as for two, and the detailed conclusions stated above for two may be taken as applying also to three or more.

The extension to any number of tandem units would mean continuing in the same way so that the labour required for n tandem units would be $8(n-1)$ multiplications and $4(n-1)$ additions of complex quantities.

Previous to the case of lines in tandem the cases examined had all involved three different complex-quantity constants only as α, β, γ; A, B, C; the fourth constant being always a repetition of the first, but in the case of two or more lines in tandem as seen from (67) and (68) and following equations the fourth constant is generally different from the first. In order to fit all cases to which the equations may be applied the four symbols α, β, γ, δ have been used from the beginning.

Conclusion: In the steady state under sine-wave conditions tandem transmission lines, or a non-uniform line, in either case without intermediate loads are structurally equivalent to an imaginary (four-constant) uniform line whose constants may be calculated and checked as explained.

IV. PARALLEL COMPOSITE LINES.

Whereas in Section II the parallelled lines were individually simple and uniform, in this case we shall consider the individual lines to be as general as in Section III, that is to say a tandem arrangement whose elements are not necessarily simple but may themselves be groups in parallel.

This problem may be handled by exactly the same method as where the items paralleled are simple transmission lines as considered in Section II, but a certain simplification was possible in that case which fails here, viz., all four constants are in general different here for each line paralleled, the exceptions being the cases of such simple uniform lines as may be paralleled with other tandem lines.

Let each of the transmission lines shown in Fig. 5 be connected in parallel at B and A, and suppose that some or all of these are lines in tandem, connected at such points as P, Q, R and S, but without loads anywhere between B and A, the other lines paralleled, if any, being simple uniform lines.

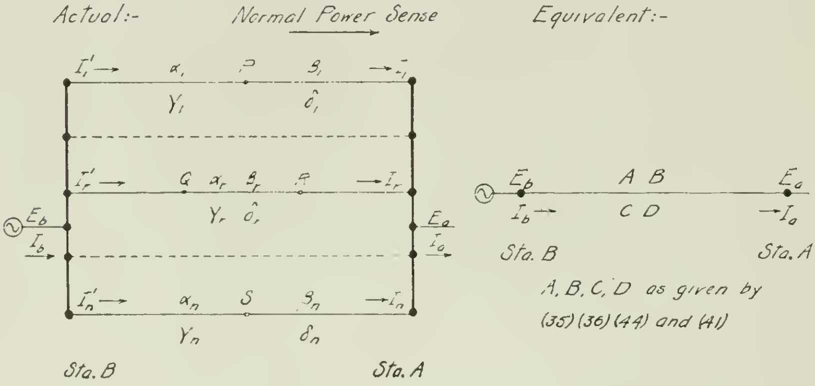


FIG. 5.

As general constants for these lines we may take α, β, γ, δ particularized by subscripts and whose values have been found by the method of Section III.

Then as the general conditions are all precisely of the same form as those of Section III, all the conclusions of section II hold here also.

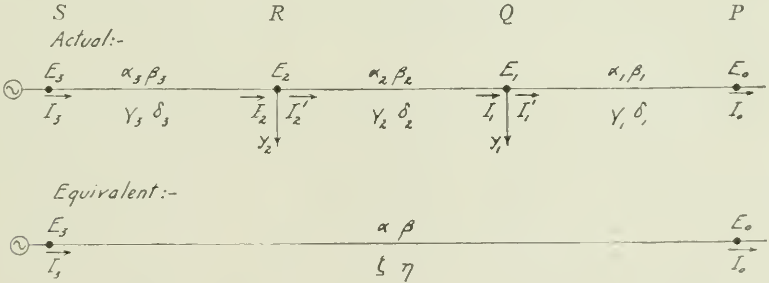
These relations in fact were purposely written with δ distinguished from α in order to make them applicable to this more general case. Instead of reprinting them the equations from (25) to (64) inclusive in Section II are to be considered as incorporated in the present section.

In the steady state under sine-wave conditions parallel, mutually non-inductive tandem transmission lines or parallel groups of individually uniform transmission line groups without intermediate loads are structurally equivalent as part of a network to a single line whose constants may be calculated and checked as explained above. The subdivision between lines of any common terminal load, zero or other, is determined for each end of the parallel group of lines.

Further, by combining the conclusions of this and the preceding two Sections it may be concluded that, for the steady state under sine-wave conditions, any combination of mutually non-inductive transmission lines without intermediate loads obtainable by any number of successive parallel or tandem groupings may be regarded as structurally equivalent in the network to a single line. If then the final grouping is parallel the equations of Section II will apply when the individual groups have been reduced to, and are regarded as, equivalent single lines.

V. INTERMEDIATE LOADS.

Suppose a line *SRQP* as in Fig. 6 to consist of parts *SR*, *RQ* and *QP*, which are each lines or groups of lines (tandem, parallel, etc.) and that they are loaded at the marked points in a manner defined by the admittances y_1 and y_2. The constants for the transmission line or group of lines between any two loads or junction points will be either three or four in number and may be calculated by the methods given above. Suppose this done in each case and the results denoted by the symbols shown in the figure.



$\alpha\ \beta\ \zeta\ \eta$ as in (85) and (86) similar to (75) and (76)

FIG. 6.

Then $\quad E_1 = \alpha_1 E_o + \beta_1 I_o \quad (79)$

Current just to right of *Q*, $\quad I' = \gamma_1 E_o + \delta_1 I_o \quad (80)$

Current just to left of *Q*, $\quad I_1 = \gamma_1 E_o + \delta_1 I_o + y_1 E_1$

$$= (\gamma_1 + \alpha_1 y_1) E_o + (\delta_1 + \beta_1 y_1) I_o$$

That is $\quad I_1 = \zeta_1 E_o + \eta_1 I_o \quad (81)$

where $\quad \zeta_1 = \gamma_1 + \alpha_1 y_1 \quad (82)$

and $\quad \eta_1 = \delta_1 + \beta_1 y_1 \quad (83)$

The check of the usual form on the numerical work is easily seen to be valid, viz.

$$\begin{vmatrix} \alpha_1 & \beta_1 \\ \zeta_1 & \eta_1 \end{vmatrix} = \alpha_1 \eta_1 - \beta_1 \zeta_1 = 1 \qquad (84)$$

That is, when an admittance on the transmitting side of a line section is taken into account together with the line itself α_1, β_1, γ_1, δ_1 are replaced by α_1, β_1, ζ_1, η_1 defined as above.

Thus the whole line *SP* may be treated by the methods of Section III on the basis of the constants $(\alpha_1\beta_1\zeta_1\eta_1)$, $(\alpha_2\beta_2\zeta_2\eta_2)$ and $(\alpha_3\beta_3\gamma_3\delta_3)$ for the respective sections. It will be seen that there is no admittance given on the transmitting side of the third section, hence the constants for this section of line remain unaltered.

Then in the same manner as for (76) and (77) of section III we find, using α for α_{321}, etc.

$$E_3 = \alpha E_o + \beta I_o \tag{85}$$

$$I_3 = \zeta E_o + \eta I_o \tag{86}$$

Here the check is
$$\begin{vmatrix} \alpha & \beta \\ \zeta & \eta \end{vmatrix} = \alpha\eta - \beta\zeta = 1 \tag{87}$$

The process may, of course, be continued step by step for any number of such sections, as in Section III. The "constants" will be found to be linear functions of each admittance when considered as separately variable, but products of admittances will occur.

In this case the line *SP* with intermediate loads given as admittances is found to act, when considered as a whole, like an undivided line with the four constants α, β, ζ, η.

Throughout Sections II, III and IV the limitation prevailed that the lines must not have intermediate loads. This limitation is now removed as shown above.

It is unfortunate that loads cannot be specified absolutely rather than by an admittance, that is to say, by giving the ratio of current to voltage as well as the phase angle. To give the load by a complex quantity *I* would, of course, mean that it was described in relation to the phase of voltage assumed standard at some one chosen point on the line and hence generally different from the phase of the voltage where the load actually is.

There seem to be substantial difficulties either way but perhaps less by assumed values of admittance than the other way. At any rate it deals with possible loads of defined power factor, though perhaps not the actual loads when the voltage becomes different from that for which they are assumed.

VI. MORE GENERAL IDEAS. MATRICES OF DETERMINANT UNITY REPRESENT TRANSMISSION LINES, ADMITTANCES, IMPEDANCES, TRANSFORMERS AND THEIR COMBINATIONS.

Throughout all the preceding sections once the constants α, β, γ, δ for the component transmission lines have been obtained, there are no mathematical ideas required beyond those of elementary algebra, and no arithmetical process is needed but the four simple rules: addition, subtraction, multiplication and division of complex quantities.

It will now be useful to introduce the more general ideas of algebraic transformations.

When as in (1) and (2) we write

$$\left.\begin{aligned} E_1 &= \alpha E_o + \beta I_o \\ I_1 &= \gamma E_o + \delta I_o \end{aligned}\right\} \qquad (88)$$

connecting E_1, I_1, with E_o, I_o, it is evident that we have to do with a "binary linear transformation".[1]

The "array" of the four constants or $\left\| \begin{matrix} \alpha & \beta \\ \gamma & \delta \end{matrix} \right\|$ regarded not as a quantity but merely as a formal display of the elements involved, is called the matrix of the transformation, and by the determinant (or resultant) of the transformation is meant the quantity Δ calculated as $\alpha\delta - \beta\gamma$ and written $\left| \begin{matrix} \alpha & \beta \\ \gamma & \delta \end{matrix} \right|$.

The transformation which reverses the effect of (88) or transforms E_1, I_1, back into E_o, I_o, is called its "inverse". This has been given already in (10) and (11), viz.,

$$\left.\begin{aligned} E_o &= \delta E_1 - \beta I_1 \\ I_o &= -\gamma E_1 + \alpha I_1 \end{aligned}\right\} \qquad (89)$$

It is found by solving the simultaneous equations of (88) for the unknowns E_o, I_o, and owes its simple form in relation to (88) to the determinant of the transformation being unity so that it does not show in the denominators of the elements of the inverse form.

In Section III we considered two transmission lines in tandem whose matrices are $\left\| \begin{matrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{matrix} \right\|$ and $\left\| \begin{matrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{matrix} \right\|$

Then the transformation for the two lines together which expresses E_2, I_2, in terms of E_o, I_o, has the matrix

$$\left\| \begin{matrix} \alpha_2\alpha_1 + \beta_2\gamma_1, & \alpha_2\beta_1 + \beta_2\delta_1 \\ \gamma_2\alpha_1 + \delta_2\gamma_1, & \gamma_2\beta_1 + \delta_2\delta_1 \end{matrix} \right\|$$

This is the product of the two matrices according to the rule "the element in the rth row and sth column of the product is formed by multiplying the elements of the rth row of the first matrix by the corresponding elements of the sth column of the second matrix and adding the products".[2]

By "first" and "second" we understand in this case "nearer to the source of power" and "next following section of line" respectively.

Thus the result of Section III may be expressed as follows: "when the matrices appertaining to tandem transmission lines are multiplied by the process of prefixing new factors which relate to transmission line

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[1] See Dickson, *Algebraic Invariants*, page 33.

[2] Dickson *Algebraic Invariants*, page 2.

Sections taken successively closer to the source of power, the continued product is the matrix of the entire line".

Now as the multiplication of matrices is known to be "associative" but "non-commutative" it follows that the multiplication may be carried out, if desired (when there are three or more factors) by taking factors together in any way which leaves the order unchanged. For instance, in the case of four factors denoted by p, q, r and s

$$pqrs = p(qrs) = (pq)(rs) = (pqr)s \qquad (90)$$

but not $pqrs = (rs)(pq)$, etc. For instance $(pq)(rs)$, denotes that the matrix product (pq) supposed already obtained is to be multiplied as first factor into (rs) as second, and not *vice versa*. This is just as in the case of quaternions.

When it is desired to determine E and I at a point farther (along the line) from the source of power in terms of conditions nearer to the source and separated by tandem lines, the "inverse" transformations like (89) are required and also in inverse order, *i.e.*, each new section of line gives successively a new prefix factor. That this order is inverse is indicated by the natural (inverted) order of the transformations in advancing from the known to the unknown when the inverse operations are used.

Analytically if p, q and r are the matrices for three sections of transmission lines, in order, advancing from the transmitting-end outwards and p^{-1}, q^{-1}, r^{-1} are the respective inverse transformations then $pqrr^{-1}q^{-1}p^{-1}$ is the "identity transformation" because of the successive coalescence to identity transformation of the central pairs. It means, in fact,

$$p\{q(rr^{-1})q^{-1}\}p^{-1}, \text{ but } pqrr^{-1}q^{-1}p^{-1} = (pqr)(r^{-1}q^{-1}p^{-1}) \qquad (91)$$

hence $r^{-1}q^{-1}p^{-1} = (pqr)^{-1}$ (92)

That is, to obtain the inverse to the transformation pqr not only must inverse transformations be used but inverted order also.

The system of checks described above for the cases discussed in Sections III and V, *i.e.*, tandem arrangements with or without intermediate loads arises as follows. The matrices concerned in Section III have their determinants as "invariants of index unity" and a law holds which may be symbolized as

$$\Delta PM = P\Delta M = P1 = 1 \qquad (93)$$

where M denotes matrix, P product and Δ determinant. This means that the determinant of the product of any number of matrices is the product of the determinants of each, and that as each of these determinants is unity the determinant of the matrix, that is the matrix-product for the entire composite line, is unity.

Again in Section V the transformation across the point Q in Fig. 6 is

$$\left.\begin{aligned} E_1 &= 1 \,.\, E_1 + 0 \,.\, I'_1 \\ I_1 &= y_1 E_1 + 1 \,.\, I'_1 \end{aligned}\right\} \qquad (94)$$

that is, the matrix of this "admittance transformation" is

$$\left\|\begin{matrix} 1 & 0 \\ y_1 & 1 \end{matrix}\right\|$$

or say (y_1) for short, for which $\Delta = 1$.

Such matrices are then interpolated as factors between those of the adjacent lines wherever "intermediate loads" are placed. Thus for the case in question taking (M_3), (M_2), (M_1) to stand for the matrices of the third, second and first sections respectively as marked in Fig. 6 we have made up the matrix for the whole line as indicated by (85) and (86) thus $(M_3)\{(y_2)(M_2)\}\{(y_1)(M_1)\}$ or, that is, as the product of three matrices obtained by pairing the "admittance transformations" (y_1) and (y_2) with those of the adjacent line-sections to the right of each. As the determinant of each of the five is unity, the determinant for the whole line (product of the five) is unity also, as well as those of the form $\{(y)(M)\}$ Examining such a pair in detail we have

$$\underset{\Delta=1}{\left\|\begin{matrix} 1 & 0 \\ y_1 & 1 \end{matrix}\right\|} \times \underset{\Delta=1}{\left\|\begin{matrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{matrix}\right\|} = \underset{\Delta=1}{\left\|\begin{matrix} \alpha_1 & \beta_1 \\ \alpha_1 y_1 + \gamma_1, & \beta_1 y_1 + \delta_1 \end{matrix}\right\|} = \underset{\Delta=1}{\left\|\begin{matrix} \alpha_1 & \beta_1 \\ \zeta_1 & \eta_1 \end{matrix}\right\|} \qquad (95)$$

As to the case of the parallel combination of transmission lines of Section II the matrix for the transformation is

$$\left\|\begin{matrix} \Sigma\alpha\epsilon \div \Sigma\epsilon, & 1 \div \Sigma\epsilon \\ \Sigma\alpha\epsilon\Sigma\delta\epsilon \div \Sigma\epsilon - \Sigma\epsilon, & \Sigma\delta\epsilon \div \Sigma\epsilon \end{matrix}\right\|$$

as has practically been stated already though in another form, and its determinant obviously unity. This form, however, is only correct where the individual determinants are each unity as is the case throughout the scope of this paper.

Again consider the case of a power transformer. It may be regarded as usual as a combination of admittance (magnetizing current), impedance (resistance and leakage reactance), and ideal transformer action.

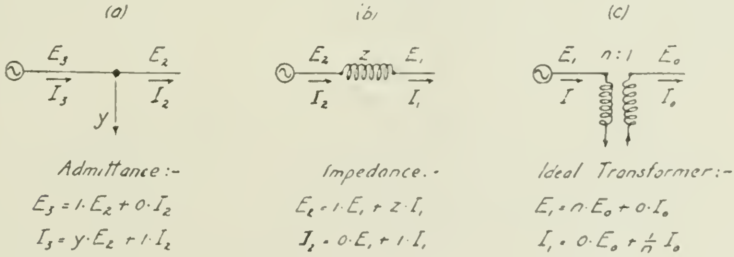

(a)
(b)
(c)
E_3
E_2
I_3
I_2
y
E_2
z
E_1
I_2
I_1
E_1
$n:1$
E_0
I
I_0
Admittance:-
$E_3 = 1 \cdot E_2 + 0 \cdot I_2$
$I_3 = y \cdot E_2 + 1 \cdot I_2$
Impedance.-
$E_2 = 1 \cdot E_1 + z \cdot I_1$
$I_2 = 0 \cdot E_1 + 1 \cdot I_1$
Ideal Transformer:-
$E_1 = n \cdot E_0 + 0 \cdot I_0$
$I_1 = 0 \cdot E_0 + \frac{1}{n} I_0$

FIG. 7.

These are indicated in Fig. 7 with the corresponding linear transformations and matrices for which it will be seen each determinant is unity.

For brevity the matrix of the transformer may be regarded as the product of the matrices indicated by (*a*), (*b*) and (*c*) respectively in the figure, that is,

$$\underset{\Delta=1,}{(a)}\ \underset{\Delta=1.}{(b)}\ \underset{\Delta=1,}{(c)} = \begin{Vmatrix} 1, & 0 \\ y, & 1 \end{Vmatrix} \times \begin{Vmatrix} 1, & z \\ 0, & 1 \end{Vmatrix} \times \begin{Vmatrix} n, & 0 \\ 0, & \frac{1}{n} \end{Vmatrix} = \underset{\Delta=1}{\begin{Vmatrix} n, & \frac{z}{n} \\ ny, & \frac{1}{n} + \frac{yz}{n} \end{Vmatrix}} \qquad (96)$$

If a more exact representation of a transformer be used, for instance, one with the primary and secondary reactances separated, thus giving the product of four matrices, still the result so far as the determinant is concerned, must always be the same, namely, unity.

A review of the matter presented in Sections II and III will disclose the fact that the only essential requisites for an item to be considered as in parallel or in series with others under the conditions of those Sections are (*a*) that it should have a matrix describing a transformation from one voltage-current pair to another and (*b*) that the determinant should be unity.

It appears then that these conditions are fulfilled by

(1) a transmission line of any length.

(2) a parallel admittance, *e.g.*, a load.

(3) a series impedance, *e.g.*, line choke coils.

(4) a transformer.

Hence all the formulae obtained above are valid when any combinations of these elements are used and not simply those consisting of transmission lines and admittances representing loads. For instance, Section II contains in its formulae the complete solution for the case of a bank of transformers whose characteristics as to ratio, admittance and impedance are all different. Also if two or more transmission lines were stepped down through individual transformer groups to a common secondary bus the transformers would simply figure in the calculation like an extra line-section in each case, a matrix such as that of (96) representing each transformer.

VII. LOOPS AND TIE-LINES.

A "loop" may be considered as a transmission line or portion of a network which (*a*) may have more than one load-point and (*b*) is connected to the transmitting side of the system at both ends.

To make a calculation in this case of voltage and current at various points any convenient load-point on it, but preferably the most important, may be chosen as the end of the section, and the portions to the right and left of this point may then be regarded as transmission lines in parallel.

This might result from the paralleling of two originally independent and intermediately loaded transmission lines at a common terminus from which, of course, other lines could proceed.

The calculation of such a loop would then naturally be treated as an instance of two lines (loaded) in parallel. The four constants $\alpha\beta\zeta\eta$ for each line whether two in number as in this case, three for a twin loop or more than three in more complicated cases, can be found as in equations (85) and (86). Then the voltage and current relations are given by (34) and (39) and division of current between the different lines at entry and at exit by (60) and (61) respectively.

If a transmission line with intermediate loads were reinforced by a new line bridging over two or more sections, the arrangement might be called a loop and the new line a tie-line.

The calculation of the looped circuit after the introduction of the tie-line would then be merely a specially simple case of that just mentioned, since one side of the loop is a simple, uniform transmission line.

Similarly, in any of the cases of combinations of tandem and parallel groupings taken up in previous sections any one transmission line in parallel with any other group may be called a tie-line. The method of calculation has already been indicated. In the stage of aggregation just previous to the last the tie-line appears as one of two or more lines in parallel and thus presents no novelty in principle.

A novelty is, however, introduced when a tie-line is made to connect points in a network, supposed already in existence, but such that the portion of the network bridged by the tie-line can already exchange power with the rest of the network at one or more points besides those at which the tie-line is intended to be connected. Thus in Fig. 8 the portion 234 can exchange power not only at 2 and 4 but at 3 as well with the rest of the network and a tie-line is now to connect 2 and 4 directly.

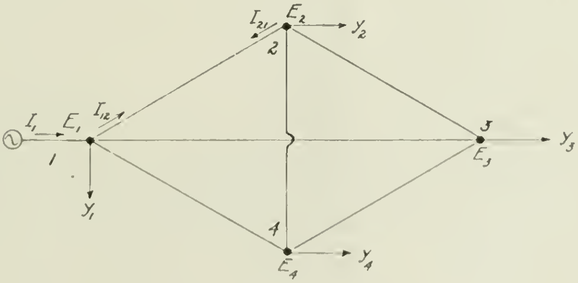


FIG. 8.

The tie-line need not be a uniform transmission line but may in what follows be any group loaded or not, and either be connected on the high tension network or through a lower voltage distribution system by transformers as in Section VI.

This problem is not capable of being solved by the previous formulae but the general method allows us to find the equations for this case without much difficulty.

In Fig. 8 suppose the four points 1, 2, 3, 4 connected by transmission lines as shown, or more generally, by any such groups of constructive elements as have already been shown to be expressible by binary linear transformations. Whether or not there are intermediate loads on any line let there be loads (admittances) y_1, y_2, y_3 and y_4 at 1, 2, 3 and 4 respectively and let E_1, E_2, E_3 and E_4 be the voltages at the corresponding points.

For the present purpose the notation hitherto used for the four constants $\alpha\beta\gamma\delta$ suffers from the limitation of an implied standard sense of power-flow, the sense of current following this. In dealing with the general network there is, in fact, an inconvenience in using a rule which requires such a distinction to be defined and observed.

In Fig. 2 consider equations (1) and (2) in comparison with (10) and (11). It will be seen that if both current senses in (10) and (11) are reversed the minus signs disappear and the constants are $\delta\beta\gamma\alpha$ in place of $\alpha\beta\gamma\delta$ in (1) and (2).

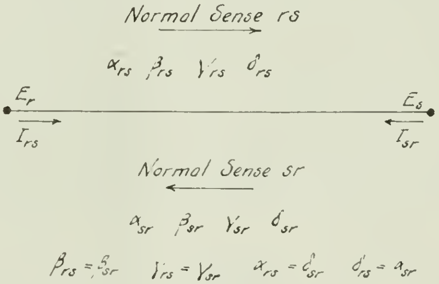

Normal Sense rs
α_{rs} β_{rs} γ_{rs} δ_{rs}
E_r
E_s
I_{rs}
I_{sr}
Normal Sense sr
α_{sr} β_{sr} γ_{sr} δ_{sr}
$\beta_{rs}=\beta_{sr}$ $\gamma_{rs}=\gamma_{sr}$ $\alpha_{rs}=\delta_{sr}$ $\delta_{rs}=\alpha_{sr}$

FIG. 9.

That is in the *form* of the equations describing entry conditions to the transmission line in terms of exit conditions β and γ persist while α and δ are interchanged. Referring to Fig. 9 where the currents and constants are marked with subscript sense-indicators we have $\beta_{rs}=\beta_{sr}$, $\gamma_{rs}=\gamma_{sr}$ but $\alpha_{rs}=\delta_{sr}$ and $\delta_{rs}=\alpha_{sr}$. Then applying the meaning of equations (12) and (13)

$$I_{rs}=\frac{\delta_{rs}}{\beta_{rs}}E_r-\frac{1}{\beta_{rs}}E_s \tag{97}$$

$$I_{sr}=\frac{\alpha_{rs}}{\beta_{rs}}E_s-\frac{1}{\beta_{rs}}E_r \tag{98}$$

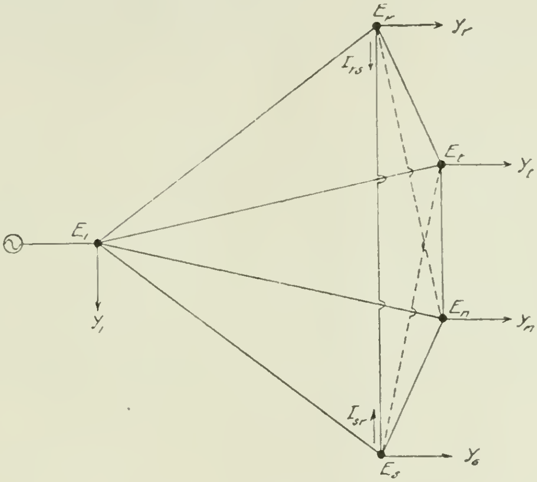
or
$$I_{sr} = \frac{\delta_{sr}}{\beta_{sr}} E_s - \frac{1}{\beta_{sr}} E_r \qquad (99)$$

Equation (99) can then, it appears, be obtained from (97) by merely interchanging r and s and thus may be regarded as a repetition for the point s of the same formula that is used for the point r.

Using (32) we may write $I_{rs} = \delta_{rs}\epsilon_{rs}E_r - \epsilon_{rs}E_s$

or
$$I_{rs} = \kappa_{rs}E_r - \epsilon_{rs}E_s \qquad (100)$$

Here $\epsilon_{rs} = \epsilon_{sr}$ but κ_{rs} in general is not the same as κ_{sr}.



Here lines rn and st are absent

$\kappa_{rn} = a_{rn} = \kappa_{st} = a_{st} = 0$

FIG. 10.

Thus at any point numbered r as in Fig. 10, taking s to stand for the number of each of the other points in turn, the current entering is $\Sigma(\kappa_{rs}E_r - \epsilon_{rs}E_s)$ which is to be summed with the exclusion of $s = r$. The sum will be the same as the negative of the load current at each load point but will be equal to $I_1 - y_1E_1$ at the first point, that is, to the negative of the load current at the first point plus the supply current for the network at that point.

So far, these considerations apply to a network with an unlimited number of branch-points and tie-lines, and writing out these equations in full we have, under the limitations explained, the general equation of the transmission network. But for simplicity of explanation suppose there is one tie-line only and the circuit consists of four principal branch-points connected by six lines (five and a tie-line). Then, writing for convenience

$$\left.\begin{array}{l} a_{rr} = \Sigma\kappa_{rs} + y_r \\ \text{for } s \neq r \\ a_{rs} = -\epsilon_{rs} \end{array}\right\} \qquad (101)$$

the equations may be written

$$a_{11}E_1+a_{12}E_2+a_{13}E_3+a_{14}E_4=I_1 \qquad (102)$$

$$a_{21}E_1+a_{22}E_2+a_{23}E_3+a_{24}E_4=0 \qquad (103)$$

$$a_{31}E_1+a_{32}E_2+a_{33}E_3+a_{34}E_4=0 \qquad (104)$$

$$a_{41}E_1+a_{42}E_2+a_{43}E_3+a_{44}E_4=0 \qquad (105)$$

Let A_{1r} be the minor of the element a_{1r} of the determinant formed from the coefficients on the left hand side of these equations, or in the general case of any number of principal branch-points. Then by the regular rules for the application of the methods of Determinants to the solution of equations we obtain, Δ being the value of the determinant:

$$\frac{E_1}{A_{11}}=\frac{E_2}{A_{12}}=\frac{E_3}{A_{13}}=\frac{E_4}{A_{14}}=\frac{I_1}{\Delta}=\mu \qquad (106)$$

Or writing $$\frac{A_{11}}{\Delta}=Z_{11},\ \frac{A_{12}}{\Delta}=Z_{12},\text{ etc.} \qquad (107)$$

$$E_1=Z_{11}I_1,\ E_2=Z_{12}I_1,\ E_3=Z_{13}I_1,\ E_4=Z_{14}I_1 \qquad (108)$$

Suppose, for instance, that the voltage is known at point 4. By (106) or (108), all other voltages E_1, E_2 and E_3 are expressible in terms of E_4, and the main supply current I_1 is fixed also.

Similarly, in the general case equal fractions of the same form but in number equal to the explicitly recognized or "principal" branch-points will occur. Thus for n principal branch-points

$$\frac{E_1}{A_{11}}==\frac{E_r}{A_{1r}}=\frac{E_s}{A_{1s}}=\frac{E_n}{A_{1n}}=\frac{I_1}{\Delta}=\mu \qquad (109)$$

Or for each value of r writing $\frac{A_{1r}}{\Delta}=Z_{1r}$ (110)

we have $$E_r=Z_{1r}I_1 \qquad (111)$$

Here the "dimensions" of A_{1r} are Y^{n-1} and those of Δ are Y^n giving Y^{-1} or Z as the dimensions of the factor Z_{1r}. This checks with the form of (111).

Here all the voltages and the main supply current become known as soon as any one voltage is assigned. Also the current entering the line at r which leads to s is

$$I_{rs}=\mu(\kappa_{rs}A_{1r}+a_{rs}A_{1s})=(\kappa_{rs}Z_{1r}+a_{rs}Z_{1s})I_1=(\kappa_{rs}Z_{1r}+a_{rs}Z_{1s})\frac{E_t}{Z_{1t}} \qquad (112)$$

where t denotes any point.

Although the final form of these equations for the most general case of network is so simple, it will be understood that the labour required to evaluate the expressions would be very great except in the simpler instances.

VIII. TRUNK LINES.

Understanding by a "trunk line" the case where more than one source of power is connected to a transmission line or network, it will be apparent that from the point of view of the treatment of loads as admittances, namely complex quantities in the first or in the fourth quadrant, it will be consistent to regard an auxiliary source of power as an admittance in the second or third quadrant. That is, if we put $y=a+bj$ the sign of a being positive indicates power withdrawn from the system, while if a be negative it will mean power supplied. Hence nothing new is introduced into the treatment described in the previous Sections except in the detailed arithmetic.

It would also be possible in order to apply other preceding results to this case to suppose that E_o, I_o define the local supply conditions instead of the local load conditions at the end of a network or part of a network.

These considerations indicate that in the foregoing treatment in previous sections from the point where "intermediate loads" are introduced and onwards, the words "or intermediate power supplies" may always be supposed added to the text of the statement.

Although the power factor may be precisely specified the amount of power supply taken account of in the calculation by the admittance method can only be fixed approximately in advance though accurately fixed when the calculation is complete. This disadvantage is of the same character as for loads similarly given as noted in a previous Section.

IX. EXAMPLE OF CALCULATION.

Four transmission lines in parallel without intermediate loads connect Niagara Falls and Dundas as shown in Fig. 1. For these it is[1] assumed that

$$Y_1=Y_2=.0001234j \qquad Y_3=Y_4=.000130j$$
$$Z_1=Z_2=14.3+16.3j \qquad Z_3=Z_4=13.61+16.71j$$

then from (6), (7) and (8)

$$\alpha_1 = \delta_1 = \alpha_2 = \delta_2 = .998995 + .0008793j$$
$$\alpha_3 = \delta_3 = \alpha_4 = \delta_4 = .999061 + .0007665j$$
$$\beta_1 = \beta_2 = 14.2904 + 16.2987j$$
$$\beta_3 = \beta_4 = 13.6014 + 16.7082j$$
$$\gamma_1 = \gamma_2 = -.0000004 + .0001230j$$
$$\gamma_3 = \gamma_4 = -.0000003 + .0001126j$$

Hence as in (14)

$$\epsilon_1 = \epsilon_2 = .0304139 - .0346881j$$
$$\epsilon_3 = \epsilon_4 = .0293031 - .0359964j$$

Then for use in (35)

$$\Sigma\epsilon = .119434 - .141369j$$
$$\Sigma\alpha\epsilon = .119434 - .141133j$$

---

[1] I am indebted to Mr. W. P. Dobson for the data and to Mr. C. K. Duff for assistance with the calculation. The arithmometer used did not require economy in the use of significant figures.

Thus

$$A = D = .999027 + .000822j$$
$$B = 3.487158 + 4.127602j$$
$$C = .0000013 + .0004712j$$

The check $AD - BC = 1$ is found to hold to the required standard of accuracy.

Assuming a load of 60,000 kw. at Niagara Falls with power factor .85 (lagging) at 110,000 volts, the voltage to neutral is $63509 = E_g$ and the current delivered at N.F. to the four transmission lines in parallel is $314.92 - 195.17j = I_g$.

Then the voltage at Dundas is $E_r = DE_g - BI_g$ as in (46). This gives $E_r = 61543 - 567j$ and the current at Dundas I_r as in (47) is $314.69 - 224.64j$.

We find, from (12) and (13) for the currents entering No. 1 and No. 2 respectively $79.46 - 47.02j$ and for Nos. 3 and 4, $78.01 - 50.56j$ amperes. The total is $314.94 - 195.17j$, a satisfactory check.

Also for currents leaving Nos. 1 and 2, $79.42 - 54.73j$ and for Nos. 3 and 4, $77.97 - 57.60j$ amperes. The total is $314.78 - 224.65j$, thus agreeing closely with the total output of the four lines.

The currents entering and leaving the separate lines may be calculated as above from (13) and (12) but on account of the relatively large error which would be introduced if the nearly equal terms are not calculated exactly it might be better to use the second forms given in (60) and (61) respectively for the purpose.

The second terms in these expressions which are denoted by $B\epsilon_r I_b$ and $B\epsilon_r I_a$ here mean $B\epsilon_r I_g$ and $B\epsilon_r I_r$ respectively and, being the larger terms, give the approximate values for the individual currents at each end. It should be observed, however, that although $\Sigma B\epsilon_r = 1$ for any number of transmission lines in parallel and hence the partial currents so calculated would necessarily add up to the total current, yet this would not prove that the individual quantities were correct. This results from the sum for the group of the first terms of these expressions being zero.

OVERLOAD LIMITATIONS OF HIGH-VOLTAGE SINGLE-TURN-PRIMARY CURRENT TRANSFORMERS

BY W. B. BUCHANAN, B.A.SC.

INTRODUCTION

This paper relates to the performance of bushing-type current transformers, commonly employed in high-voltage transmission systems because of the difficulty of insulating other types. Very serious limitations are imposed by the one-turn primary combined with comparatively small line current, and excessive length of magnetic circuit necessary to provide space for insulation. Facts have been investigated which vitally affect the action of selective and overload protective apparatus for generating and substations.

The investigation was based on a protection problem before the Hydro-Electric Power Commission of Ontario. As the work promised information of general value, it was undertaken on broader lines than required by the immediate problem. The paper reports those portions of the investigation of general interest.

Assistance was given the writer by Profs. Rosebrugh and Price on various phases of the work, and by Mr. C. K. Duff in preparation for publication.

I. FIELD OF THE INVESTIGATION.

The rapid development of the high tension alternating-current transmission and distribution of power during recent years has presented many difficult and intricate problems, and in connection therewith the problem of providing adequate protection to generating and transmitting apparatus, at the same time ensuring continuity of service through the parts unaffected by the trouble, has been growing in magnitude and complexity.

In the general problem of protection the condition of voltage or current in the line or apparatus itself is made the criterion as to whether or not it may safely be continued in service. For automatic protection these quantities must be used directly, or be represented as accurately as possible by smaller proportionate values in the scheme of protective apparatus, or for purposes of power measurement. The high voltages and large currents commonly used make it commercially impossible to use the line voltage and current directly on account of insulation and heating problems, and the great difficulty of standardizing control and metering apparatus. These conditions lead to the use of potential and current transformers which are intended to give results at each and every instant indicating exactly the condition in the power line.

The high tension voltage is obtained by the use of potential transformers stepping down from higher values to that for which the meters or relays are built. The low tension equivalent of the high tension voltage is not difficult to obtain with considerable accuracy, and in a manner providing for proper insulation. As systems of very high voltage are of constant potential and excessive rises of potential on a system are of short duration, the most unsatisfactory conditions under which these transformers operate are easily determined and provided for.

On the other hand, the transmission of power which is continually changing in value and power factor at a constant potential requires line currents which are continually changing in value and which may in case of trouble to apparatus, rise to values which are many times that for which any part of the apparatus was designed. The current transformers which are intended to give currents to the relay or meters which are proportional to these line currents at all times must be connected in series in each line, and thus containing elements having line potential, must have their primary windings well insulated from the secondary windings and core.

The difficulty and expense of building such a transformer with a number of primary turns greater than one for voltages above twenty-five thousand volts has led to the extensive use of a transformer with a single-turn primary winding for protection with relay equipment. Current transformers with a single turn primary are also used on bus-bars and line-leads to supply indicating meters, though it is generally known that they have considerable errors which are inherent in their construction, and the readings are usually taken as being approximate.

Since this current transformer with a single-turn primary winding is the first link in many schemes of protection it appeared advisable to undertake a more extensive study of the different factors entering into its operation than has hitherto been published, and the effect of these factors on the results obtained under the abnormal conditions which become its particular field of operation. Actual checks on the true ratio of primary to secondary currents are seldom taken up to the loads corresponding to five or ten times the primary current for which the transformer is designed, hence the actual performance under such conditions is more often assumed than accurately determined. Standard inverse-time-limit overload relays are marketed with attached performance data covering their operation at currents up to 30 or more times normal-load current. Relay adjustments based on these values are commonly accepted without regard to the overload characteristic of the current transformer supplying the current, whereas great change in ratio may make such settings most unreliable.

The theory of the operation of transformers is well established, and the following investigation discloses no new principle, but the results obtained will probably be interesting, if not entirely new, to most engineers. Closely associated with the characteristics of the transformers themselves is the character of the secondary loads supplied, and the complications introduced by various standard methods of interconnecting the secondaries of the current transformers on polyphase systems.

The performance of the single-turn primary type of current transformer under heavy overload conditions and variations that may be introduced by various three-phase connections is the study involved in the following pages.

Throughout a considerable range of reading the writer has found but little indication of the magnitude of the errors involved in this type of series transformer, though some caution with respect to their use is given by the manufacturers. The published literature on the measurement of ratio and phase angle errors of current transformers is almost entirely confined to the type of much greater precision, that in which the primary winding consists of a larger number of turns, or the primary current is large. Some values, however, are given in the Bulletin of the Bureau of Standards, Vol. 6, No. 1 (Reprint No. 116, p. 37) which show that this type of transformer is in a class quite different from the usual type.

II. METHOD OF TEST AND SOME RESULTS.

In this work the object was not highly accurate evaluation of the errors of a particular current transformer, but a series of typical results sufficiently exact for curves showing the characteristics of the bushing type transformer with extra long magnetic circuit and few primary ampere-turns at rated load.

For measuring ratio, precision null methods are much in vogue on account of their ability to accurately measure small errors. In this work it was found necessary to revert to the primitive method of primary and secondary ammeters for measuring the two currents directly, and therefrom obtaining ratio. The reasons were: (1) The ratio errors were found so great that the ammeter method was quite suitable as to accuracy. (2) Null and most other precision methods are not suitable where primary and secondary wave forms are much different. The ammeters give ratio of r.m.s. values regardless of distortions, within the ability of the meters as to frequency errors. As most protective apparatus depends for action only on r.m.s. current, this method was chosen.

For measuring phase angle, a method was devised of using a polyphase wattmeter. The term "phase angle" as applied to primary and secondary currents of different wave forms is rather meaningless since it inherently refers to sine waves of one frequency. Phase angle of current transformers is of little interest except for wattmeters or watt-relays. In service the voltage circuits of such instruments are connected to voltages of commercial wave form, usually of approximate sine form. In this method the voltage circuits of the wattmeter were connected to voltages of commercial form, so that effects of time difference between primary and secondary currents as measured by the wattmeter would influence the observations approximately as they would affect watt-relays.

Fig. 1 shows the circuits, and Fig. 2 the corresponding vector diagram. The wattmeter was a Weston polyphase for 150/75 volts and 5/2.5 amperes in each phase. Keys were provided in each voltage circuit so that either could be closed or opened as required. Weston portable current transformers *P* and *S* were used to reduce primary and secondary currents to values suitable for the wattmeter and ammeters. These instrument transformers were so chosen that overload currents for the tested transformer were normal currents for the instrument transformers. The ability of these transformers to give a secondary current exactly proportional at every instant to the primary current, regardless of distortions, is shown in oscillogram No. 1 in which (*a*) is alternator voltage, (*b*) current through primary of a Weston current transformer, (*c*) current through its secondary and meter burden. These transformers are sufficiently accurate to be used without correction in any measurements in this investigation. The use of resistor *R* will be referred to later. The method will readily be understood from the vector diagram Fig. 2 and the following explanation.

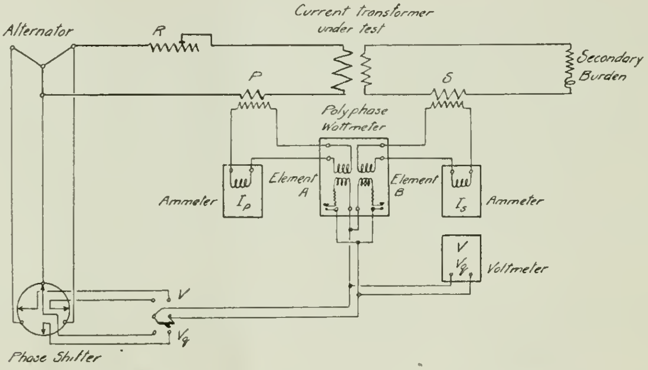


FIG. 1. Meter connections for measuring ratio and phase angle.

Having arranged the secondary burden as desired, and the supply circuit for necessary currents as indicated by the ammeters, the ratio of r.m.s. currents is obtained from ammeter readings I_p and I_s. For phase angle, the voltage switch is closed to V_q, and with voltage key B closed the phase-shifter is adjusted to make element B of the wattmeter indicate zero, thus putting V_q in quadrature with current I_s. Closing voltage key A instead of B then gives from element A a volt-ampere indication $V_q I''_p$, which divided by V_q as read on the voltmeter determines component I''_p of the primary current in quadrature with the secondary current I_s. The sine of the phase angle θ is I''_p divided by I_p as given by the ammeter.

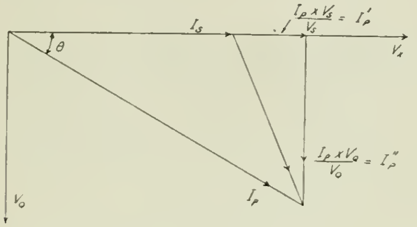

I_s
$\frac{I_p \times V_s}{V_s} = I'_p$
V_s
θ
$\frac{I_p \times V_Q}{V_Q} = I''_p$
I_p
V_Q

FIG. 2. Vector diagram corresponding to test connections in FIG. 1.

If it be desired to check ratio by the wattmeter, the voltage switch is reversed to voltage V from the polyphase phase-shifter in quadrature with V, and therefore in phase with I_s. Element A then indicates volt-ampere product VI'_p, which divided by V from the voltmeter determines component I'_p of the primary current in phase with the secondary current I_s. The tangent of the phase angle θ is I''_p divided by I'_p, and primary current I_p is $\sqrt{I'^2_p + I''^2_p}$. I_s can be taken from the ammeter, or by reading from element B of the wattmeter the product VI_s and dividing by observed V. Thus ratio may be obtained from the two ammeters, or from the wattmeter and either ammeter, or from the wattmeter.

With approximately rated currents, ratio by all the above methods checked very closely, but with heavy over-loads large inconsistencies were found due to great distortion of the current waves. Strictly the results can agree only with sine waves of the same frequency for voltage and both currents. Practically they agree quite closely if the waves are not much distorted.

The tests were made on a current transformer, see Fig. 3, with an annular-ring core of rectangular section 16 square inches, and mean length of magnetic circuit 50 inches, laminations being one-piece punchings. The secondary winding was 8 turns of copper strap uniformly distributed around the core length, and wound tightly over thin core insulation. The normal primary winding was a copper rod passing axially through the centre of the ring. The nominal rating would be 100 to 5 amperes at 25 cycles with about 1.3 ohms non-inductive secondary burden.

Tests up to 10 or 20 times full load primary current involved 10 or 20 times 100 primary amperes, currents which could not be supplied by available apparatus. Since primary resistance and leakage reactance have no effect on ratio or phase angle of a current transformer, experiments were tried with various numbers of primary turns, for example with 50 turns in which nominal rated current would 1/50 of 100 or 2 amperes instead of 100 amperes with 1 turn. It was found that the same ratio and phase angle were obtainable with one axial primary conductor, or any desired number of turns with correspondingly reduced current, provided these turns were uniformly distributed around the core.

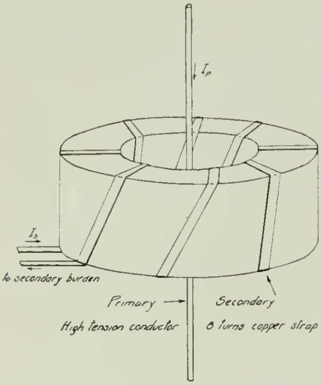


FIG. 3. Sketch of core and windings of a high-voltage bushing-type current transformer.

Some trials were made with the 50-turn primary connected directly to the alternator, that is with resistor *R*, Fig. 1, removed, and some with resistance in circuit. Alternator voltage was varied to adjust current as required. The results were interesting, but not normal. They are tabulated in Table I, and corresponding oscillograms, 2 to 9 inclusive, are assembled at the end of this paper.

TABLE I.

Oscillogram Number	Source of Supply Voltage	Secondary Burden	Alternator Voltage	Primary Amps. for 1 turn	Secondary Amps. 8 turns	Turns Ratio	Current Ratio
2	One 15 kva. 110 V. alternator	1 ohm non-inductive	80 (*a*)	200 (*b*)	9.25 (*c*)	8	21.6
3	"	"	115	400	10.9	"	36.7
4	"	"	105	600	11.3	"	53.1
5	"	"	112.5	800	12.1	"	66.1
6	"	2 ohms non-inductive		400	5.55	"	72.0
7	"	"		1000	6.2	"	161.0
8	Two 15 kva. alternators in parallel	"		1000	5.8	"	172.0
9	"	"		1000	5.65	"	177.0

Current transformer for 110,000 volts, 100 to 5 amps. at 25 cycles with 1.3 ohms burden. Core section 16 sq. ins. Mean length of magnetic circuit 50 ins. The turns and current ratios are reduced to values for one primary turn. Tests made with 50 distributed turns in primary. Currents are r.m.s. values.

The facts that the turns ratio is 8, and the current ratio at nominal rating is about 20, show that this transformer is very unlike the usual switchboard type. The enormous change in the current ratio with increasing current is due to approaching core saturation. Details with regard to these matters will be discussed in section IV.

In tests 2 to 7 inclusive some resistance *R* was inserted in the primary circuit.

No. 8 was similar to No. 7 except that resistance *R* was not used.

No. 9 shows the result of duplicating test No. 8 but with two 15 kva. alternators in parallel instead of one alternator, for the purpose of reducing the armature reaction per alternator and consequent distorting effect on the wave form of the voltage.

These oscillograms show primary current waves having more or less the characteristic form of the exciting current wave of a constant voltage transformer. This is the result of allowing the primary of the current transformer to absorb all or a large part of the alternator voltage, thus permitting the primary current wave form to be determined largely by magnetic conditions in the core and impedance of the secondary burden. Thus the secondary current through the non-inductive burden is of much the same wave form as the supply voltage. The reason is that primary counter voltage and secondary generated voltage must be approximately of the same form as the alternator voltage wave, because primary counter voltage at each instant must be approximately equal and opposite to alternator voltage at the same instant, since primary *IR* drop is small. To generate the necessary core flux for that voltage with core saturation during a part of each alternation requires excessive rushes in the magnetizing component of primary current during the intervals of saturation.

Step-down transformer methods of supply for any current transformer worked to core saturation will introduce the same kind of distortion. The only way to obtain a primary current wave approaching that of an ordinary power circuit is to have supply voltage large compared to primary terminal voltage required by the current transformer, and consume the difference in a resistor, or a reactor without core saturation which will not distort the current seriously with regard to the form of the supply voltage.

For these and other reasons it appeared advisable to use a distributed primary winding reduced from 50 to 8 turns, considering nominal rated primary current as $\frac{1}{8}$ of the current for 1 primary turn, and consume the large difference between alternator voltage and primary terminal voltage of the current transformer in a resistor as shown at *R*, Fig. 1.

Some results of tests under these conditions are listed in Table II, and corresponding oscillograms, 10 to 17 inclusive, are assembled at the end of the paper.

TABLE II.

Oscillogram Number	Source of Supply Voltage	Secondary Burden	Primary Amps. for 1 turn	Secondary Amps. 8 turns	Turns Ratio	Current Ratio
10	One 15 kva. 110 V. alternator	1.26 ohms non-inductive	100	5	8	20
11	"	"	200	8.1	"	25
12	"	"	300	9.7	"	31
13	"	"	400	10.5	"	38
14	"	"	600	11.7	"	51
15	"	"	900	12.9	"	70
16	Two 15 kva. alternators in parallel	"	1200	13.5	"	89
17	"	"	1600	13.75	"	116

Same core as for tests in Table I. Tests made with 8 distributed turns in primary. Burden adjusted to give nominal ratio 20, at nominal rating, 100 to 5 amperes. Turns and current ratio reduced to values for one primary turn. Currents are r.m.s. values.

The upper wave in each oscillogram is the supply voltage, the middle wave the primary current, and the lower wave the secondary current.

Without the resistor *R*, Fig. 1, in circuit, the alternator is free to deliver any wave form of current required by the cycle of actions in the current transformer and its burden. With relatively large resistance *R* in circuit, the resistor largely controls the wave form of the primary current, and therefore it will be approximately of the same form as the alternator voltage wave. At each instant during a cycle, the requirements of the core, including the excessive demands during intervals of core saturation, must be supplied by the difference between primary and secondary ampere-turns at the same instant. If the primary current be not permitted by the resistor *R* to follow the distorted forms shown in oscillograms 2 to 9, the core requirements must be met by such distortion of secondary current as will leave from instant to instant the necessary difference in ampere turns. Obviously also the core requirements will not be the same since the secondary output is quite different in the two cases.

This series of oscillograms brings out the following interesting features:

(1) That a current transformer fails to deliver current to a secondary burden during such instants of time in each alternation as the core is saturated.

(2) The secondary current resulting from the induced secondary voltage is, with a non-inductive burden, confined to the interval during which the core flux is changing from saturation in one sense to saturation in reverse sense.

(3) The wave form of the primary current during the intervals of core saturation will then have a very little effect on the secondary current. Therefore a flat topped wave of primary current may cause a secondary current nearly identical with that obtained from a primary current of say twice the effective value but having a peak occurring during those time intervals. Thus it is possible to obtain a wide range of values of ratio with a given secondary current.

(4) In Nos. 14 to 17 of this series of oscillograms a pronounced notch is shown in the wave of the supply voltage. This is a very interesting example of the effect of armature reaction in an alternator.

The voltage was obtained from one 15 kva., three-phase, 110 volt, 25 cycle alternator for oscillograms 14 and 15, and oscillograms 16 and 17 were obtained with two such machines operated in parallel. The primary equivalent of the secondary current in these cases is in effect a component leading the main primary current in quadrature. Hence during such intervals as the secondary current assumes appreciable values the armature reaction resulting therefrom causes a rise in the generated voltage which immediately falls off when the secondary current drops because of core saturation. The difference in the ratio obtained by using one or two alternators to supply a given primary current under such conditions is quite appreciable and is referred to in the preceding paragraph. On this account it was necessary to maintain uniform conditions of loading for corresponding tests in order to ensure results that would check properly.

With an inductive load in the secondary circuit of approximately seventy per cent. power factor the secondary current takes the wave form shown by waves (*c*) in oscillograms 18 to 21 inclusive. Numerical values of these waves are as follows in Table III:

TABLE III.

Oscillogram Number	Source of Supply	Primary Amps. in 1 turn	Secondary Amps. in 8 turns
18	Two alternators in parallel	900	12.6
19	Two alternators in parallel	1200	13.2
20	One alternator	930	11.7
21	Two alternators in parallel	1600	13.7

It is evident from the form of these secondary current waves that an analysis of a wave into its harmonic components would show a relatively large ratio of any harmonic of the lower orders to the fundamental. The study of the effects thus resulting from various three-phase connections, in some of which certain harmonies can freely flow and in others of which they are almost prohibited, was found to be of sufficient importance to be considered in a separate section, see section VII.

Frequency also is a very important factor, particularly under heavy overload conditions. The ability of a transformer working between positive and negative core saturation to deliver secondary current is directly proportional to the frequency at which it operates. Hence a primary current of exactly the same wave form and effective value but 4 per cent. higher in frequency than the primary current in another test may be expected to give a secondary current about 4 per cent. higher than occurs in that other test. This was demonstrated in tests not reported here in detail.

From the features previously outlined it will be readily seen that anything like precise work is not required in this investigation. Ratios are sufficiently accurate when measured by high-grade ammeters, and phase-angles which vary anywhere from 10 degrees to 60 degrees are not difficult to measure with reasonable accuracy in the manner previously outlined. See curve of measured phase angles in Curve Sheet No. 3 at the end of this paper.

III. FUNDAMENTAL THEORY AND THE EFFECT OF VARIOUS FACTORS DETERMINING RATIO AND PHASE ERRORS.

The fundamental actions by which a current transformer delivers a current from its secondary proportional to the current in the primary are the same as obtain in the constant potential power transformer. Since its purpose, however, is to maintain proportionality between primary and secondary currents, more attention must be given to reducing the exciting current to a minimum, as this is the factor which is the cause of the errors found in this transformer.

The theory underlying the operation of constant-voltage transformers is so well known that it is not necessary to repeat it here in detail. There are, however, a few factors not appreciated by the average engineer as clearly as they should be.

A secondary burden, however small, requires a terminal voltage which increases with the current flowing. The generated voltage in the secondary winding to cause that current is greater than the terminal voltage across the burden by the amount of resistance and leakage reactance drops in the secondary winding, and varies in proportion to the current. In order to generate the voltage the core must carry an alternating flux also proportional to the current. The exciting ampere-turns required to cause the flux varies in a definite manner with regard to the working flux (see Figs. 6 and 7) and therefore in a general way with regard to the secondary current. The magnitude and phase relation of the exciting ampere-turns with respect to the secondary ampere-turns determine the character and magnitude of the errors of the transformer as a device for giving a secondary current which is a definite fraction of the primary current.

It is evident that with the exception of the leakage reactance of the secondary winding the various quantities required to calculate these errors may be determined without difficulty.

In the constant-voltage power or potential transformer, in which the ratio of primary to secondary voltage is an important factor, it can usually be assumed that the total leakage reactance is divided between primary and secondary windings approximately in proportion to the square of the turns in each. In calculations involving both reactances, the error from this assumption is small, because if the assumed value is too low for one reactance, it will be correspondingly too high for the other reactance, and the two errors approximately compensate each other.

In the current transformer, however, the ratio of primary to secondary currents and phase angle are the points of outstanding interest, and these quantities are not influenced by primary resistance and reactance but are affected by the secondary resistance and reactance. The extent to which an error in evaluation of secondary reactance will affect the calculated values of current ratio and phase angle depends upon its magnitude as a part of the total impedance in the secondary circuit. For strict accuracy it is necessary to know the exact value. It thus becomes important to determine the conditions under which the foregoing assumption as to the division of reactance between primary and secondary windings is warranted, and to know the variations from these conditions which would make it inaccurate.

A study of the magnetic field distribution of a few examples will show that the primary reactance may vary through a very wide range of values without appreciably changing the value of the secondary reactance, and that in the special case where the secondary is closely and uniformly wound on a core of toroidal form as in the bushing type transformer the reactance of the secondary winding is so small as to be negligible for the purposes of the tests covered in this investigation.

Probably the most simple case to study in connection with secondary reactance is that in which the primary and secondary windings are wound on opposite sides of the core, and it is convenient to make the windings of equal number of turns. By so doing we can pass the same current through both windings in series so connected that their magneto-motive forces are in opposition around the core. Such a condition represents the effect of the secondary ampere-turns and its equivalent primary ampere turns in actual operation. Referring to Fig. 4, it is evident that two neutral zones *C* and *D* will occur at which no magnetic field will exist, and at points *e*, *f* and *g* different densities will be found, with a maximum at the centre of each winding. These magnetic fields give rise to the reactance in the primary and secondary windings respectively and with equal and symmetrical distribution of windings they would reasonably be expected to be equal. Suppose, however, that winding *B* is larger in diameter than coil *A* and therefore has considerably greater length of conductor, otherwise remaining unaltered. Then the same ampere-turns will set up the same field intensity at various points internally as before, but the area enclosed by *B* has been increased, hence also its total flux, while that of *A* has not been altered. Actual tests made by the writer on such a model shows a reactance for *B* of 0.0367 ohms while that for *A* was 0.03 ohms at 25 cycles.

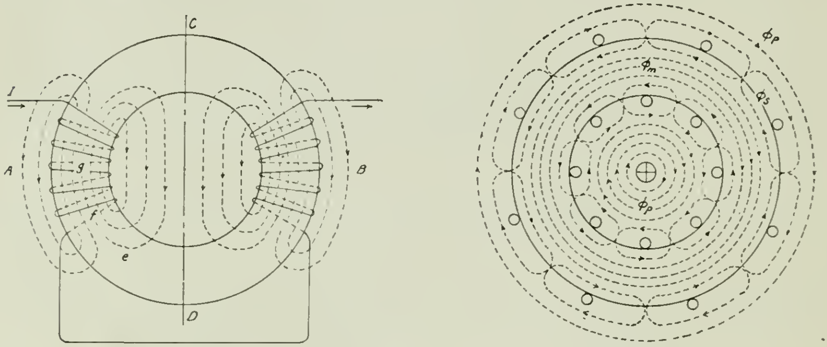


FIG. 4. FIG. 5.

Diagrams of distribution of leakage magnetic fields.

When the primary winding is of a single turn passing through the centre of the core and the secondary is uniformly wound on the core, as shown in Fig. 5, the magnetic field distribution may be illustrated by a section through the core at right angles to the primary winding. A study of this field shows that all the flux encircling the primary which does not enter the toroid enclosed by the secondary winding is linked with the primary winding only; that all the flux that passes completely around within this toroid is linked with both primary and secondary winding; and that the secondary leakage flux is limited to that of the field of a single conductor taken from the conductor to a neutral zone at a distance of one-half the spacing between conductors. This reduces the internal reactance of the secondary winding to a very small amount, the order of magnitude of which is obtained by calculation and checked by experiment described in the Appendix.

With an accurate method of determining the internal reactance of the secondary winding it becomes possible to obtain test results from which the ratio and phase error of transformers in which secondary reactance is not negligible may be calculated. The secondary current is definitely related to the induced secondary voltage by the value and character of the total secondary impedance. The exciting current is also definitely fixed with respect to the same secondary voltage by the magnetizing requirements of the core. Then if these relations can be determined by tests the results calculated therefrom should be as accurate as the methods of test and calculation used. Such a method may be briefly outlined as follows:

The phase relation of the exciting current with regard to the induced secondary voltage is obtained by making a test with an open-circuited secondary. The exciting current I_e in the primary (or a special exciting winding) is read, the open circuit secondary voltage E_s, and a wattmeter so connected that I_e passes through its current coil and the secondary voltage E_s is applied to its potential coil. If necessary a correction may be made to the wattmeter reading to eliminate the effect of power taken by the potential coil of the wattmeter and the voltmeter. These quantities, however, are likely to be relatively very small, and in most cases may be neglected. The net wattmeter reading divided by the open-circuit secondary voltage will give the component of current in phase opposition to E_s, and from this and I_e as read by an ammeter the quadrature component may be found. These components will represent ampere-turns and also amperes in the case of a single turn primary.

For calculation it is convenient to plot the two components of primary exciting ampere-turns as ordinates on a base of induced secondary voltage, from which any intermediate values may be obtained. Then assuming a horizontal vector as in Fig. 6 to represent the phase position of E_s, the secondary current I_s will lag behind it by an angle of which the tangent is the total reactance in the secondary circuit divided by the total secondary resistance. This ratio may not be constant for all values of secondary currents. If the impedance of the burden varies with current, the locus of secondary current will be a curve instead of a straight line. Then, adopting a suitable scale of ampere-turns, lay off the secondary ampere-turns in phase with the secondary current. Find the induced voltage necessary to force this current through the total impedance of the secondary circuit. Corresponding to this value find the components of the exciting ampere-turns and draw the vector representing this in its proper phase relation. The number of primary ampere-turns required is then the vector sum of the primary equivalent of the secondary ampere-turns and the exciting ampere-turns. Finding the true ratio and phase angle then becomes a simple problem in the solution of triangles.

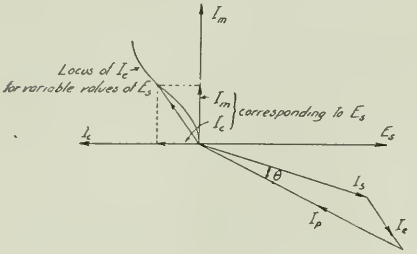


FIG. 6. Vector diagram of a current transformer.

The foregoing method of calculation was checked against results obtained by the wattmeter method of measuring ratio and phase angles (section II). For the purpose of the check tests the internal reactance of the secondary was measured by the use of an exploring coil wound midway between the secondary turns, and was found to be 0.037 ohm. The sum of the reactances of wattmeter, ammeter and leads at 25 cycles was 0.0097 ohm, giving a total secondary reactance of 0.046 ohm. The total resistance of secondary circuit was 0.366 ohm. The final results were as follows in Table IV:

TABLE IV.

Secondary Current	By Wattmeter Measurement			By Calculation		
	I_p	Ratio	Phase Angle	I_p	Ratio	Phase Angle
3 amp.	3.9	1.3	12° 45′	3.87	1.29	12° 45′
4 amp.	5.03	1.26	10° 35′	5.05	1.26	10° 55′
5 amp.	6.1	1.22	9°	6.17	1.23	9°
6 amp.	7.25	1.21	7° 40′	7.27	1.21	8°

Measurements made at 25 cycles on a ring core of 10 sq. ins. section and 28 ins. mean length.

The currents and ratios given are on a basis of twenty turns each in secondary and primary. The accuracy obtained indicates that this method of obtaining the results should be as satisfactory for the transformer with a single-turn primary as an actual test while serving a secondary burden. The tests are easier to make, and once obtained may be used for any secondary load, the impedance of which is known. A further advantage is that an approximate idea of the practical limit of the secondary current may be obtained from the available secondary voltage without the necessity of making heavy overload tests on the transformer.

The locus of the exciting current vector in its proper phase relation to the induced secondary voltage vector, which is constant in direction, and to the secondary current vector, illustrates graphically the meaning of the expressions derived by Dr. Wirz giving the conditions for zero phase error.[1]

The reason for the reversal of curvature in the curve of exciting current I_e is as follows: As core density rises from very low values, power requirements for core losses rapidly become predominant because they grow roughly as density squared while reactive requirements for magnetizing grow roughly as density. As saturation is approached the situation is rapidly reversed since density and core losses rise very slowly while magnetizing requirements grow enormously.

From such a diagram as Fig. 7 it is clear that the power factor of the secondary circuit may be lowered until the secondary current vector produced in a reverse direction will intersect the locus of the exciting current in two points. Thus two values of exciting current are determined for which the corresponding values of secondary current are obtained with zero phase error at a definite value of secondary impedance.

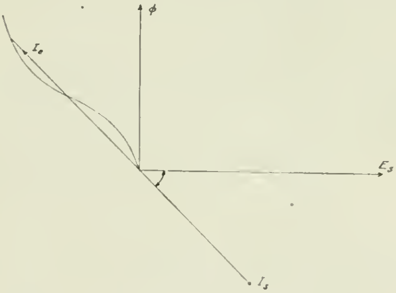


FIG 7. Locus of exciting current vector.

It is also evident that the secondary ampere-turns can never equal the primary ampere-turns on account of the exciting current required. For this reason, it is impossible to obtain a current ratio which is exactly equal to the inverse ratio of the turns. Hence to obtain a transformer having a certain nominal ratio at a specified burden and power factor, it is necessary to use a ratio of secondary to primary turns which is less than the ratio of primary to secondary currents desired. For if I_p represents the primary current, T_p the primary turns, I_s the secondary current, T_s the secondary turns, and I_e the exciting current in actual values, with dotted symbols indicating the vector quantities, then I_p/I_s is the current ratio of the transformer. But the ratio of turns $T_s/T_p = (\dot{I}_p - \dot{I}_e)/I_s$ since $\dot{I}_p T_p - \dot{I}_e T_p = \dot{I}_s T_s$, hence $T_s/T_p = \dot{I}_p/\dot{I}_s - \dot{I}_e/\dot{I}_s$.
If, as appears to be standard practice, we call the departure of the ratio of turns from the nominal ratio of the transformers the "compensation",

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[1] A Theory and Calculation of current transformers, Dr. E. Wirz, Bulletin of the Swiss Electrical Engineering Society, Nov. 1913, Vol. 4.

then it is evident the compensation must be made greater when any factors are altered which tend to increase I_e, *e.g.*, increase of core length, increase of secondary impedance, or decrease in the number of primary turns. A decrease in core area also necessitates a higher flux density to supply a given secondary voltage.

In the precision current transformer the effect of the exciting current may be made as small as desired by the use of many turns, the practical limit being determined by the expense of construction. Further remark on this type, however, is not within the scope of this article.

IV. LIMITATIONS IMPOSED BY THE USE OF A SINGLE-TURN PRIMARY.

The bushing-type current transformer for large currents at low voltage can be designed with good characteristics. The characteristics of the high-voltage type are seriously impaired by a group of interdependent limitations:

(1). Insulation requirements, which make very desirable the single axial primary conductor, demand a very long magnetic circuit in order that ample space may be available for insulation between the high-tension primary conductor, and the core with secondary winding.

(2). The long core requires very considerable exciting ampere-turns even though the core section be made large to reduce core density.

(3). These ampere-turns must be provided by one primary turn, thus making the magnetizing and core-loss currents the largest possible from the point of view of primary turns.

(4). High-voltage lines of usual capacity carry comparatively small current, say 50 to 400 amperes, of which the current required to excite the core will be no negligible fraction.

Current ratio will therefore differ considerably from the ratio of turns, and a large phase angle will be found with usual types of secondary burden.

The secondary turns in this type of transformer will never be as large as the nominal ratio of the transformer on account of the large compensation required, and if designed for a high impedance in the secondary circuit they may be twenty-five to fifty per cent. less in the low ratios. Then since the induced secondary voltage depends on the product of the total flux and the secondary turns, E_s varies with A, B, and T_s where A is the core area in square inches, B the flux-density in lines per square inch, and T_s the secondary turns.

The following numerical examples will serve to illustrate the effect introduced by this limitation. Assuming that a flux-density of 100,000 lines per square inch will be reached when the secondary current reaches its practical limit, and a frequency of 25 cycles per second, we have from the well known equation for transformer e.m.f.

$$E_s = \frac{4.44 \times A \times B \times T_s \times f}{10^8} = 0.11\ A \times T_s.$$

Then given a core area of 10 square inches and 25 turns in the secondary, E_s may become $.11 \times 10 \times 25 = 27.5$ volts. If the total secondary impedance is 1.5 ohms, then we would have a practical limit to the secondary current of $27.5/1.5 = 18.4$ amperes, or with 1 ohm impedance of 27.5 amperes. Large errors in ratio and phase angle will be present. These values only represent the point at which the secondary current ceases to rise at a rate at all comparable with the corresponding rise in primary current. A transformer of 25 secondary turns, provided with 25 ampere-turns for excitation, would thus give a nominal ratio of 150/5.

To deliver a secondary current 20 times the full load current of 5 amperes, or 100 amperes, though 1 ohm impedance would require 100 volts. This with a core of 10 square inches cross-section would necessitate $100/(0.11 \times 10) = 90$ turns. Allowing 50 ampere-turns for excitation this gives a nominal ratio of 500/5. This then is the lowest ratio with one primary turn and a core of 10 square inches area could be expected to deliver 20 times rated secondary current or 100 amperes.

To minimize these overload limitations the requirements are:

(1) Iron of higher permeability.
(2) Larger core areas.
(3) Higher ratios, or its equivalent of operating at lower secondary currents.
(4) Lower secondary impedances.

(1) The core loss and magnetizing requirements of the iron used in these cores need not be as exacting as for that used in the more precise instrument transformers. At normal loads the ratio and phase angle errors with a long core are so large that a small difference is immaterial, and it appears to be more important to provide permeability for the highest flux-densities possible.

(2) It is evident that the maximum current which the transformer can deliver from its secondary will vary directly with the core area. Practical limitations, however, are introduced in stationary types by reason of the limited available space and cost of material.

(3) The theoretical discussion shows that better characteristics are obtained by the use of a greater number of turns in the secondary but this alters the nominal ratio of the transformer. We may reduce the secondary current, however, and if we have to design a transformer for 100 primary amperes we would expect better overload characteristics if it were designed for 1 ampere in the secondary than the standard

value of 5 amperes with the same secondary impedance. This would be equivalent to the standard rating of 500/5 with a lower terminal voltage demand on the transformer.

(4) The theory also points out the advisability of using as low values of impedance in the relays and other parts of the secondary circuits as possible. The limiting feature in this respect seems to depend on the construction of the relay. Relays are on the market which have lower impedances than those with which the tests referred to throughout this article were made, and hence better results would be expected if in all other points they prove as satisfactory.

Characteristic curves were taken to illustrate the change in ratio occurring due to change in the impedance of the secondary circuit for different values of secondary current. These are shown on curve sheet No. 1. The curves show how the ratio of primary to secondary currents rises more rapidly above the point when approximately 20 volts are induced in the secondary winding; also that the same transformer may be given a large number of nominal ratings according to the impedance of the circuit. The number of secondary turns being 25 it is evident that the impedance must be zero to obtain ratios 100/4, 125/5, 150/6, 200/8 or 250/10. It is possible, however, to obtain 125/4 at 0.4 ohms, 150/5, at 0.7 ohms, 150/4 at 2.1 ohms, etc.

V. CURRENT TRANSFORMERS OF SIMILAR CHARACTERISTICS.

From the general theory certain relations may be deduced which become quite simple for the case of a series transformer with a single-turn primary. Some attention must be paid to these relations in the selection of current transformers of different ratios from which similar characteristics are required, as in differential relay protection for power transformers. *Assuming a fixed value of flux density* we may compare the performance of transformers of different designs, but the same quality of iron as follows:

It is evident that the exciting current I_e will vary directly as the length of the core l and with that only for any given flux density.

Under the same conditions the generated secondary voltage E_s varies directly with the area of the core and the secondary turns.

The secondary current I_s depends on the generated secondary voltage E_s and the total impedance Z_s of the secondary circuit.

The secondary ampere-turns then vary directly as the core area, the square of the total secondary turns, and inversely with the total secondary impedance, or I_sT_s varies as AT^2_s/Z_s for given core density.

Then since for the same ratio and phase errors I_e must vary directly as I_sT_s, l must vary as AT^2_s/Z_s, or combining results, Z_s varies as AT^2_s/l

when all quantities vary and the same ratio and phase angle errors are required, the ratio error being based on the ratio of turns.

The expression $Z_s = KAT^2_s/l$ will show the difficulty of selecting a pair of single turn primary current transformers of different nominal ratios to furnish similar characteristics. A ratio of secondary turns of 5 to 1 would require secondary impedances in the ratio of 25 to 1 with length and area of the core unchanged.

In general the selection of transformers of similar characteristics is simplified very much by the adoption of a constant value of secondary turns, length and area of the core, and by making the primary winding of a number of turns which will correspond with any change in the nominal ratio desired. This method seems to offer the only satisfactory solution to the selection of transformers having different ratios but from which similar characteristics are required. Obviously this plan is impossible if single-turn primaries are necessary.

(*b*) *With Parallel and Series Connected Secondaries.* In arranging to obtain desired characteristics it may be necessary to use combinations of available current transformers. With this in view, also the considerations above as to similar characteristics, a few tests were made. The results are shown in Curve Sheet No. 2, and were obtained under various conditions as indicated in the following table:

TABLE IV.

Curves	Impedance Z_s	Secondary Turns	Core Area	$K = \frac{Z_s l}{AT^2_s}$
1	1 ohm non-inductive	25	10 sq. in.	.008
2	2 " "	25	10 " "	.016
3	4 " "	25	10 " "	.032
4	4 " "	50	10 " "	.008
5	8 " "	50	10 " "	.016
6	2 " "	25	20 " "	.008
7	4 " "	25	20 " "	.016
8	8 " "	25	20 " "	.032
9	8 " "	50	20 " "	.008
10	2 " "	25	10 } 2 transformers with primaries in series and secondaries in series	.008
11	4 " "	25	10	.016
12	8 " "	25	10	.032

These practical results verify the foregoing theory in the following particulars:

(1) Coincidences of curves 6 with 1, 7 with 2, and 8 with 3 show that the secondary impedance varies with the core area, other quantities remaining unchanged.

(2) Coincidences of curves 4 with 1, 5 with 2, and 9 with 6 show that the ratio error curve is the same throughout when the total secondary impedance varies as the square of the secondary turns. In these cases the higher ratio involves the lower current scale, since the ampere scales have been so chosen that for any given point on the curve $I_s T_s$ is constant.

(3) Secondaries connected in series and primaries in series show similar characteristics at a secondary impedance double that with a single transformer. Compare curves 10 with 1, 11 with 2, 12 with 3. It will be observed that this shows thôse curves the same, or nearly so, in experiments for which the values of K are the same.

(4) A slight indication of the variation in results that may occur under test is shown by comparing curve 5, obtained by supplying the load fröm two alternators in parallel, with 2, 7, or 11 which were taken with one alternator only, and with which 5 should coincide. This was referred to in section II.

(5) Two current transformers operating with secondaries in parallel would give a characteristic curve similar to that of a single transformer with twice the external secondary impedance, but with the current scale doubled. Thus two transformers each giving a curve such as No. 4 with an effective secondary impedance of 4 ohms will, with paralleled secondaries, each separately give the results shown by No. 5 and combined with have a characteristic similar to No. 1.

The effect of slight air-gaps at joints in the magnetic circuit has been neglected because they are avoided in most transformers of low ratio by making the core of one-piece punchings.

VI. APPLICATION TO PROTECTIVE DEVICES.

Such characteristics of current transformers as have been considered assume various degrees of importance according to the class of service for which they are used. It is evident that the magnitude of errors involved is such that only approximate results are obtained unless unusual care is exercised as to working conditions, hence they are not used for refined measurements. In order that an accuracy of even 5% in ratio may be assured with low ratio transformers of this type it is necessary that the transformer with its connected load should be calibrated and treated as a unit.

1. *Wattmeters.* These instruments are affected by both ratio and phase angle errors, hence the true power could only be found by the use of suitable corrections of considerable magnitude, one of which also requires the power factor of the load for its determination.

2. *Ammeters and current relays operating instantaneously.* These depend only on the ratio, hence may be calibrated with the transformer, and give satisfactory results up to saturation of the

core. The corrections applied to readings based on the nominal ratio of the transformer, however, will assume considerable magnitude over the useful range of secondary currents. This applies also in the case of the definite time limit overload relay.

3. *Inverse-time-limit overload relay.* The foregoing conditions also apply to this relay, and in addition it will be found that the inverse-time-limit characteristic of the combination will approach a definite-time-limit characteristic from the point of the curve where saturation of the transformer core begins.

Curve Sheet No. 5 shows a typical inverse-time-limit curve of a commercial relay, and the results obtained from its combination with a bushing type transformer. Curve No. 4 (*a*) is the ratio curve obtained from a bushing type current transformer with a nominal ratio of 100/5. Curve 5 (*a*) is a typical curve of a commercial inverse-time-limit overload relay of time on times-current-tap-setting tending to a definite time at heavy overload. The curve selected gives three seconds at a current fifty per cent. greater than the current tap used. With the 5 ampere current setting on the relay the overall characteristic is represented by curve 5 (*b*), and with the 8-ampere setting by 5 (*c*) with time in seconds as ordinate and effective primary amperes as abscissa.

It is evident from these curves that careful attention must be given to the effect of transformer ratio errors if satisfactory selective action is to be obtained. When the current transformers are all of the same type and ratio, so that the same or proportional errors are introduced in each case, the effects of these errors are largely screened out, but this would be a very special case. The following example will illustrate the importance of this feature in operation.

Assuming a transmission line with distributing stations approximately as shown in Fig. 8, the current transformer ratios and relay settings may be selected as indicated in the figure. In order that continuity of service be assured for all but extreme conditions, the main line relays would probably be given a setting of not less than two hundred per cent. normal load capacity. Then if in stations Nos. 1 and 2 transformers and relays of the characteristics given in curves Nos. 4 and 5

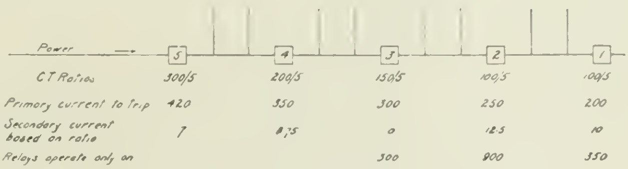

FIG. 8. Transmission line, branches, and sub-stations.

are used, the actual primary current for which No. 1 is set is 350 amperes instead of 200, and for No. 2 approximately 900 amperes. Station No. 3 may be equipped with transformers of 150 to 5 ratio, and having a higher number of turns in their secondary windings their characteristic curves will be somewhat better with equal secondary loads, but not nearly so good as those of the higher ratio transformers in stations Nos. 4 and 5. It will thus be seen that it may be possible to trip out the line at stations 3, 4, or 5 under ordinary heavy overload conditions which should have tripped out station 1 only. A heavy short circuit developed in the section fed through station 1 will change the inverse-time-element characteristic to a greater extent than in the previous case, adding to the previous probability the further chance that the relays in station 5 will operate their trip coils more quickly than any of the other stations due solely to the better characteristics of the current transformers of the higher ratios. This may be explained as follows:

Comparison between curves 5 (*b*) and 5 (*c*) show how the time element is altered by changing from the 5 ampere to the 8 ampere current tap setting on primary amperes as abscissa. The practical limit of the secondary current from this transformer was found to be about 13.5 amperes, and hence the time of operation for the largest primary current possible could not be expected to fall appreciably below the time required for 13.5 amperes to operate the relay. The definite time characteristic then approaches a value in seconds given by $13.5/5=2.7$ times or $13.5/8=1.7$ times current-tap setting according to whether the 5-ampere or the 8-ampere tap be used. Obviously if the current transformer could not give 10 amperes, such a setting on the relay would never give operation. It should be noted, however, that the impedance of the relay may be altered with the change of current setting, depending on the type of relay, and if such be the case, the results are likely to be slightly better than that illustrated.

The foregoing numerical results are based on single phase operation. The modifications introduced in the case of polyphase connections being of special character are considered in section VII.

4. *Reverse-watt relay.* This relay is also entitled to some consideration in this connection. The criterion by which its usefulness is to be judged is that it must be capable of operating on a predetermined amount of power flowing in a direction the reverse of normal flow. Operating on a wattmeter principle any and all factors affecting wattmeters will affect the relay, hence not only mean effective values of current and voltage but their wave forms are important.

The mean effective values of voltage and current operating the relay depend on the resistance of the fault, impedance of the lines, and capacity of the system on which the fault occurs. Relays are made reasonably sensitive to small values of either quantity of proper wave-form, but the method of supplying them may seriously affect the results obtained. The following example from actual test-results illustrates what may happen, and probably does in some cases, where bushing type transformers are used to supply reverse-watt relays.

Oscillogram No. 18 shows the wave form of (*a*) supply voltage, (*b*) primary current to the transformer, (*c*) secondary current from the transformer to the relay. The total impedance of the secondary circuit was slightly less than 1 ohm at 0.7 power factor. The current transformer had a nominal ratio of 100 to 5 amperes, on the basis of single-turn primary. The primary voltage had a mean effective value of 96 volts, the primary current was 112.5 amperes through 8 turns, equivalent to 900 amperes with single-turn primary, and the secondary current was 12.6 amperes through 8 turns. Sixteen equidistant ordinates were drawn in one-half cycle, and the mean of the products of instantaneous voltages and currents measured at these ordinates was found to give 760 watts. This represents a component of current in phase with the supply voltage of $760/96 = 7.9$ amperes out of a total current of 12.6 amperes, practically all of which should have been in phase with the supply on account of resistors in the primary circuit, and at the same time the primary current was equivalent to 900 amperes through 1 turn. As a check on the above wattmeter readings were taken later with conditions closely approximating those just discussed with the following results: supply voltage 110 at 25 cycles; primary current 900 amperes through 1 turn; secondary current 13.2 amperes; power by wattmeter, reading the product of supply voltage and secondary current, 896 watts. This gives a component of current in phase with the supply voltage of $896/110 = 8.1$ amperes, a fair check on 7.9 above.

A factor tending to ensure operation of watt relays would be the transmission line reactance which tends to cause the primary current to lag behind the potential from which it is supplied. The secondary current then leading this in phase would be pulled more into phase with the supply voltage, and as the secondary current wave contains a fundamental harmonic which is a large percentage of the total current the torque of the relay would be increased.

These relays are usually set for instantaneous action, hence the time element seldom requires consideration. If such were required, however, it would be based on the torque as obtained by the wattmeter element, which would depend on the primary power factor as well as wave form distortion.

5. *Balanced feeder protection.* Since in feeder protection the current at one end of the cable is practically the same as at the other end, current transformers of the same ratio are likely to be used. In order that there

be no tripping current from their secondaries, however, it is necessary that they be of the same type or sufficiently similar in characteristic to keep the tripping current to a value below that required for opening the circuit over the complete range of operation. Ratio and phase angle errors both require consideration, and it is not always readily apparent that each transformer may be giving exactly 5 amperes but due to a phase angle difference of 10° a differential current of 0.87 amperes may appear. This current, however, cannot be obtained directly from the ratio and phase angle curves of the two transformers taken independently unless the impedance of the path of the differential current, in reality the relay circuit, is very small compared with that of the normal secondary circuit. The tendency of a higher impedance is to decrease this difference in secondary currents and its maximum value is obtained from the curves as suggested above.

If the method of opposing secondary voltages be adopted it is necessary to take certain precautions in order that normal conditions be not misrepresented. Oscillograms 36 and 37 illustrate the precautions to be taken. In these cases cores of the same mean length were used, excited with the same current. The products of area and secondary turns were equal, and throughout the full range of open-circuit secondary voltages the values obtained were very close to each other. Upon checking for polarity, however, a voltage was measured as given in the following results where V_x and V_s are the two open circuit voltages and $V_x - V_s$ their vector difference.

Oscillogram	Voltages V_x	V_s	$V_x - V_s$	Excitation
36	(*b*) 40·5	(*c*) 40·5	(*a*) 2·75	
37	44·5	(*c*) 44·5	(*a*) 3·65	(*b*) 190 amp. turns

Wave (*a*) in each shows the general characteristic of this voltage and apparently its presence is due mainly to the fact that the transformers are slightly different in the core loss per unit volume. As this voltage is practically in quadrature with the applied primary voltage it represents very little power available, hence will not operate a device requiring much energy, the limiting amount being of the order of the difference between the core losses of the two transformers at that density. If a more sensitive device be used and trouble occurs this factor may be of sufficient importance to warrant consideration.

6. *Transformer protection.* In order that a transmission system be protected from the effects of the failure of a power transformer, current transformers are required in the high and low tension sides and their secondaries connected in series.

These secondaries may be connected in phase opposition, thus depending on the difference in the secondary voltages to operate the relay and requiring approximately equal open circuit voltages over the full scale under normal operation. It is evident then that the two current transformers should reach saturation point in their cores under the same load conditions to ensure this requirement. In general this would probably mean such a heavy overload that the circuit should be opened somewhere, but the relays in this case are likely to be set for instantaneous action and would operate more quickly than those delayed by time elements. Hence if overload conditions of the current transformers be not carefully studied, an operation may occur which will make it difficult, if not impossible, to localize the real cause of the trouble.

With secondaries connected directly in series, exactly similar currents must be given by the current transformers, or the differential current will flow through the relay circuit. Different ratios are thus required on the high and low tension sides with the probability of much greater difference in ratio and phase angle errors unless care is taken in the selection of the transformer and their loads. For this purpose it is immaterial what the ratio and phase errors may be, provided they balance at all loads with the power transformer in good condition.

The unavoidable errors in the series transformers of single-turn primaries make it very difficult, if not impossible, to select a pair of such transformers that will fulfil the above requirements. The high tension side, the side of the lower ratio current transformer, is the more likely to be supplied with a bushing type transformer having a greater mean length of core than that on the low tension side. This fact tends to increase the errors which already discount the low ratio transformer for this purpose.

As far as the writer is aware, the bushing-type transformer is not recommended for power transformer protection, and from the results obtained on overload tests it appears that incorrect operation would undoubtedly follow its use in any simplified form.

Normally each single phase transformer is protected by itself as a unit, and no additional complications are introduced. A special case may occur in star-delta or delta-delta three-phase power transformers.

7. *Miscellaneous.* It is evident from the results of the foregoing tests that a model transmission line, constructed for the purpose of making tests on relays and their combinations, should be equipped with current transformers which have characteristics similar to those in actual service or the results obtained may be quite misleading.

A very important field of application for a single-turn primary series transformer with a core which may be separated and clamped around a conductor is in checking up currents on bus-bars, feeders, etc. For this purpose the burden of the transformer should be as small as possible and an ammeter used giving a full-scale reading on small currents say of one ampere, which really means a high ratio transformer. Then with the transformer and ammeter calibrated as a unit fairly correct results for the true primary current may be obtained. The phase angle error is likely to be too large, however, for even approximately correct wattmeter readings at low power factors.

VII. ALTERATIONS INTRODUCED IN RATIOS AND WAVE FORM BY VARIOUS COMMON THREE-PHASE CONNECTIONS.

From the wave form of the secondary current obtained under heavy overload conditions on single phase tests and the large third harmonic that appears in the analysis of the current wave some study of the effect of the common three phase connections seemed advisable. Three transformers were selected very similar in characteristics and each with the connected load of ammeter, relay and leads gave results closely approximating curve (*a*), Curve Sheet No. 6. A nominal ratio of 125/5 was thus obtained through secondary circuits of approximately 0.9 ohms resistance and 0.7 ohms reactance each and these transformers were used throughout this series of tests.

(1) *Star-connected secondaries with and without a common lead between star points*, Fig. 9. In the tests the resistance of the neutral wire was very low, practically nothing more than a 25-ampere ammeter and a knife-switch.

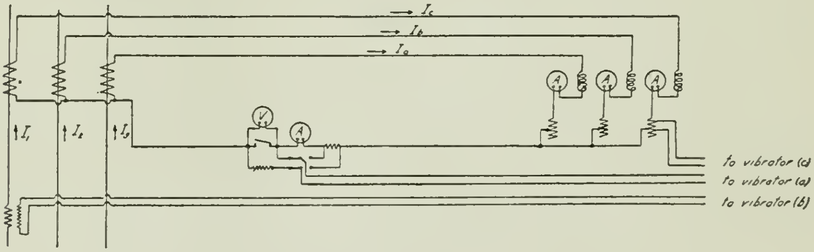

FIG. 9. Star connected secondaries.

The curves of ratio are plotted as (*b*) and (*c*) on Curve Sheet No. 6, (*b*) giving the ratio with neutral wire closed, (*c*) with neutral wire switch open. Corresponding tests on (*b*) and (*c*) were taken with very short time intervals between and comparisons between these values are much more accurate than reference to (*a*), the single phase ratio curve as the values of the latter were altered appreciably by the heating of the secondary circuit.

These curves (*b* and *c*) show that the ratio is altered by the connection used, a higher value of secondary current being obtained by using the fourth wire. Oscillograms were also taken with the following conditions:

Osc.	Primary Amp.-turns	Secondary Current I_c	Voltage V_d	Current in 4th wire I_d
22	(*b*) 425 A.T.	(*c*) 10.35 amperes	(*a*) 8.5 volts	Open circuit
23	(*b*) 420 "	(*c*) 10.8 "	Closed	(*a*) 12 amperes
24	(*b*) 950 "	(*c*) 12.9 "	(*a*) 11.1 volts	Open circuit
25	(*b*) 950 "	(*c*) 13.7 "	Closed	(*a*) 18.2 amperes

Waves 23 (*c*) and 25 (*c*) show currents similar in wave form to that obtained on single phase test (Nos. 18 and 19); 22 (*c*) and 24 (*c*) are the corresponding currents without their third harmonics. The third harmonics of the three waves of 23 (*c*) and 25 (*c*) add to make up the currents in the fourth wire shown by waves 23 (*a*) and 25 (*a*).

Normally, the subtraction of the third harmonic and its multiples of 6.1 amperes from the original wave of 13.7 amperes would leave an effective value of $\sqrt{13.7^2-6.1^2}$ or 12.3 amperes. Test results, however, show this value to be 12.9 amperes and the current wave as shown in 24 (*c*). The impossibility of holding other conditions, including primary current, unchanged, however, prevents anything but approximate results being obtained, though a tendency to lower values is evident.

The large value of the third harmonic and its multiples obtained in the fourth wire suggests its use as an indicator of heavy overload conditions. If the loads fed by the primary currents are balanced and fairly free from third harmonics it might be possible to make use of the inherent weakness of low ratio transformers without objectionable interferences. Where an open circuit voltage of 11.1 volts is available and a capacity of 18.2 amperes which is larger than that obtained in any of the lines and due to a distinctly faulty condition, it should be possible to insert a relay of low impedance which would operate only under the most extreme conditions. Such a relay operating on the effective third harmonic would require a minimum of reactance for satisfactory service.

(2) *Reversed V connection or star-connection with two transformers.* The secondaries of two transformers connected as in Fig. 7 normally give a third current I_a equal to I_b or I_c when loads and power factors of the lines 1, 2 and 3 are balanced. The current transformers in lines 1 and 2 are practically operating single phase, but the distortion of the secondary currents I_c and I_b under heavy overload conditions gives a resultant current I_a greater than I_c or I_b due to the addition of the third

harmonics and their multiples. Oscillogram No. 26 shows: (*c*) the typical secondary current with inductive load 12.8 amperes; (*b*) primary current giving 770 ampere turns, (*a*) the current I_a in the third wire of 15.7 amperes. As may be noted from the following experimental results, this variation in value of I_a from that of I_c and I_b commences when saturation in the cores begins and distortion in the secondary wave form commences.

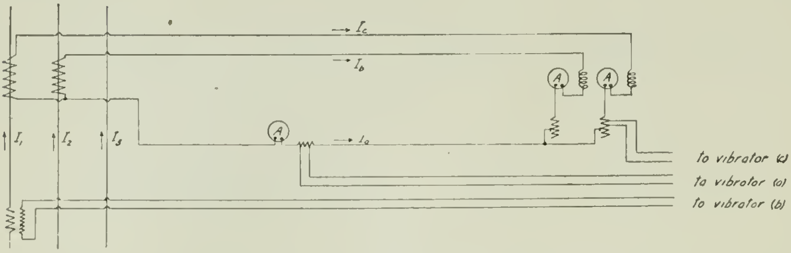

FIG. 10. Reversed *V* connection of secondaries.

Primary Ampere Turns			Secondary Currents		
I_1	I_2	I_3	I_c	I_b	I_a
440	435	430	11 amperes	11.2 amperes	12.8 amperes
590	590	570	12.1 "	12.2 "	14.4 "
675	680	650	12.6 "	12.65 "	15.2 "
770	770	720	12.8 "	12.9 "	15.7 "

(3) *Open delta or V connection.* The currents in the lines as shown in Fig. 8 normally have the currents I_c and I_b the same as in single phase operation, but current I_a is the vector difference between I_c and I_b or numerically $\sqrt{3}$ times either I_c or I_b at balanced load and power factor.

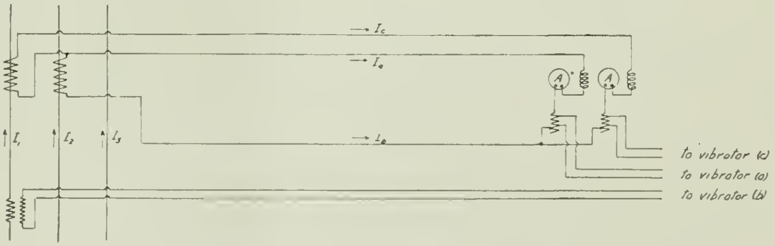

FIG. 11. Open delta or *V* connection of secondaries.

Oscillogram 27 (*a*) shows this current I_a in which according to theory, the third harmonics of I_c and I_b have been eliminated while the fifth harmonic remains in the same ratio of amplitude to the fundamental. Wave 27 (*c*) shows $I_c = 12.1$ amperes; wave 27 (*b*), $I_1 = 700$ ampere-turns; and 27 (*a*), 19.1 amperes in root-mean-square values. The latter is thus lower by 1.8 amps., *i.e.*, $\{(12.1 \times 1.73) - 19.1\}$ than would be obtained with corresponding effective values of pure sine waves and is due to the exchange of the third harmonics of current and their multiples between transformers.

(4) *Closed delta connection.* This connection is not much used for protective apparatus as far as the writer is aware, but is common in metering equipment. A possible application, however, requiring the use of the overload characteristics is in connection with a three-phase transformer star-delta connected or delta-delta where it is not possible to insert the current transformers inside the delta and a representation of the actual current in each phase is required. When a delta connected source of supply and the current transformer secondaries are arranged as shown in Fig. 12 the system works out as follows:

Current through relay or meter say $I_d = I_a - I_c =$ equivalent of $I_1 - I_3$ = equivalent of $(I_x - I_z) - (I_z - I_y) = I_x + I_y - 2I_x$; but with pure sine waves $I_x + I_y + I_z = 0$, hence $I_x + I_y = -I_z$, therefore I_d is the equivalent of $-3I_z$.

Similarly I_e is equivalent of $-3I_x$ and I_f of $-3I_y$.

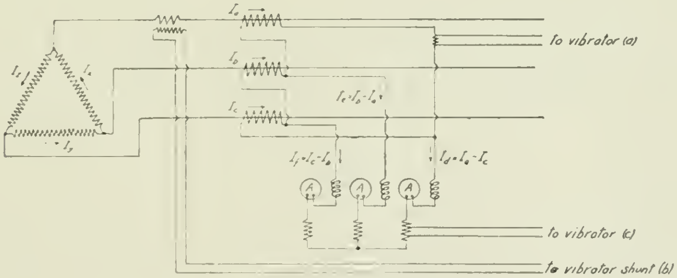

FIG. 12. Closed delta connection of secondaries.

In the above and in following pages where sums or differences of quantities are referred to, it is to be understood that the symbols denote vectors or complex quantities.

The foregoing results, being based on the assumption of a pure sine wave, no indication is given of what may occur with distorted secondary current wave forms such as occur when the current transformer core is worked beyond the saturation point. Actual tests show conditions to be very much altered and oscillogram No. 28 was taken with wave $(a) = I_a$ of 24.9 amperes, $(b) = I_1$ giving 670 ampere turns, $(c) = I_d$ of 6.5 amperes. The currents in other parts of the circuit at the same time were as follows, where I_1, I_2, I_3 are currents through the one-turn primaries:

I_1	I_2	I_3	I_a	I_b	I_c	I_d	I_e	I_f
670	625	735	24.9	25.4	24.3	6.5	6.1	6.0 amperes

The impedance of the delta is evidently very low to permit the large third harmonic of current shown by oscillogram 28 (*a*) to flow. This may be accounted for by the fact that since the cores are in a saturated condition most of the time the presence of the iron has little effect on the reactance.

The results obtained in this test indicate that the resultant values of the harmonics in wave-forms are quite arbitrary values, and because they appear in certain proportions under one condition is no guarantee that they will maintain those proportions under other conditions. It is necessary from time to time to remember that after all we have not these harmonics as we express them, but a single value of current at each instant.

(5) *Z connection.* In this connection, as indicated in the diagram Fig. 13, the closed loop forming the path of the circulating current is through the three secondary windings and meters, if meters be used as in Fig. 10. If the primary currents be balanced and the current transformers have the same characteristics two of the third harmonics of e.m.f. in the secondary voltages will be in the same sense around the loop, while the third will be in the opposite sense. We would thus expect better results than from the case of the normal delta connections and the results of the following test show this to be the case. On account of the fact that the harmonics cannot be considered alone in their effects it is practically impossible to make such deductions from the case of the closed delta as will give correct numerical results in the Z-connection. An examination of the test results and corresponding oscillograms will assist in an understanding of the phenomena involved.

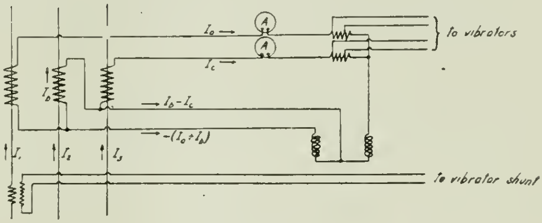

FIG. 13. *Z* connection of secondaries.

Primary Amp. Turns				Secondary Currents					Oscillogram	
Conditions	I_1	I_2	I_3	I_a	I_b	I_c	(I_a+I_c)	(I_a+I_b)	(I_b-I_c)	
A	(*b*) 370	350	310	23	22	24.2	7.5	(*c*) 9.5		No. 29
B	320	310	280	18.5	18.5	17.8	6.1	8.8	13.5	
B	440	440	400	(*a*) 27.0	26.5	24.8	(*b*) 7.1	(*c*) 10.6	16.4	No. 30

Condition "*A*" involved a closed loop of approximately 0.25 ohms resistance which was increased to 0.55 by inserting approximately 0.15 ohms non-inductive resistance in each of lines *a* and *c* corresponding to meter coils as condition "*B*". This amount of resistance has apparently but little effect on the current from the transformers.

Oscillogram No. 29 shows (*a*) supply voltage; (*b*) primary ampere turns $I_1=370$; (*c*) secondary current $I_a+I_b=9.5$ amperes. Oscillogram No. 30 shows (*a*) secondary current $I_a=27$ amperes; (*b*) relay current $I_a+I_c=7.1$ amperes; (*c*) relay current $I_a+I_b=10.6$ amperes.

The numerical results just given have little practical significance except to show that extreme variations in results do occur by reason of the interconnected secondaries, and that these variations are generally such as tend to render the combination less effective than would be the case if single-phase operation could be adopted.

VIII. SUMMARY.

The true ratio of the current transformer with a single turn primary is found to depart much more from the ratio of turns than that in which the primary winding consists of a number of turns. The phase angle at non-inductive secondary loads is also much greater. While compensation may be made for one value of secondary current at one secondary load the general effect of increased core-length is an increase in the errors.

A serious limitation in the lower ratios is the inability of the transformer to develop a voltage great enough to deliver heavy overload current through the usual secondary circuit impedances. The maximum available secondary voltage is a direct function of the frequency, area of the core and number of secondary turns and is accompanied by considerable distortion in the secondary current wave. Beyond this the primary current takes a "spill-over" effect due to the saturated condition of the core, without a proportionate increase of the secondary current. This weakness in operation thus results in a wide variation in the ratio when operating with saturation of the core and also results in a change in the time-element characteristic based on primary currents when such a transformer supplies an inverse-time limit overload relay.

Variations in the results are obtained on overload tests, in different common methods of interconnecting transformer secondaries for supplying meters and relays, due to the distortion of the secondary current wave form.

A method of calculating the ratio and phase angle from actual test values is given which gives quite accurate results for any purpose for which these transformers are suitable.

The foregoing discussion of the application of single turn primary transformers shows the advantage to be gained by working at as low secondary currents as possible, and hence using as high ratios as permissible particularly in the values below 300/5. In general the limitations are to be found in the lowest value of current at which the type of relay to be used will operate satisfactorily and in the probable minimum value of current overload in the primary for which adjustments are likely to be made. Supposing then that these values are 6 and 300 amperes respectively, then a ratio of 250/5 would be a reasonable one to use notwithstanding the fact that normal load current may never exceed 100 amperes.

APPENDIX.

Secondary Leakage Reactance.

(1) *By Measurement.* A study of the magnetic field distribution of a secondary winding which is wound uniformly in the form of a toroid will show that the flux of self-induction is limited to that which will pass between the centre of the conductor itself and a neutral line midway between adjacent turns. Any flux which exceeds this limit is picked up by successive turns, is carried completely around the length of the core and hence becomes a mutual flux.

This means that if we wind an exploring coil on a transformer of this type, placing it midway between turns, it will have the same number of turns as the secondary winding, and will give the nominal induced e.m.f. of the secondary winding. Another similar winding of fine wire placed centrally on the strip, if the secondary be of flat conductor, or otherwise wound closely to the secondary, will give a very close approximation to the real-induced e.m.f. of the secondary.

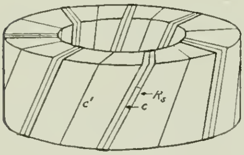

FIG. 14. Core with secondary and test windings for measuring secondary leakage reactance.

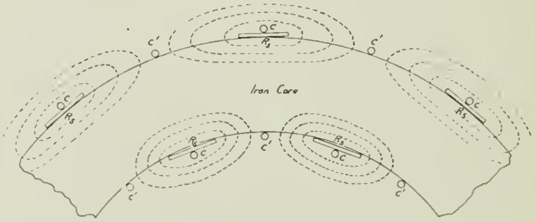

FIG. 15. Diagram showing section of circuits, and the secondary leakage flux.

This disposition of windings is illustrated by Fig. 14 and a section at right angles to the conductors or parallel to the core is shown in Fig. 15 with the resultant field distribution. The conductors represented by R_s, c and c' then have terminal voltages of the following values: R_s supplies the external load, hence its voltage may be represented by V_s, Fig. 16. Winding c has the same induced voltage but is not affected by the *IR* drop, hence gives the real induced e.m.f. in R_s. Winding c' gives the nominal induced e.m.f. of R_s for it differs from that obtained in c only through the effect of the flux of self-induction affecting the voltages of c and R_s.

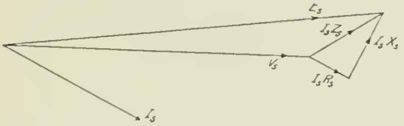

FIG. 16. Vector diagrams of voltages in windings shown in FIG. 14.

Hence tying corresponding ends of c, c' and R_s together as in Fig. 17, reading the voltage differences between opposite ends we may obtain all the results illustrated by the vector diagram Fig. 16, *i.e.*,

$$V_{1,2} = V_c - V_s = IR_s \quad V_{2,3} = V_{c'} - V_c = IX_s$$

A secondary winding was used having a resistance of 0.0088 ohms and a current of 25 amperes at 25 cycles. With a 7.5 volt Weston voltmeter $V_{1,2}$ was found to be 0.2 volt corresponding to a resistance of 0.008 ohm. The reading, however, was too low on the scale to warrant expectation of greater accuracy than 10%. $V_{2,3}$ did not show any effect that could be detected under the same conditions.

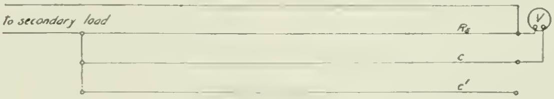

FIG. 17. Connections of windings shown in FIG. 14.

An alternating current galvanometer was then used to obtain some approximation of this value as follows: the field was supplied from a phase shifting transformer from which either of two voltages V_p or V_q in quadrature with each other were available. The phase of V_q was so adjusted that zero reading was obtained with the voltage IR_s applied to the moving coil of the instrument and V_q to the field coils (or in short form, $IR_s \times V_q = 0$) which means that the current produced in the moving coil by the voltage IR_s was in quadrature with the field flux produced by the voltage V_q and hence in phase with the field which would be produced by V_p. Thus readings are obtained by $IR_s \times V_p$ and $IX_s \times V_q$ but not with $IX_s \times V_p$ nor $IR_s \times V_q$ (see Fig. 18). It was found necessary to use a resistance in series with the moving coil giving a multiplying factor to the readings. It would appear also, on account of the long air-gap in the main field that the deflection would be proportional to the voltage applied to the field windings. The following results were obtained:

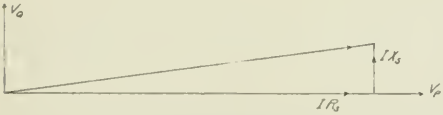

FIG. 18. Vector diagram showing how reactance test data were used.

I_s	V_q	Deflection	Multiplier	V_p	Deflection	Multiplier
8.1	82	1 div.	40	49	10.5 div.	100
15	100	1.5 "	100	58	32 "	100

From which in the first case

$$X/R=\frac{49}{82}\times\frac{40}{1050}=.0228 \text{ at 25 cycles}$$

and in the second case

$$X/R=\frac{58}{100}\times\frac{150}{3200}=.027 \text{ at 25 cycles}$$

From direct measurement $R=.0088$ ohms hence $X=.00022$ ohms approximately at 25 cycles. In the above results numerous causes of small errors may have been present, but since the main point of interest in this test is to determine the order of magnitude of the secondary reactance and it being such a nearly negligible value, these errors are unimportant.

(2) *By Calculation.* The exact calculation of the reactance of the secondary in this case would be very laborious. An approximate calculation allowing for reluctance in semicircular paths, through air only, indicates that the reactance at 25 cycles should be in the neighbourhood of 0.0006 ohms. The direct measurement whose accuracy is open to question had yielded a result 0.00022 which is of the same order of magnitude. This is so small in comparison with one ohm, the minimum burden impedance used in the tests, that it is practically negligible.

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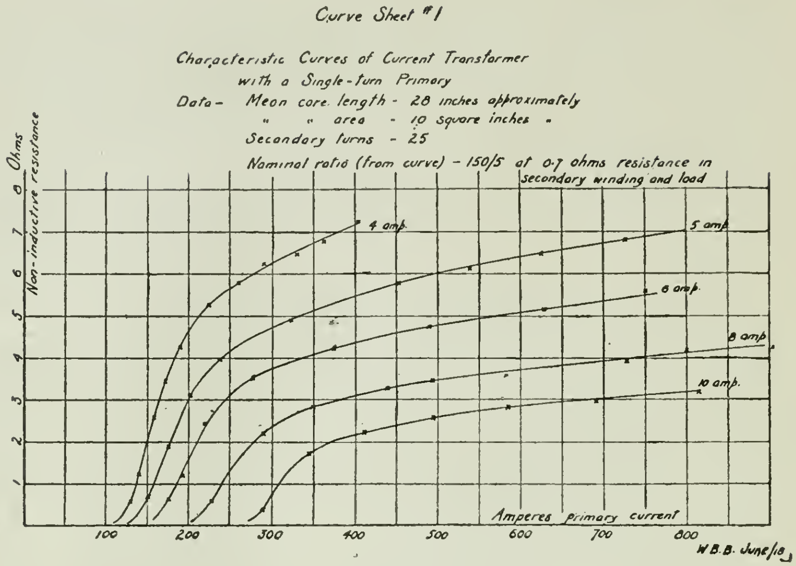
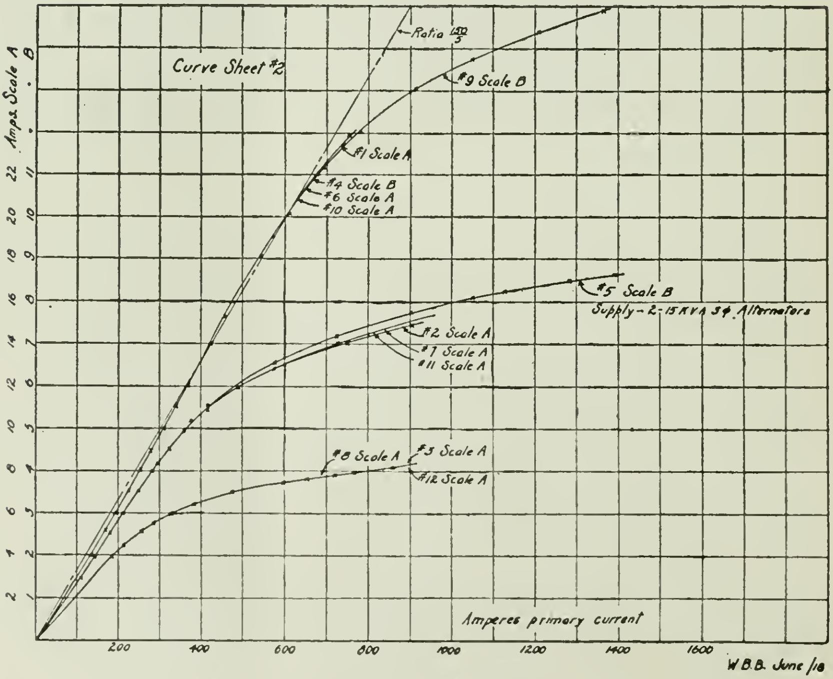


CURVE SHEET NO. 1.



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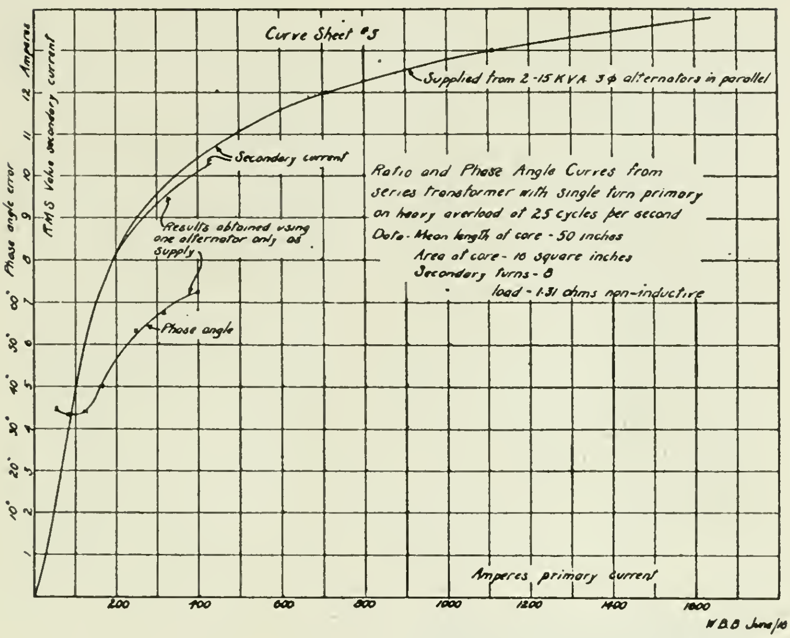

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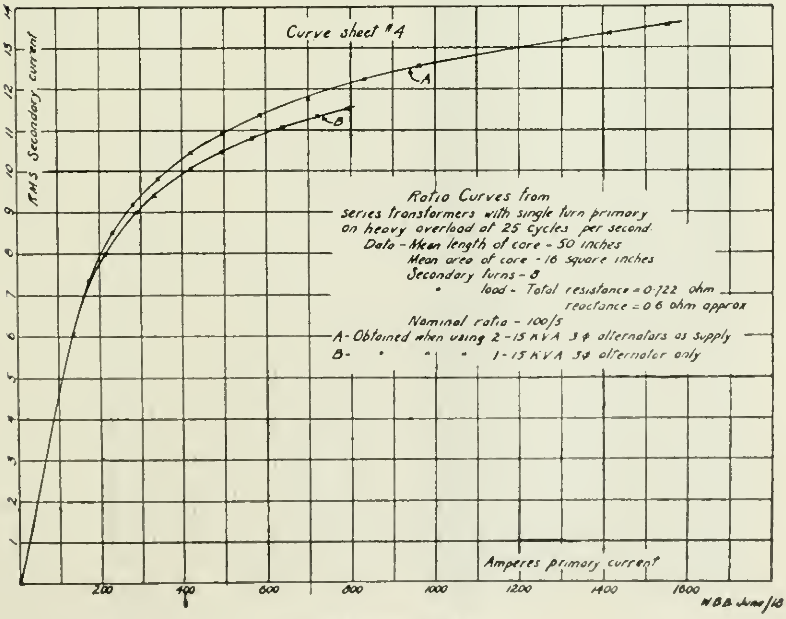

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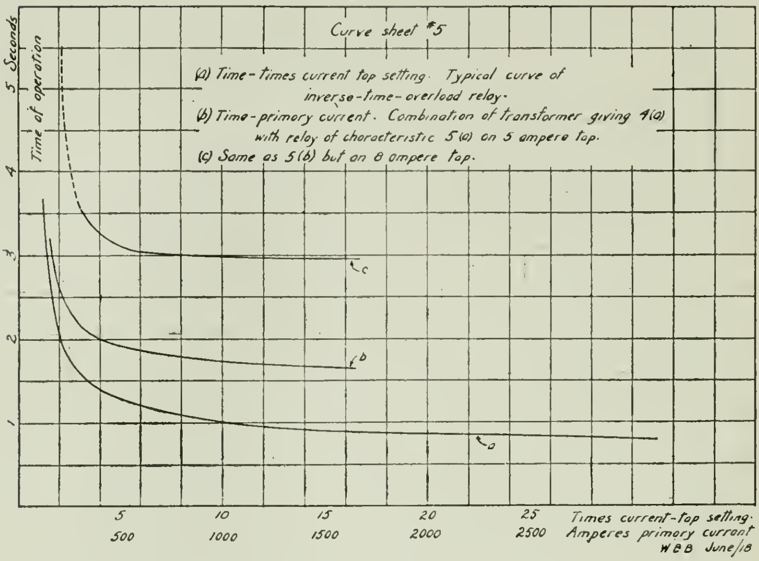

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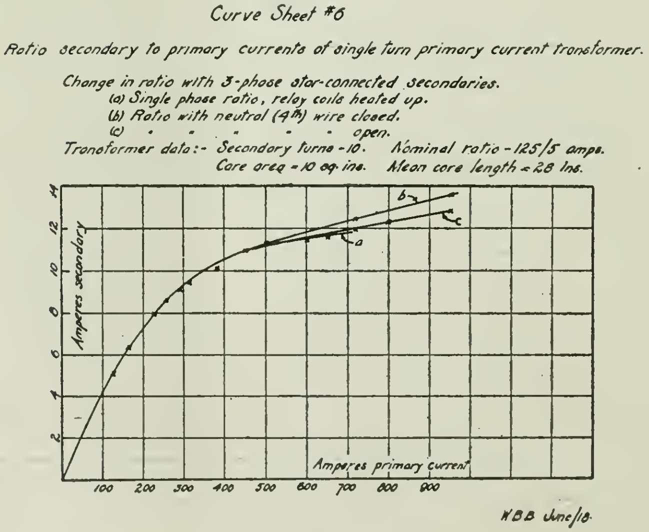

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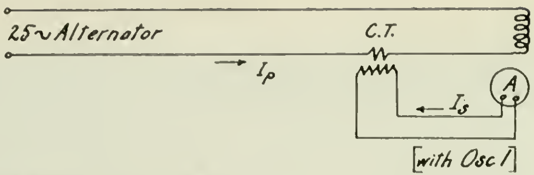


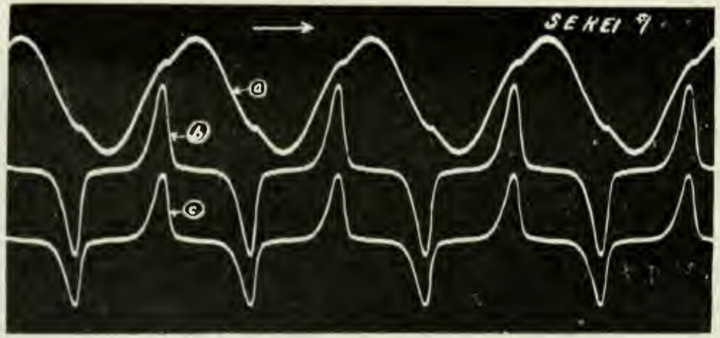
EXPLANATION NO. 1.

In the following oscillograms, numerical values given are mean effective values. The source of supply used was a G. E. 15 k.wa., 110 volt, 3 phase, 25 cycle alternator with about 8% regulation, except where otherwise stated.

Osc. 1 (*a*) Alternator voltage.
(*b*) Primary current of Weston transformer.
(*c*) Secondary current of same transformer.

Weston current transformers were used in the primary and secondary circuits of the current transformer under test.

The ability of these transformers when operated within rated load to transform badly distorted currents with sufficient accuracy for the purposes of this investigation is shown.



Oscillogram 1.

EXPLANATION NO. 2-9.

For connection diagram, see Fig. 2.

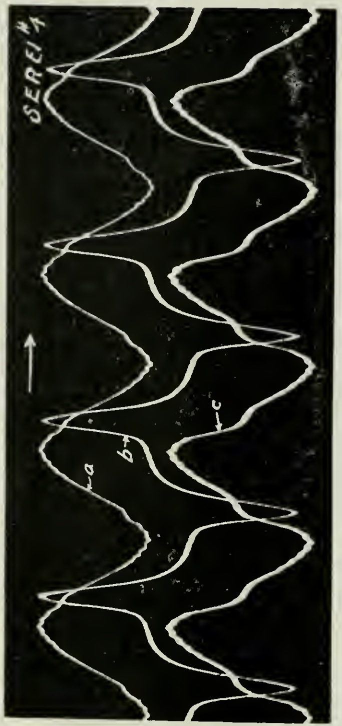
Osc. 2-9 (*a*) Alternator voltage.
(*b*) Primary current.
(*c*) Secondary current.

Nos. 2 to 9 are a series showing the development of distortions due to saturation in the core of the current transformer under test. The tested transformer had a core of 16 sq. ins. cross-section and 50 ins. mean length,—a simple annular ring of rectangular section. The primary excitation was supplied through 50 turns uniformly distributed over the core, and the secondary winding was of 8 uniformly distributed turns.

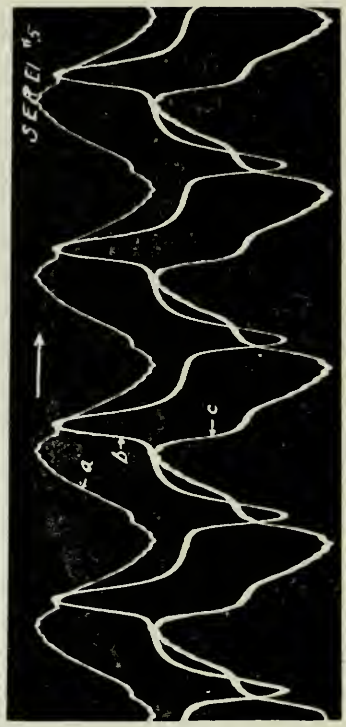
The large number of primary turns necessitated a high impressed voltage, almost equal to that of the alternator, the current transformer primary being the greatest impedance in the circuit. Hence it takes a current like the exciting current of a constant-voltage transformer while the secondary current remains more nearly the shape of the impressed voltage.

In Nos. 2 to 7 there was some resistance in series with the primary from the laternator. In Nos. 8 and 9 there was no resistance in series with the primary, otherwise similar to No. 7.

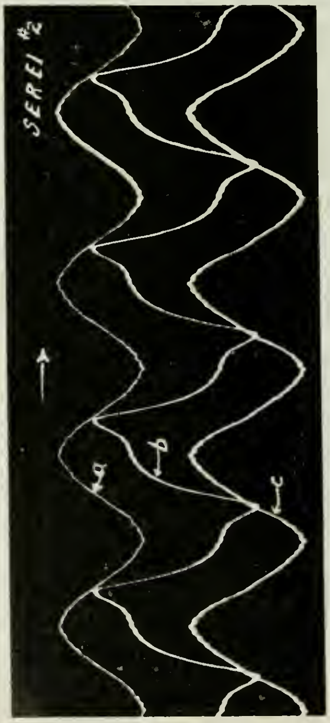
In Nos. 2 to 8 one alternator was used but in No. 9 the supply was from two alternators in parallel,,otherwise similar to No. 8.



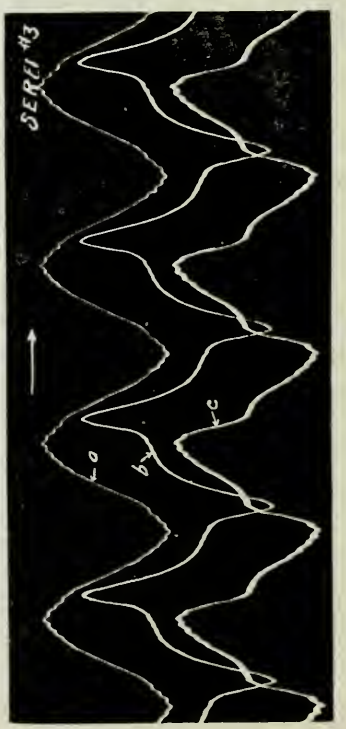
Oscillogram 4.



Oscillogram 5.



Oscillogram 2.



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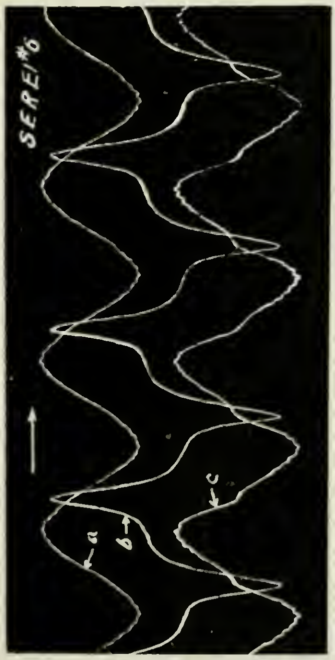
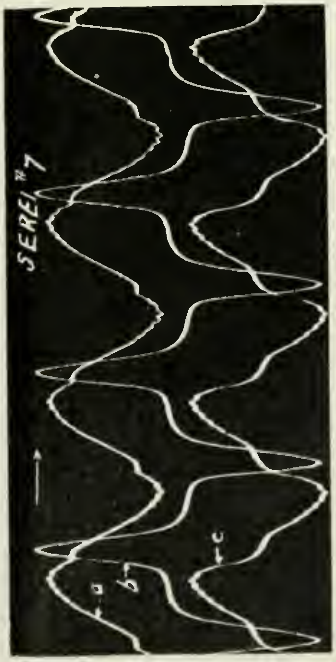
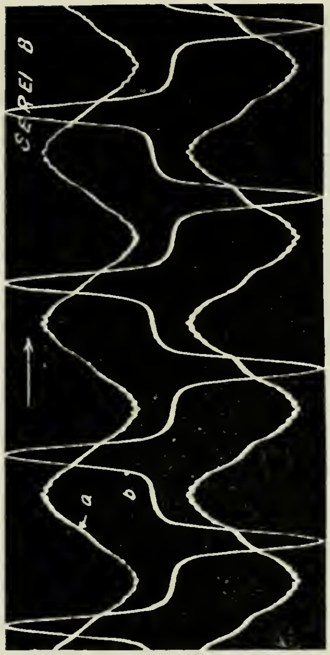


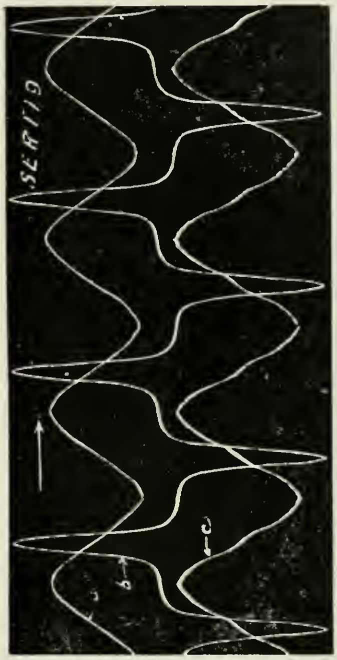
Oscillogram 6.



Oscillogram 7.



Oscillogram 8.



Oscillogram 9.

EXPLANATION 10-21.

For connection diagram, see Fig. 2.

Osc. 10-21 (*a*) Alternator voltage.
(*b*) Primary current.
(*c*) Secondary current.

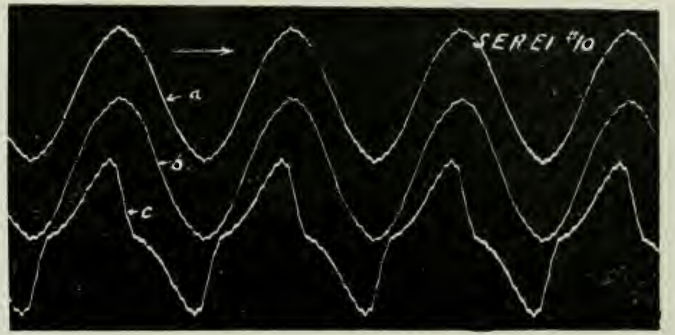
In Nos. 10 to 21, distortion of primary current is avoided by use of an 8 turn primary uniformly distributed around the core and a resistor in series with the primary.

In Nos. 10 to 17 the secondary burden was approximately 1.4 ohms non-inductive. Nos. 18 to 21 show the secondary wave form resulting with a secondary burden of 70% power factor.

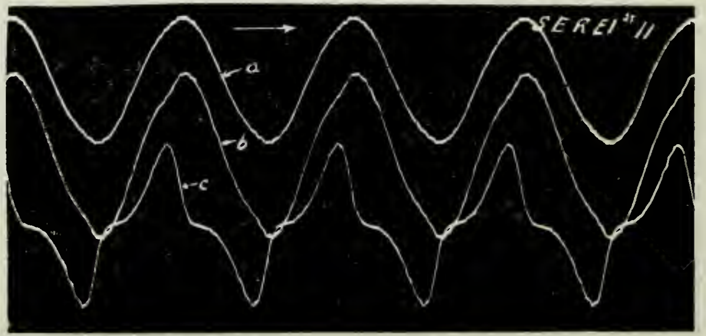
In Nos. 16, 17, 18, 19 and 21, two alternators in parallel were used as supply.

The pronounced notch in the alternator voltage wave is due to armature reaction of the primary current.

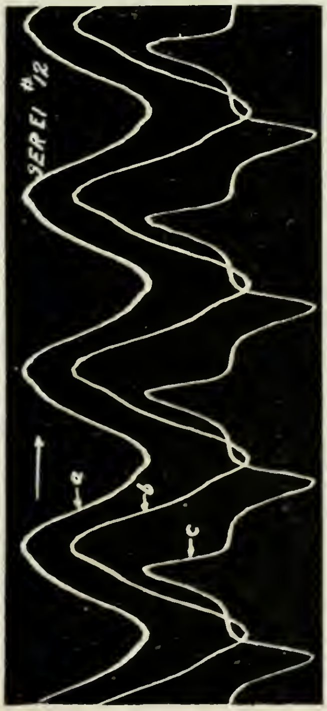
Note that the current transformer fails to deliver a secondary current during the intervals in which the core is saturated; that the secondary current, with non-inductive burden, is confined to the intervals during which flux is changing from positive to negative saturation or *vice-versa*; that the form of the part of the primary current wave occurring during core saturation will have very little effect on either wave-form or value of the secondary current.



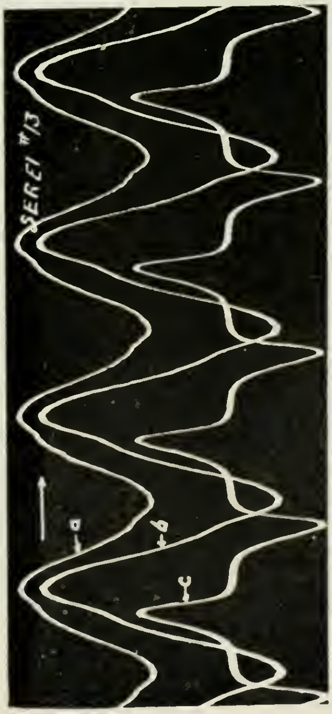
Oscillogram 10.



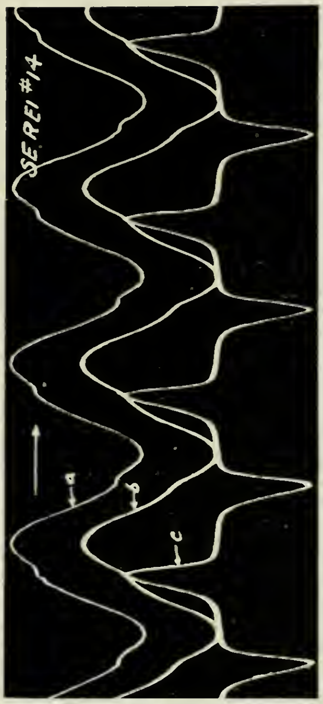
Oscillogram 11.



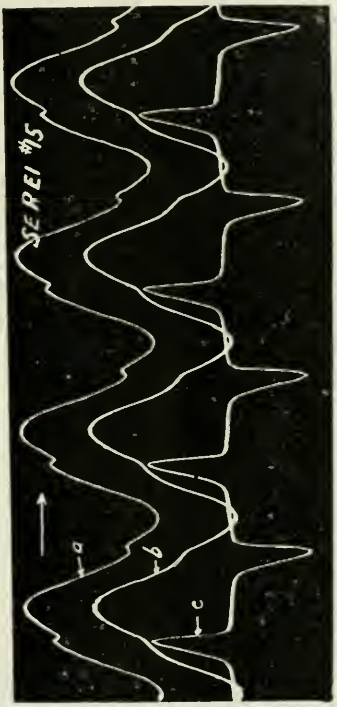
Oscillogram 12.



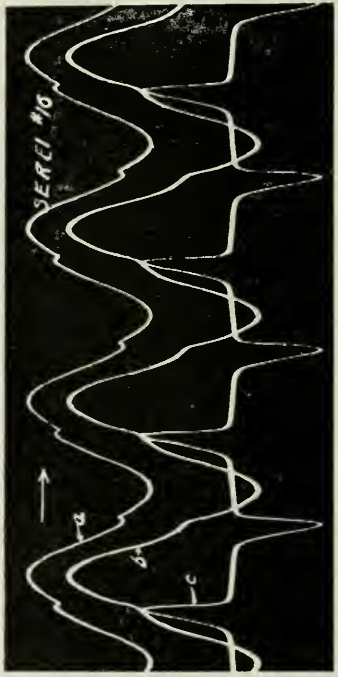
Oscillogram 13.



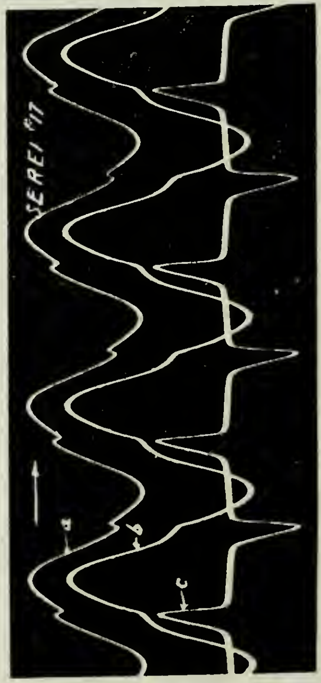
Oscillogram 14.



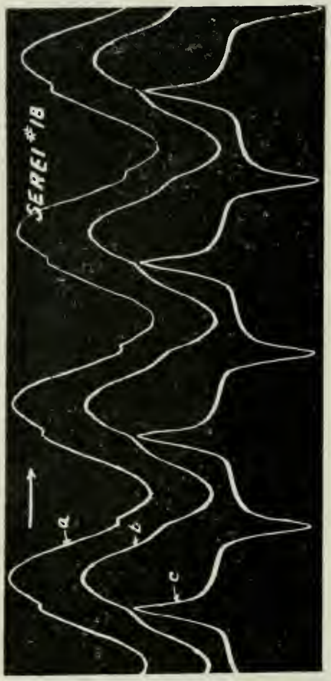
Oscillogram 15.



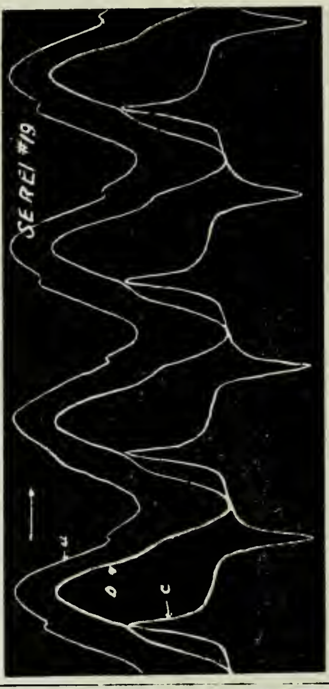
Oscillogram 16.



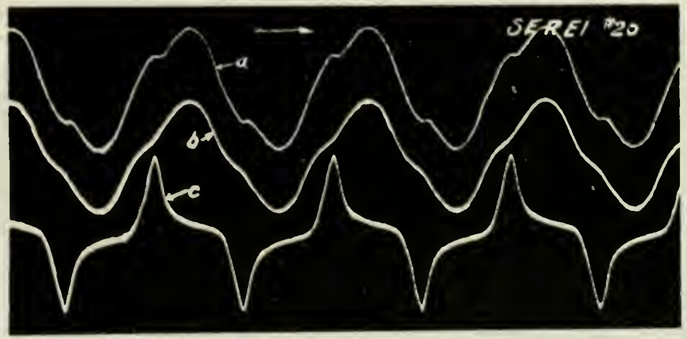
Oscillogram 17.



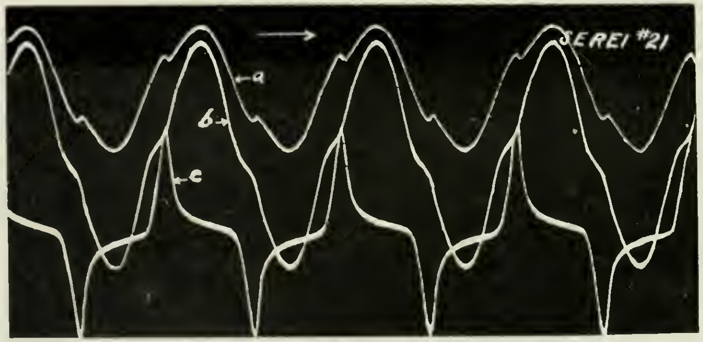
Oscillogram 18.



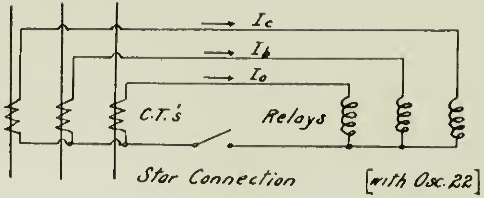
Oscillogram 19.



Oscillogram 20.



Oscillogram 21.



EXPLANATION 22-25.

Osc. 22-25 Star-connected secondaries.

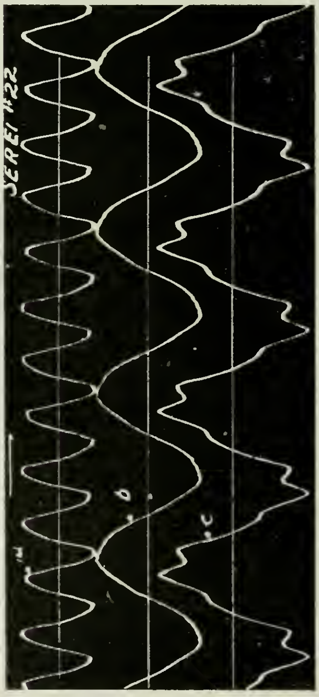
Osc. 22 and 24 Neutral switch open.

- (*a*) Voltage between neutrals.
- (*b*) Primary current.
- (*c*) Secondary current I_c.

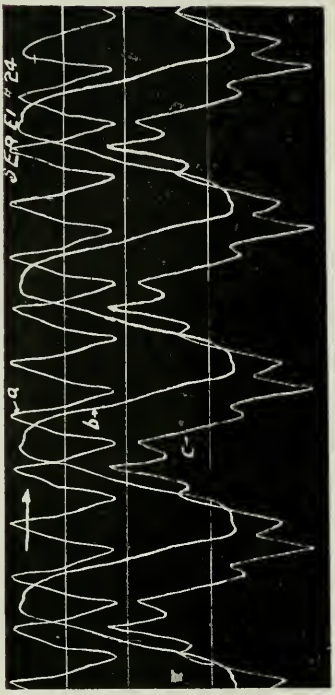
Osc. 23 and 25 Neutral switch closed.

- (*a*) Current in neutral.
- (*b*) Primary current.
- (*c*) Secondary current I_c.

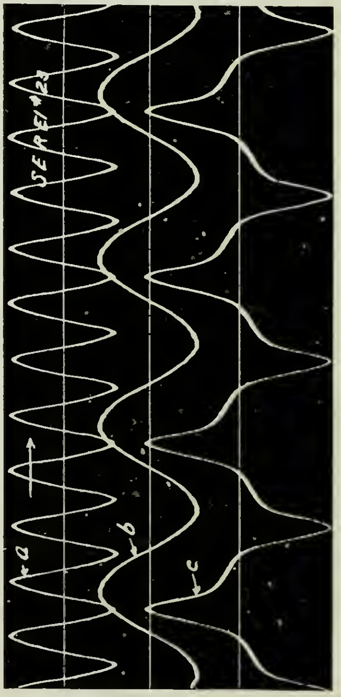
The transformers used in Nos. 22 to 30 have cores of cross section 10 sq. ins. and mean length 28 ins. with 10 secondary turns. A nominal ratio of 125/5 is obtained with secondary burden of approximately 0.9 ohms resistance and 0.7 ohms reactance or 1.14 ohms impedance.



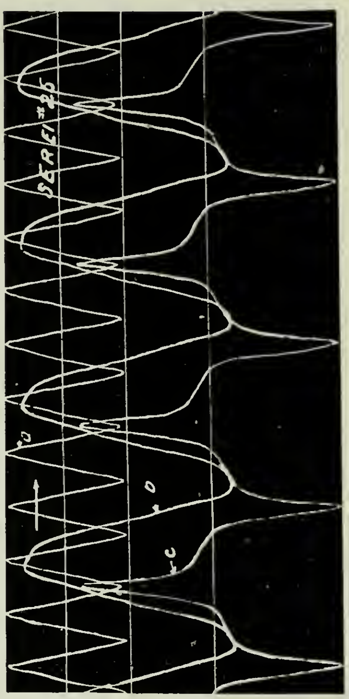
Oscillogram 22



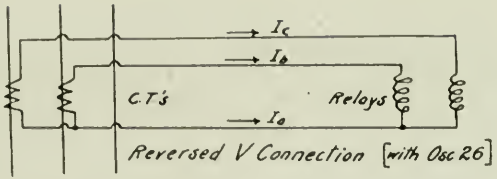
Oscillogram 24.



Oscillogram 23.



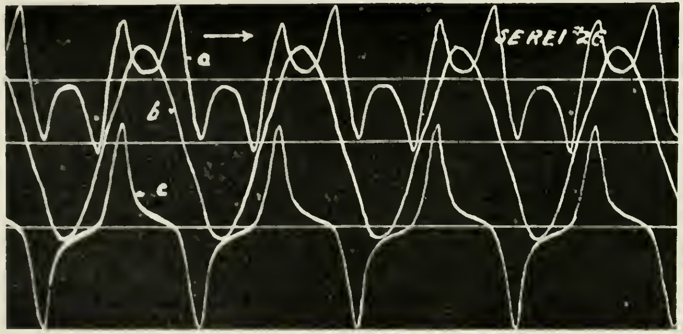
Oscillogram 25.



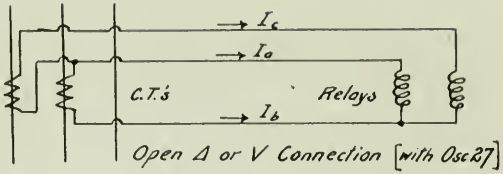
EXPLANATION 26.

Osc. 26 (*a*) Current in third wire, I_a is 15.7 amps.
(*b*) Primary current amps. reduced to one turn, 770.
(*c*) Secondary current I_c is 12.8 amps.

Above saturation, the current in the third wire, I_a, is greater than the two secondary currents I_b and I_c due to the addition of their third harmonics.



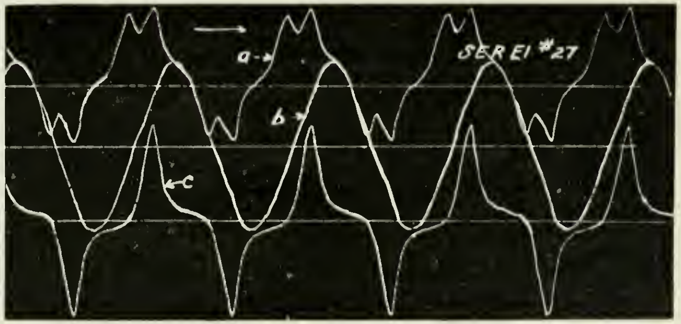
Oscillogram 26.



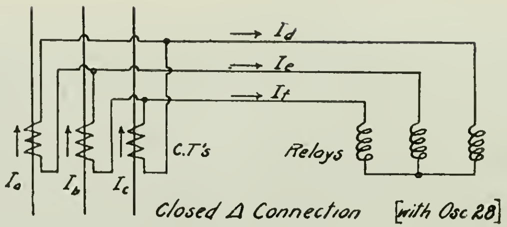
EXPLANATION No. 27.

Osc. 27 (*a*) Current I_a is 19.1 amps.
(*b*) Primary current amps. reduced to one turn, 700.
(*c*) Secondary current I_c is 12.1 amps.

Third harmonic is eliminated in I_a while fifth harmonic remains.

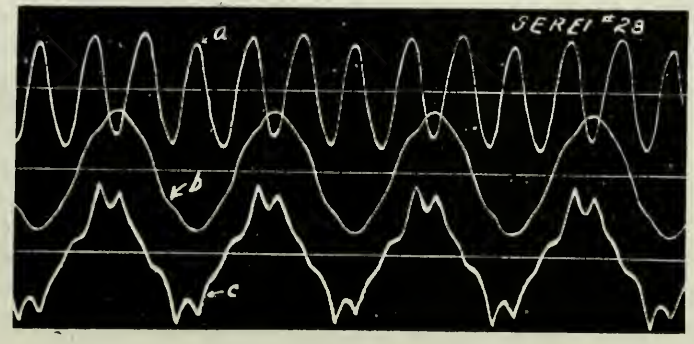


Oscillogram 27.

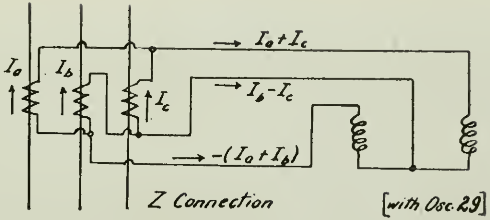


EXPLANATION No. 28.

Osc. 28 (*a*) Delta current I_a is 24.9 amps.
(*b*) Primary current amps. reduced to one turn, 670.
(*c*) Relay current I_d is 6.5 amps.



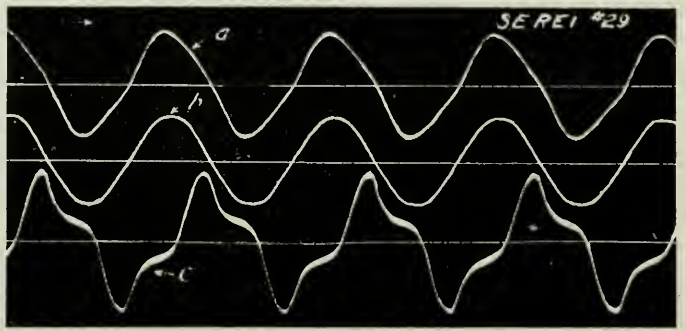
Oscillogram 28.



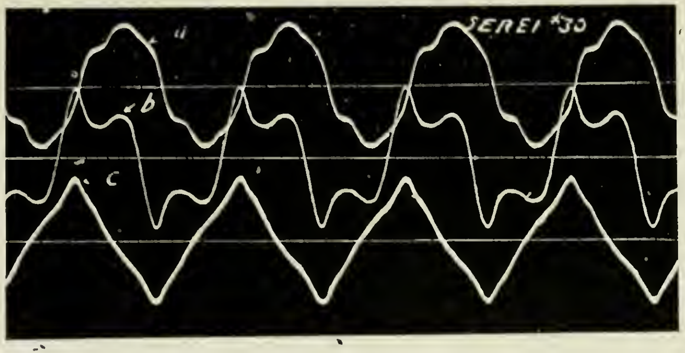
EXPLANATION NO. 29-30.

Osc. 29 (*a*) Alternator voltage.
(*b*) Primary current amps. reduced to one turn, 370.
(*c*) Secondary current $I_a + I_b$ is 9.5 amps.

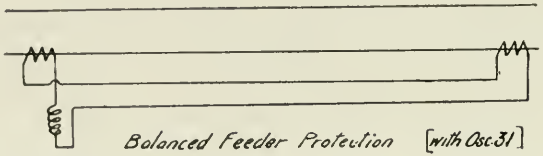
Osc. 30 (*a*) Secondary current I_a is 27 amps.
(*b*) Relay current $I_a + I_c$ is 7.1 amps.
(*c*) Relay current $I_a + I_b$ is 10.6 amps.



Oscillogram 29.



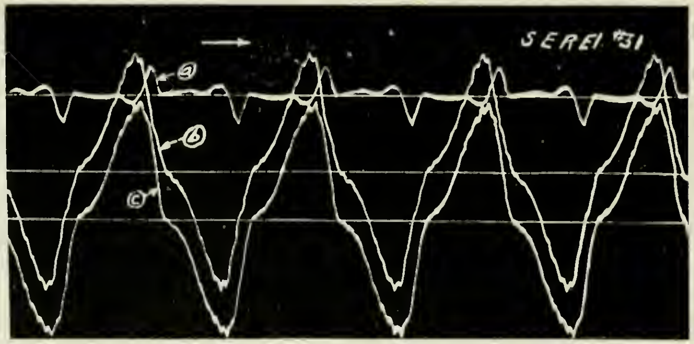
Oscillogram 30.



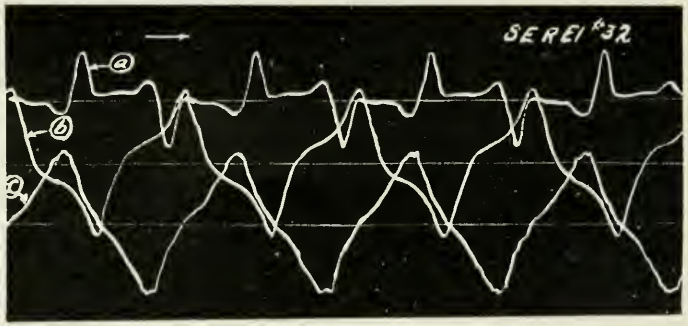
EXPLANATION NO. 31-32.

Osc. 31 (*a*) Difference between open-circuit voltages is 2.75 volts.
(*b*) Open-circuit voltage of transformer No. 1 is 40.5 volts.
(*c*) Open-circuit voltage of transformer No. 2 is 40.5 volts.

Osc. 32 (*a*) Difference between open-circuit voltages is 3.65 volts.
(*b*) Primary current amps. reduced to one turn, —190.
(*c*) Open circuit voltage of transformer No. 1 is 44.5 volts.



Oscillogram 31.



Oscillogram 32.

A PROCESS FOR ELECTROLYTICALLY REFINING NICKEL.*

BY G. A. GUESS, M.A.

If an anode of crude nickel, which may contain copper and iron, is electrolyzed in a bath of nickel sulphate in which is suspended finely divided calcium carbonate there is deposited on the cathode, if suspended in a sack diaphragm, metallic nickel practically free from copper and iron. This is the basis of a process developed in the Metallurgical laboratories of the University of Toronto. The action of the finely pulverized limestone appears to be represented by the following equation:

$$2\ CuSO_4 + 2\ NiSO_4 + 3\ CaCO_3 = 2\ CuO,\ 2\ NiO,\ SO_3 + 3\ CaSO_4 + 3\ CO_2$$

The copper is precipitated as a double basic sulphate of copper and nickel, which is quite insoluble. The function of the diaphragm is to protect the cathode from mechanical pollution by contact with the insoluble copper.

Since an equivalent amount of nickel is combined with the copper in the insoluble basic sulphate, it is obvious that the copper content of the anodes should be as low as possible. Preliminary treatment of the converter matte to remove copper is necessary. Two methods at once suggest themselves for doing this. The first is by leaching the roasted matte with dilute sulphuric, as is done in the preliminary operation for the Mond process. The second method would be a fusion with coke and saltcake, which is the preliminary step in the Orford process. The findings herewith submitted are the results of refining anodes produced by the first method.

The matte used had the following analysis: Ni 56% Cu 24% S 18%. It was crushed to 40 mesh and roasted. The calcines were leached with 8-10% sulphuric acid solution. There was extracted 50% of the copper and a small amount of nickel. The dried residues were mixed with charcoal and fused and cast into anodes. These anodes contained Ni 79.8% Cu 18.0% Fe 2.0%. The anodes were 5½×5½ inches, and had two lugs which were drilled and tapped for copper rods. Each anode weighed approximately 1,900 grammes, and two anodes were used in each tank.

The cathodes were sheet lead or sheet aluminum 6×6 inches. There were three in each tank suspended in a bag of light canvas which was pulled over a wooden frame in order to keep it distended.

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* Patented in the United States and Canada, British and Foreign patents applied for.

The electrolyte was a solution of nickel sulphate usually kept at from 5.5 to 6.5% nickel (110 to 130 lbs. nickel per fluid ton. The temperature of the electrolyte was maintained between 40 and 50 degrees C. In starting a new tank 3.5% (70 lbs. per fluid ton) of ordinary calcite crushed to 200 mesh was added to the electrolyte. The solids were kept in suspension by circulation with a pump which discharged into a small lead tank which sat on a hot plate, and from which the electrolyte flowed by gravity back to the stoneware tank. Limestone at 200 mesh was added hourly to the electrolyte. The amount added was based on the rapidity of solution of the anode. Calculated back to the original matte the quantity of limerock used ranged between one-half and three-quarters of a ton of limestone per ton of matte. More limestone was needed than the equation called for of course. A small amount of glue was used in the electrolyte.

A current density of 13 amperes per sq. ft. of cathode gave excellent nickel. The voltage required was 2 volts. The ampere efficiency at the cathode was 98%, at the anode sometimes slightly over 100%.

In practice there would be removed regularly from circulation a portion of the accumulated mud in order to keep the percentage of solids constant. This mud after washing showed on drying Ni. 8.28%, Cu 9.10%, S 8.88%, CaO 30.1%, Fe 1.4%. Fusions were made of this mud with silica as flux in graphite crucibles. The product was a slag 48%, SiO 38%, CaO and a matte containing Ni. 36.1%, Cu 37.8%, S 19.5%.

The procedure suggested is to roast this matte, then reduce and cast into anodes. These anodes will be used in copper refining tanks using the copper rich and acid liquor from the leaching operation. Good cathode copper can be produced until the copper content of the electrolyte is reduced to 1.5 to 1.7% copper. Not enough of this mud was obtained to furnish material for such anodes in a copper tank. Anodes made from other sources were used. A current density of 12 amperes .35 volts was used. Insoluble lead anodes were used after the copper was reduced to 1.47%. The voltage for the same current was now 2.2 volts. From 1.47% Cu to .054% Cu the ampere efficiency was 66.2% to the final reduction to .0115% Cu the ampere efficiency was 46.7%.

The copper house in such a refinery would receive the leach liquor from the roasted matte, would electrolyze this with anodes made from the mud in the nickel house. When the copper content of the electrolyte reached 1.5% the solution would be electrolyzed with lead anodes to remove most of the copper. It would then go to evaporating pans and crystallizing tanks. The mother liquor containing the acid and the rest of the copper would be used for further leaching. The crystals would go to the nickel house.

In the nickel house would be received the crude nickel anodes. Electrolytic nickel would be produced and a lime mud which would carry away a chemical equivalent of nickel. This nickel would be returned from the copper house as nickel sulphate. In addition to this there is required an amount of nickel as nickel sulphate, the chemical equivalent of the copper in the anodes, since this amount of nickel is plated at the cathode while the copper in the anode is being dissolved. The copper house is required to supply then 117.4 lbs. of nickel as nickel sulphate for every 63.5 lbs of copper in the anodes. To obtain this some of the nickel house anodes may be electrolyzed in the copper house to supply any nickel sulphate not made by the amount of nickel leached out from the roasted matte. The amount of acid required is similarly twice the chemical equivalent of the copper in the nickel house anodes.

The rare metals platinum and palladium which occur, do at present prices form an important constituent[1] of these mattes. These metals are retained in the slime which forms an adherent crust on the anodes, and does not tend to fall off. The slime if desired may be obtained for the most part free from the lime mud.

No laboratory work was done using a salt cake fusion. Anodes made after such a fusion might be taken as having from 2 to 5% copper. The amount of mud made would be correspondingly small and this might be returned for salt cake fusion with the original matte. The nickel sulphate required would come from the electrolytic refining of the copper made from the "Tops".

Nickel cathodes free from iron and containing as little as .01355% copper were made. A small amount of unprecipitated copper was always found in the electrolyte. It was always very small, often only .001% Cu. The double basic sulphate is easily filtered, *i.e.*, it is easily retained on any filter paper. Bags of light canvas were found more satisfactory than heavy material.

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[1] Ontario Nickel Commission Report, page 486.

University of Toronto

FACULTY OF APPLIED SCIENCE AND ENGINEERING
SCHOOL OF ENGINEERING RESEARCH

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CONTENTS

Section No. 1.

PAGE

The Toronto Aerodynamic Laboratory 3
J. H. Parkin.

Alignment of Balance and Model 16
J. H. Parkin and H. C. Crane.

Air Speed Indicators for Wind Channel 26
J. H. Parkin.

Studies with a Full Scale Four Foot N.P.L. Type Wind Channel of the Proper Form of Intake to Eliminate Disturbances 60
J. H. Parkin and H. C. Crane.

Wind Tip Research 78
J. H. Parkin, H. C. Crane and S. L. Galbraith.

Balance Portion for the Rudder of the F.3 Flying Boat 111
J. H. Parkin, H. C. Crane and S. L. Galbraith.

Macdonald Aerofoils 115
J. H. Parkin, H. C. Crane and S. L. Galbraith.

Rudder Balance for F.3 Flying Boat 118
J. H. Parkin, E. V. Ahara and J. S. E. MacAllister.

Section No. 2.

The Bellis and Morcom Spring Governor 125
J. H. Parkin.

Section No. 3.

The Imhoff Sewage Tank in America: A Decade of Operation Reviewed 143
Peter Gillespie.

Section No. 4.

PAGE

Effects of Magnetic Leakage in Current Transformers...... 167
H. W. Price and C. K. Duff.

Method of Measuring Ratio and Phase Angle............ 191
H. W. Price and C. K. Duff.

Through-Type Portable Current Transformers........... 202
H. W. Price and C. K. Duff.

Determination of Turn-Ratio of Current Transformers...... 211
C. K. Duff.

Minimizing the Errors of Current Transformers by Means of Shunts.................................... 216
H. W. Price and C. K. Duff.

Section No. 5.

True Net Sections of Riveted Tension Members.......... 233
C. R. Young.

Section No. 6.

Investigation of the Proportioning of Concrete Mixtures.... 245
Peter Gillespie and A. R. Duff.

Aeronautical Research Paper No. 1.
September 1920.

THE TORONTO AERODYNAMIC LABORATORY

J. H. Parkin*
Assistant Professor of Design

Introduction.

The experimental investigation of aerodynamic phenomena and problems in connection with aircraft is carried out in two principal ways, by means of tests of small scale reproductions or models in a wind channel and by tests on full scale aircraft in flight. There are also the less commonly used whirling arm method used chiefly for propeller investigations and the open air car and track for testing full size elements or complete machines (see references at the end of this paper). Each of these methods, the model experiments and the full-scale tests, together with theoretical mathematical investigations, is necessary to a proper development of the science and each supplements the others.

While the full scale work is ordinarily beyond the means of universities, model researches and mathematical analyses fall well within the scope of university work and afford a very promising and interesting field of endeavour. By proper co-operation between the universities carrying on such work and the government and aircraft manufacturers engaged in full scale construction and operation, aeronautical progress can be greatly facilitated with a minimum of wasted effort.

The investigation of aeronautical problems in a wind channel is analagous to the testing of ship models in an experimental basin. Wind channel testing is based on the principle of relative motion which, in brief, is that the reaction between the air and a body is the same, in magnitude and position whether the body be moving through still air or the air flow past the stationary body, the relative velocity being the same in magnitude and direction in both cases.

The most obvious advantage of the wind channel method of investigation is that the testing is done in an air stream remarkably steady and uniform in velocity and direction, features all conducive to a high degree of precision, together with the ability to repeat the exact conditions at will. Other important advantages are convenience both as to ease of taking readings, time and bodily comfort; relatively low cost both of

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[*] In charge Aerodynamic Laboratory and Research.

materials and power, and the possibility of readily making many tests difficult or impossible to carry out on full scale.

The essential requirements in a wind channel are steadiness and uniformity of distribution of velocity over the working section. Efficiency of operation is of secondary importance.

The extent to which the results of model experiments in a wind channel are applicable to free flight of full scale machines is of primary importance in this work. The principle of dynamical similarity requires that for similar flow the value of $\frac{vl}{\nu}$ be the same for model and aircraft The medium being the same this requires that vl be constant for both· Since the smallest machine is probably five times the scale of the largest model and flight speeds vary from about the same to two or three times channel speeds a strict application of this principle would necessitate impossible channel air speeds. Hence the scale or vl effect must be determined.

The general conclusion to be drawn from investigations so far carried out (but this phase of the work has been scarcely touched) of the scale effect for different aerodynamic forms is that model tests at $vl=30$ ($l=6''$ chord and $v=60$ *f.p.s.*) can be applied to full scale work without correction, while the corrections for smaller values are slight. Obviously the larger the model that can be tested the greater the accuracy with which it may be constructed and the forces measured, while, on the other hand the bulk and cost of apparatus and buildings and difficulty of making tests is greatly increased.

TYPES OF WIND CHANNEL.

There are two principal types of wind channel used in aerodynamic research, the Eiffel or French and the N.P.L. or British. In the Eiffel channel, developed by G. Eiffel, a column of air is drawn across an air tight experimental chamber between a bell-shaped collector and an expanding diffusor, the latter terminating in a suction fan. The chamber is relatively large, containing model, balance and observers, and the air stream unconfined.

The N.P.L. channel was developed at the National Physical Laboratory in England. In this channel the air is drawn through a square tube by a propeller, entering at an elementary bellmouth and being returned to the room through a distributor. Only the model is inserted in the channel and the air stream is confined.

The second type of channel is superior to the first from the standpoint of steadiness although the former may possess advantages in the way of ease and convenience of operation. Experiments by Lieut.

Castellazzi (Rendiconti dell' Instituto Centrale Aeronautico 1917) showed that an increase in efficiency of well over 50% resulted from enclosing the air stream. Recently in tests carried out at Langley Field (N.A.C.A. Report No. 73, p. 20) with models of wind channels it was shown that the enclosed type was superior to the unconstrained type in the way of steadiness of flow. Other tests at McCook Field (N.A.C.A. Report No. 83, Part I.) demonstrated the losses that occur due to eddies in an unconfined air stream.

In both types of channel the air follows a closed circuit through channel, returning through room to re-enter the channel. A third less commonly used type of channel provides this closed circuit in the channel itself as in the Göttingen and Washington Navy Yard channels. Such channels are less satisfactory than those with free return flow.

TORONTO PLANS.

The University of Toronto has installed a wind channel and proposes the investigation of aerodynamic problems as part of the work of the School of Engineering Research. The equipment is such that, besides researches initiated at the University, aerodynamic problems of all kinds, the rating of instruments and other similar work can be undertaken for aircraft manufacturers, aeronautical transportation companies and others engaged in aeronautical work.

TORONTO LABORATORY.

Although a description of the equipment of the aerodynamic laboratory, consisting of units elsewhere developed and described, may seem incongruous in a research bulletin, no further justification for its publication need be made than to remark that one of the chief factors (outside of personnel—a very important but somewhat variable element) on which those interested may base their estimates of the probable value of the results of researches carried on, is a complete knowledge of the equipment and methods used.

The installation at Toronto is after the British plans. The channel is of the standard 4 ft. N.P.L. construction and the balance of an improved N.P.L. design. As both channel and balance have been fully described in various reports and periodicals (see references at the end of this paper) it is proposed to give only a brief general description here of both channel and balance, emphasizing the points of difference or improvements effected over the standard installation.

CHANNEL PROPER (see. Fig. 1)

The wind channel proper is an airtight tube of wood, 4 ft. square in section inside, and 24 ft. long. There is a bellmouth collector of 13″ radius at the entrance followed by a sheet-metal honeycomb of 3″ square cells, 30″ long, 3′ 6″ from the bellmouth, and at the outlet end of the tube a second similar honeycomb. The first honeycomb is for the purpose of straightening out the air into parallel streams, in addition to which it has a certain damping effect on the pulsations (N.A.C.A. Report No. 83) while the second honeycomb eliminates the whirl in the air due to the propeller.

The working section of the channel, provided with a plate-glass window and door, is located 15 ft. from the entrance or 9 ft. behind the first honeycomb. The standard of the balance projects through the floor at this section.

At the outlet end the tube is connected to a galvanized iron transition section or propeller race in which the cross-section changes from 4 ft. square to 5′ 8″ dia. circular in a length of 4 ft. The circular section is 33″ long and projects 9″ into the distributor. The propellar rotates in the cylindrical portion.

DISTRIBUTOR

The distributor is a feature exclusive with the N.P.L. channels. The distributor consists of a long box, with perforated sides, into one end of which the propeller discharges, and the other end being closed, generally abutting against a wall of the building. The function of the distributor is to break up the violent propeller slip stream and return the air to the room without disturbance. At Toronto the distributor is 6′-3½″ square inside and 20′-0″ long. The four sides are made up of graded checkerwork gratings, of 1¼″ stock, so constructed as to form 1¼″ square orifices, one diameter long, the number of openings per square foot being graded from a maximum at the propeller end to a minimum at the wall.

In later distributors at the N.P.L. the checkerwork has been abandoned in favour of the simpler and equally effective slot construction.

FRAMEWORK

The tube, propeller race and distributor are built in a framework of structural steel. In the design of the steelwork for the Toronto channel the standard plans were departed from in minor particulars. The more common angle shape is used instead of the T., the centre line of the channel is raised to 9′ 6″ above the floor, instead of 8′ 6″ to provide headroom and clearance over tanks in the laboratory, and the frames are of slightly heavier construction and greater rigidity.

INTAKE

A feature of the Toronto installation is the intake arranged at the entrance for the purpose of eliminating disturbances in the air due to the unsymmetrical position of the channel in the building, the presence of large obstructions, and openings in ceiling and walls. The intake was developed at Toronto and a full description of it and the research resulting in its design is given in Aeronautical Research Paper No. 3.

Briefly, the intake is a short distributor using the slot type of construction, all four sides and the end being latticed. It is provided with eight radial blades on the front and sides. The intake performs the same function on the entrance as the distributor on the exit, namely, the breaking up of eddies arising from the obstructions and other disturbing elements in the room causing the air to enter in a relatively steady stream of uniform velocity over the working cross-section.

The steadiness is such that tests have been run without damping of any kind in the balance and the velocity is uniform within practically ½% over the working cross-section.

PROPELLER.

The air is drawn past the model by means of a four-bladed propeller. Two propellers have been used at Toronto, one a 5′ 6″ dia. ×3′ 0″ pitch Lang propeller, the other of standard N.P.L. design for this channel 5′ 6″ dia. ×2.25′ pitch with broad thin blades. The latter is more efficient, slightly less noisy, and appears to give a steadier current. It is now regularly used in the channel. The propeller turns at about 800 r.p.m. for an air speed of 40 f.p.s.

PROPELLER DRIVE

The requirements of the drive for the propeller of a wind channel are: freedom from speed fluctuations (steadiness) with flexibility, *i.e.* variable speed control for securing different air speeds.

Electricity has so far been practically exclusively used for wind channel drives because of its numerous advantages. Direct current must be used since speed variation is not convenient with alternating. The exact arrangement of the drive and control varies in different installations. It may be arranged either direct connected as in the British channels, where the motor is at the end of the distributor and drives the propeller through long shaft and flexible couplings, or chain driven from the motor outside the distributor as at the M.I.T. and Toronto. The latter arrangement possesses some advantage in that the motor is placed on the floor requiring no heavy stand while a disadvantage is the tendency of the chain to surge. At Toronto, at a certain speed, various pulsations are found to synchronize resulting in very marked surging in the drive.

The drive at Toronto is by means of an 18-20 H.P. variable speed shunt motor, through inclined silent chain to the propeller shaft carried on ball bearings. The bearings for the propeller shaft are carried on supports forming part of the first two distributor frames. To prevent vibrations resulting from this arrangement being transmitted from the distributor to the channel proper there is absolutely no connection between the propeller race and the distributor; each is quite independent of the other.

The power consumed at an air speed of 40 f.p.s. is 4.7 H.P., representing an overall efficiency of the channel of 47% based on the kinetic energy given the air.

CONTROL.

The question of the control of the motor speed and hence the air speed is of importance. The air, due to the various disturbing elements in the building, tends to pulsate, and to these pulsations may be added pulsations due to speed fluctuations of the motor. The former may be reduced by suitable intake arrangements, the latter depend on the steadiness of the impressed voltage on the motor and the perfectness of the speed control equipment.

The ideal voltage would be an absolutely constant one, such as storage battery voltage. The cost of storage battery supply is ordinarily prohibitive. Such a supply is, however, used for the 4 ft. N.P.L. channel of the East London College (see R. & M. No. 578, Nov. 1918, p. 1). Investigations at the N.P.L. (see R. & M. No. 67, p. 51, para. 25) seemed to indicate that the change in steadiness due to change from storage battery to machine voltage is slight. This would depend, of course, upon the steadiness of the particular machine voltage used in making the comparison, which, it is believed, was comparatively good. The ordinary voltage from central stations, traction companies and other similar sources is subject to fluctuations of a more or less serious nature, depending on the governing of the prime movers and the other loads on the circuit.

Various methods are used for the speed control of the motor, principally manual, as a satisfactory automatic control has been difficult to devise (see N.A.C.A. Report 72, p. 12) although an automatic control has been successfully used in Germany. Manual methods so far used or considered are:

(*a*) Control of shunt motor by combination of armature and field rheostats, rough adjustments to the mean general speed being effected by armature resistance and fine adjustments, such as those for correcting line voltage variations by field rheostats. This is the method at present employed at Toronto.

The disadvantage of the method is the distinct time lag. Since no sufficiently sensitive tachometer has been developed variations in speed are indicated by variations in air speed shown on the micromanometer. Hence between the change in voltage giving rise to the speed change and the appearance of the resulting change in the manometer reading there is an appreciable lag and it is only possible to "chase" the variations.

(*b*) The insertion of a heavy armature resistance in the circuit has the effect of damping out speed fluctuations due to variable voltage, acting as an electrical flywheel. This is employed at Toronto and is found effective. As much armature resistance as possible, while still securing the necessary speed, is always put in the circuit.

(*c*) A method suggested to overcome the lag difficulty in (*a*) is the manipulation of a fine armature resistance in conjunction with a voltmeter to keep the impressed voltage constant.

An improvement on the latter method, suggested by Mr. H. C. Crane, measures the voltage by means of balancing a standard cell against the voltage across a portion of a shunt across the line, the flow of current shown on a sensitive ammeter when the circuit is not in balance, *i.e.* when the voltage is other than that desired. However, as the speed of operation of the motor with given load depends on its temperature this method would give a constantly changing speed until the motor reaches its highest temperature.

(*d*) WARD-LEONARD CONTROL.—The three unit Ward-Leonard type of control is used at the M.I.T. In this system virtually the prime mover speed is controlled, and there are no loads other than the propeller motor on the circuit. The system is expensive and complicated.

A mechanical method of damping out fluctuations in speed by means of a heavy flywheel is worth consideration. The propeller has a certain flywheel effect.

The only automatic control known to have been perfected so far is that of the German channels at Göttingen and Friedrichshafen (Zeppelin). In these channels the speed is measured by means of a weighing manometer. One arm of the manometer balance carries a pointer moving between two contacts controlling a servo motor. The latter moves a rheostat varying the speed of the channel motor in such a way as to return it to normal. Ordinarily a light spring contact controlling a fine rheostat is first made but for large speed deviations the pointer overcomes the resistance of the flat spring contacts and makes the main contact. When the fine rheostat approaches its limit a contact is made moving the main rheostat one step (see N.A.C.A. Report and Aviation and Aero. Engng., Nov. 8, 1920, p. 259).

SOURCES OF ELECTRIC CURRENT AT TORONTO.

There are available at Toronto three sources of electric current supply.

1. That from the University central power and heating plant. This is rather unsteady during academic terms due to variations in loads thrown on the circuits. The voltage is very steady between the close of the term and the shutting down of the plant for the summer. Little attention is required to control the air speed during this period.

2. That from the Toronto Electric Light Co. which is used when the University plant is shut down during the summer. This is somewhat unsteady but it has been found possible to secure a steady airspeed even under these circumstances by the methods described.

3. That from the constant voltage line of the Electrical Department. The voltage on this line is controlled by Tyrrell Regulator to within 1 volt either way on 220 volts, but is constantly flickering between these limits. This source of supply has not been given any extended trial as yet owing to various limitations.

AERODYNAMIC BALANCE.

There are a number of types of aerodynamic balance in use, only two of which are well known or widely used. These are again the Eiffel and N.P.L. types. The Eiffel balance is of the three moment type; that is, readings are taken of moments about three axes. Ordinarily two axes only are used and readings taken with the model in normal and inverted positions. This type of balance is subject to inherent errors due to its construction, lacks sensitivity, is heavy, slow in manipulation (three sets of reading must be taken) and the calculation are lengthy.

The N.P.L. balance on the other hand is simple in construction and use, while the precision is as high as that of the more complicated types. The balance has been fully described in references listed at the end of this paper.

In brief the N.P.L. balance consists of three arms, mutually at right angles at the intersection of which is located a hardened steel conical pivot. The whole balance is ordinarily carried on this single pivot and hence is free to rock in two planes. The vertical arm is the standard projecting through the oil seal in the floor of the channel and carrying the model. The model, except in special experiments, is mounted with its Y axis vertical, that is, standing on one wing tip. The two horizontal arms are the lift and drag arms perpendicular and parallel respectively to the channel axis. On these arms are hung weights to balance the lift and drag forces on the model (see Fig. 2).

The vertical arm is continued below the pivot where it carries a dashpot for damping out pulsations, sensitivity weights, clamping lock and conical seat for a second adjustable pivot. When both pivots are functioning the balance is free to rotate about a vertical axis through them and the torque is measured on a calibrated spring device attached to an extension of the lift arm. When making ordinary lift and drag measurements rotation is prevented by a locking spring and strut attached to the same arm.

For direct lift measurements (vertical forces) additional arms are provided as described later.

The precision of these balances is such that they will tilt for a force of 0.0001 lb. on the model which corresponds to the movement of 0.01 inch of a jockey weight weighing 0.036 lb. The precision in practice is less than this and is usually governed by the steadiness of the wind and skill of the observer.

NEW FEATURES IN THE BALANCE AT TORONTO.

The balance at Toronto was built by the Cambridge Scientific Instrument Co. of Cambridge, England, to the latest N.P.L. designs and incorporates several improvements over the original design.

TORSION MEASURING DEVICE

In the original balance a torsion spring located in the vertical arm of the balance, below the main pivot, was used for the determination of moments about the vertical axis. In the Toronto balance a flat cantilever spring is used, acting through a steel strut on the continuation of the lift arm on the pedestal side of the balance. This arm carries the usual cylindrical counterweight with stud for weights when negative lifts are to be measured, the locking strut and C spring and beyond the latter the conical sockets for a similar strut and C spring for connecting with the torsion cantilever spring. The cantilever spring is provided at its tip with conical sockets for the strut and C spring and at the base is attached to a carrier fitted with vertical knife edges bearing in sockets in a bracket on the pedestal. The tip of the arm of the carrier bears against a micrometer screw and is kept in contact with the latter by a light spring and strut on the other side of the knife edge.

By adjusting the micrometer screw the spring carrier and spring are rotated about the knife edge deflecting the cantilever spring (either way). The spring, due to its deflection, exerts a push or pull through strut or C spring on the balance arm producing a moment balancing, the moment due to the model. The rotation of the micrometer head from its zero position necessary to again establish zero is a measure of the torque. The latter may be found from the spring calibration. Evidently this is a null method and there is no alteration in the incidence of the model. The zero position is indicated, as in the original balance, by means of microscope with cross hair and lined mirror on moving part of balance.

To dampen out oscillations while making torque measurements, in addition to the four sections of the lower dashpot, a small adjustable damper in an oil chamber in the top of the pedestal may be used. The The moving part of the damper is attached to the counterweight already referred to and the degree of damping may be varied by varying the size of the apertures in the damper.

Three springs are provided with the balance for the measurement of torques of different magnitudes. The calibration of these springs has been carried out as follows:

A bracket fitted with two knife edge bearers was attached to the stationary arm below the drag beam. The knife edge of a balance beam rested on the bearers (see Fig. 3). The balance beam carried at its extremities scale pans, and at the centre a vertical arm carrying above the knife edge two conical points bearing against the sides of the drag beam 18″ from the main pivot. A light counterweight below rendered the beam stable.

After establishing the zero, weights were placed on either scale pan and the movement of the micrometer screw to restore equilibrium noted. The dimensions of the system were such that the torque on the balance in inch-pounds in terms of weight on pan in grammes was $T=0.04286W$. Readings were taken for increasing and decreasing torques on both sides of zero and large scale calibration charts drawn from which the formulas for the springs were determined.

The calibration device was found sensitive to 0.02 gramm (0.00004 lb.) corresponding to a torque of 0.00085 in pds. on the balance, to which the latter was sensitive.

VERTICAL FORCE MEASUREMENT.

The interior sliding rod of the earlier balance for the measurement of vertical forces has been superseded by a movement of the whole balance. The balance, for this purpose, is arranged to have a parallel movement about two knife edges which are the fulcrums of two auxiliary removable levers and rest in bearers in brackets on the balance pedestal. The short arm of the lower lever has a conical centre taking the steel point at the extreme lower end of the balance, and the long arm carries a heavy counterweight sufficient almost to counterbalance the weight of the moving part of the balance. The upper lever, located just above the plane of the lift and drag arms, similarly, has a conical socket at the end of the short arm, taking a steel centre carried in a yoke in the balance just above the seat of the main pivot. The long arm of this lever is provided with jockey weight, scale pan and zero line mirror by which the additional weights necessary to establish a balance are added and the measurement of the vertical force effected.

The disadvantage of the interior sliding rod was the great accuracy of construction necessary in order to permit of free sliding without rotation. By the present arrangement this difficulty is overcome and while the weights to be balanced are large in comparison with the direct lift to be measured sufficient accuracy is readily obtainable.

SETTING OF MODEL.

The elimination of the sliding rod enables a chuck to be used permitting the accurate setting of the model with regard to the main pivot. The chuck has a shoulder against which the end of the model spindle rests. The shoulder is exactly 21″ from the main pivot. Spindles for standard aerofoil models (18″×3″) are then made 6″ long so that the centre of the aerofoil is readily and accurately placed 36″ from the main pivot.

SETTING UP BALANCE.

The channel and balance have been carefully set up and the channel alignment and balance setting checked by approved methods. It is believed that the method used to check the alignment of balance and air stream direction, making use of the Alignment Apparatus (see Appendix), has permitted the setting of the balance to within 0°.025. The alignment apparatus and protractor of balance have frequently been cross-checked to within 0°.025 or 1′.5.

VELOCITY MEASUREMENT.

The velocity is measured by means of side plate and Chattock micromanometer. The side plate, consisting of a 4″ dia. disc of brass with five 0″.02 holes drilled in the centre, is let into the wall of the channel 2 ft. upstream from the model (to reduce model interference effects). The plate is connected through rubber tubing to a standard 13″ Chattock Micromanometer (described in detail in Aero. Res. Paper No. 2). The Chattock gauge is regarded by some as too sensitive for wind channel work. With the Toronto installation, possibly owing to the steadiness of the air current, this gauge has proved most satisfactory.

STANDARDS.

The standards adopted at Toronto are those of the British laboratories namely:

Air—Pressure	29.921 ins.	760 m.m.
Temperature	60.°F	15.°6C.
Density	0.07635 lbs./c. ft.	0.001223 gms./c.c.
Speed	40 ft. per sec.	
Gravity	32.174 ft./sec/sec.	980.66 cms./sec/sec.

TECHNIQUE OF OPERATIONS.

The routine followed at Toronto in testing standard 18″×3″ aerofoil models is briefly as follows:

The aerofoil is provided with a tapering (3/16″ to 5/16″) steel spindle tapped into the end of the aerofoil 1″ from the leading edge.

The alignment of the balance is checked and the protractor set at zero. The model is then placed in the chuck and lined up by means of the alignment apparatus. The sensitivity weights of the balance are adjusted.

The zeros of the lift and drag arms are established for about the average weight likely to be used on each. The lift and drag arms are then balanced, and the balance rotated through 180° and again balanced. The lift and drag arms are set at the average of the two readings and equilibrium secured by adjusting the weights above the arms thus bringing the centre of gravity of the balance into the axis. This is checked at different angles. The aerofoil is then set at the first angle of incidence usually $-6°$, making use of a large reading glass to get the setting accurately.

The Chattock gauge zero is adjusted and the reading for the air speed to be used set.

The motor is then started, brought to proper speed and maintained there as nearly as possible while lift and drag measurements are made for the desired range of incidences. The run is repeated, making torque (centre of pressure) measurements.

The arrangements in use at the present time make use of two observers. The Chattock gauge is set up on a high table (see Fig. 2) so that its microscope is immediately adjacent to the cross line of the lift beam and between latter and stationary beam below. The eye can thus observe both Chattock and lift beam with little movement, practically no change of focus and minimum lag. The electrical control rheostats are within easy reach from this position.

One observer controls the speed with the right hand, keeping it as nearly constant as possible as shown by the Chattock gauge and signalling the other observer when right, at the same time manipulating the weights on the lift beam with the left hand. The other observer attends to the drag beam, which requires greater care owing to the small magnitude of the forces, and records the observations.

It is proposed in the near future to use either two gauges on the side plate, one for each observer, or to use a projecting arrangement as referred to by Pannell (R. & M. 242, 1915-16, p. 37), by which the bubble of the Chattock is thrown on a screen and can be observed by both observers without eyestrain.

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Appendix.

ALIGNMENT OF BALANCE AND MODEL.

J. H. PARKIN AND H. C. CRANE

THE PROBLEM.

The accurate trueing up of the balance and model involves first of all the determination of the true wind direction, which may or may not be parallel to the channel axis. Having the wind direction, the balance arms must be set exactly parallel and perpendicular to the wind direction. Finally a reference line or other device must be provided for the convenient and precise lining up of the models to be tested.

METHOD.

The problem is a difficult one since errors in both assumed wind direction and balance alignment occur simultaneoulsy. The small magnitude of the quantities being solved for (the effect of relatively small errors is shown later), coupled with the fact that the determination is based on experimental results into which may enter relatively large experimental errors further complicates the problem.

The approved method for determining both the wind direction and balance alignment is to mount a flat plate or aerofoil in the balance at different angles of incidence to the true wind direction, with the balance as nearly as possible in alignment, and take readings of lift and drag. The plate or aerofoil is then reversed and the readings repeated. From the two sets of readings the errors of alignment are calculated.

The aerofoil is preferable to the flat plate because of the low efficiency of the latter which renders detection of errors difficult. In addition the aerofoil has an advantage in connection with the elimination of spin which will be referred to later.

At Toronto a standard aluminum aerofoil was used. It was set up first of all with chord parallel to channel axis (see Fig. 4) by means of the Alignment Apparatus, described later. In position I the underside was to the left looking downstream. A complete set of L and D measurements were made in this position for incidences from −4° to +20°. The aerofoil was then placed in position II, underside to the right and the readings repeated. The L, D and L/D curves for both positions were then plotted to a large scale and used in determining the correct alignments. The use of the curves, besides advantages which will be evident later, lends to a reduction of trouble from experimental error through showing at once, readings in error. For correct alignment of both model and balance both sets of curves are congruent, generally they differ in both shape and position.

THEORY.

Let $F(\theta)$ and $f(\theta)$ be the true lift and drift at an apparent angle of incidence θ (see Fig. 4). If the assumed wind direction be in error β from the true wind, the actual incidences in positions I and II will be $(\theta-\beta)$ and $(\theta+\beta)$ and the true lifts and drags $F(\theta-\beta)$, $F(\theta+\beta)$, and $f(\theta-\beta)$, $f(\theta+\beta)$ respectively. Let the balance misalignment from the true wind direction be α. Then if the apparent lifts and drags in positions I and II be L_1, D_1 and L_2, D_2,

$$F(\theta-\beta)=L_1\cos\alpha+D_1\sin\alpha \tag{1}$$
$$F(\theta+\beta)=L_2\cos\alpha-D_2\sin\alpha \tag{2}$$
$$f(\theta-\beta)=D_1\cos\alpha-L_1\sin\alpha \tag{3}$$
$$f(\theta+\beta)=D_2\cos\alpha+L_2\sin\alpha \tag{4}$$

Since α is small, $\cos\alpha=1$ and $\sin\alpha=\alpha$ without serious error.

$$F(\theta-\beta)=L_1+D_1\alpha \tag{5}$$
$$F(\theta+\beta)=L_2-D_2\alpha \tag{6}$$
$$f(\theta-\beta)=D_1-L_1\alpha \tag{7}$$
$$f(\theta+\beta)=D_2+L_2\alpha \tag{8}$$

Also β is very small so that expanding $F(\theta\pm\beta)$, the squares of β may be neglected and $F(\theta\pm\beta)=F\theta\pm\beta\frac{d}{d\theta}.F\theta=F\theta\pm\beta F'\theta$ substituting in the above and solving for α.

$$\alpha=\frac{-2\beta F'\theta-L_1+L_2}{D_1+D_2}$$

or

$$\alpha=\frac{+2\beta f'\theta+D_2-D_1}{L_1+L_2}$$

In the latter equations $F'\theta$ and $f'\theta$ can be determined from the slope of the curves and the two unknowns α and β thus found.

In actual practice, however, the degree of accuracy in the determination of the angles is well within the experimental error to be expected and the magnitude of the latter will frequently obscure the true angles. The following method using the characteristic curves has been developed at Toronto to obviate this difficulty.

EFFECT OF ERRORS ON CURVES.

It will be well first to examine the effect of the different errors of alignment on the characteristic curves. The relative effect of balance misalignment on the L and D curves will be first considered.

On the assumption that the assumed wind direction is correct, equations 5, 6, 7 and 8 may be written

$$F(\theta)=L\pm D\alpha=L\pm(L\sin\theta\left(\frac{D}{L}\right))$$

$$f(\theta)=D\pm L\alpha=D\pm(D\sin\theta\left(\frac{L}{D}\right))$$

the upper signs applying for position I.

For any of the standard aerofoil sections such as would be employed for this purpose, $\frac{L}{D}$ is generally greater than 10 (see Figs. 5, 6 and 7), except where L is small and beyond the critical angle. Hence for an assumed error in balance alignment of 1°, actually it is more of the order of 0.2 to 0.3 degrees, the per cent. corrections are:

$$\text{on } L-\frac{\pi}{180}\times\frac{1}{10}\times100=0.174\%$$

$$\text{and on } D-\frac{\pi}{180}\times\frac{10}{1}\times100=17.4\%.$$

On the L curve, as usually plotted, the error of 0.174% is not visible and in any case is much smaller than the experimental error likely. However, the correction on the D curve, being comparatively large and much greater than the experimental error, shows up prominently, as is indicated in Fig. 5, which shows the effect when $\alpha=0.°2$. Evidently the balance misalignment is best determined from the drag curves.

The effect of an error in assumed wind direction is equivalent to a greater or less incidence and results in a displacement of all the curves to right or left without changing the maximum or minimum values of the forces (coefficients) or ratios (L/D). The largest difference is thus at the steep portion of the curves as is shown in Fig. 6.

The combined effect of simultaneous errors in both model setting and balance alignment is shown in Fig. 7.

MODEL ALIGNMENT OR WIND DIRECTION.

Both curves of L and D are affected by errors in model alignment as has just been seen, the D curves being also affected by balance misalignment. The L curves are relatively unaffected by the latter errors except at small and large angles. At low angles of incidence, while the percentage error may be large as just shown due to small L/D, the absolute error is negligible since lifts are generally of small magnitude, while lifts beyond the critical angle are not used. Hence over the working range the L curve is unaffected by errors in balance alignment, (α) and any displacement will be due to error in model alignment if the effect of spin be neglected.

The effect of spin in the air current is to displace points on the curves vertically an amount proportional to the slope. If, then, the wind direction be determined from the curves, since the working position of the L curve is nearly straight, the spin correction is eliminated for all curves having the same slope, and except the correction be large, which could only occur in channels of poor design, the spin effect is negligible.

Further, the lift forces being relatively larger than the drag forces may be measured with greater accuracy.

Hence the L curves are best used for the determination of error in assumed wind direction.

The error may be read directly from the curves. It is one-half the horizontal distance to the angle scale used between the L curves for positions I and II.

The final wind direction determined at Toronto was found to be 0.°2 from the channel axis.

BALANCE ALIGNMENT.

Having determined the true wind direction attention is turned to the drag curves which it has been shown are preferable for the determination of balance setting errors. The D curves are generally not coincident except where L, and consequently correction on D due to balance misalignment is zero.

If now at, say, an angle of incidence of 8° where the L/D is still good, the drag of fair magnitude and still some distance from the critical angle, the balance be rotated by means of the adjustment screw provided for that purpose, a series of drag readings for one setting may be secured. By calculating the angle of rotation corresponding to the movements of the adjusting screw and correcting the angle of incidence for the consequent change in incidence, a calibration curve of the adjusting screw will be found similar to that shown in Fig. 7. The correct setting of the balance can then be determined directly from this calibration curve, it being that which gives a point on the curve midway between the two drag curves.

The drag curves used in the above may be the original ones taken with both model and balance alignment in error, in which case care must be taken to first correct for the previously determined model misalignment by shifting the curves together by an amount equal to that angular error, and from the new positions finding by the above auxiliary curve method, the error in balance setting. This is generally more convenient than running a second pair of tests, with the model properly lined up giving D curves on which the only error is that of balance misalignment.

This method of determining the errors in setting of model and balance were found very convenient and effective at Toronto and have resulted in conjunction with the Alignment Apparatus in securing settings of both model and balance to 1/40° or 1½ minutes.

ALIGNMENT APPARATUS.

The commonly employed method of aligning aerofoil models is to bind a perfectly straight batten to the face of the model and sight it against a reference line marked on the channel floor. This method possesses obvious disadvantages although it is claimed to be possible to align a model to within 0.°05 by its use. It is believed that the Alignment Apparatus, designed and used at Toronto, overcomes many of the disadvantages of the batten method and permits of greater precision.

The apparatus in principle is simply a pair of accurate squares, one set along the true wind direction with its arm therefore perpendicular thereto, on which slides the second square whose arm is therefore exactly parallel to the wind direction. The latter arm is used to line up the model.

As constructed, see Figs. 8 and 9, a flat, slightly eccentric *A* frame is made up of 1″×¼″ flat. The long straight side is about 6′ 4″ long overall and in each end has tapped, 6′ 0″ apart brass plugs or feet. Screwed to the *A* frame are two blocks of steel, carefully reamed to take a ½″ dia. cold rolled rod, and so placed that the axis of the rod is exactly perpendicular to the line of the feet. The rod is 36″ long and is held in the block by set screws. Sliding on the rod is a 3″ steel sleeve reamed an accurate fit on the rod, and carrying a steel blade 1″×1/8″×12″ at right angles to the rod.

Two brass sockets, tapped for 5/16″ screws, are fitted into the wall of the channel, 2′ 0″ above the floor at the steel frames on each side of the working section, in order to secure permanence and rigidity. The feet of the apparatus are reamed for 5/16″ screws.

In use, the apparatus is fastened to the sockets in side of the channel by means of cap screws, and supported by a leg of 3/8″ rod tapped into the outer steel block (see Fig. 8). The model is set loosely in the balance chuck and the sliding blade brought up to it thus setting it accurately. The chuck is tightened and the alignment checked by means of the blade and a light. The apparatus is then removed, the holes in the sockets being filled with flat-headed machine screws.

It has been repeatedly shown during the work of aligning the balance and model that the Alignment Apparatus enabled the model to be set to within 1/40° and that the protractor of the balance, if a reading glass was used, could be set to this same accuracy.

The apparatus, when set in the floor of the tunnel, and after a determination of the true wind direction in the vertical plane, may be used to set models to the correct angle of pitch. For this purpose an accurate protractor is necessary.

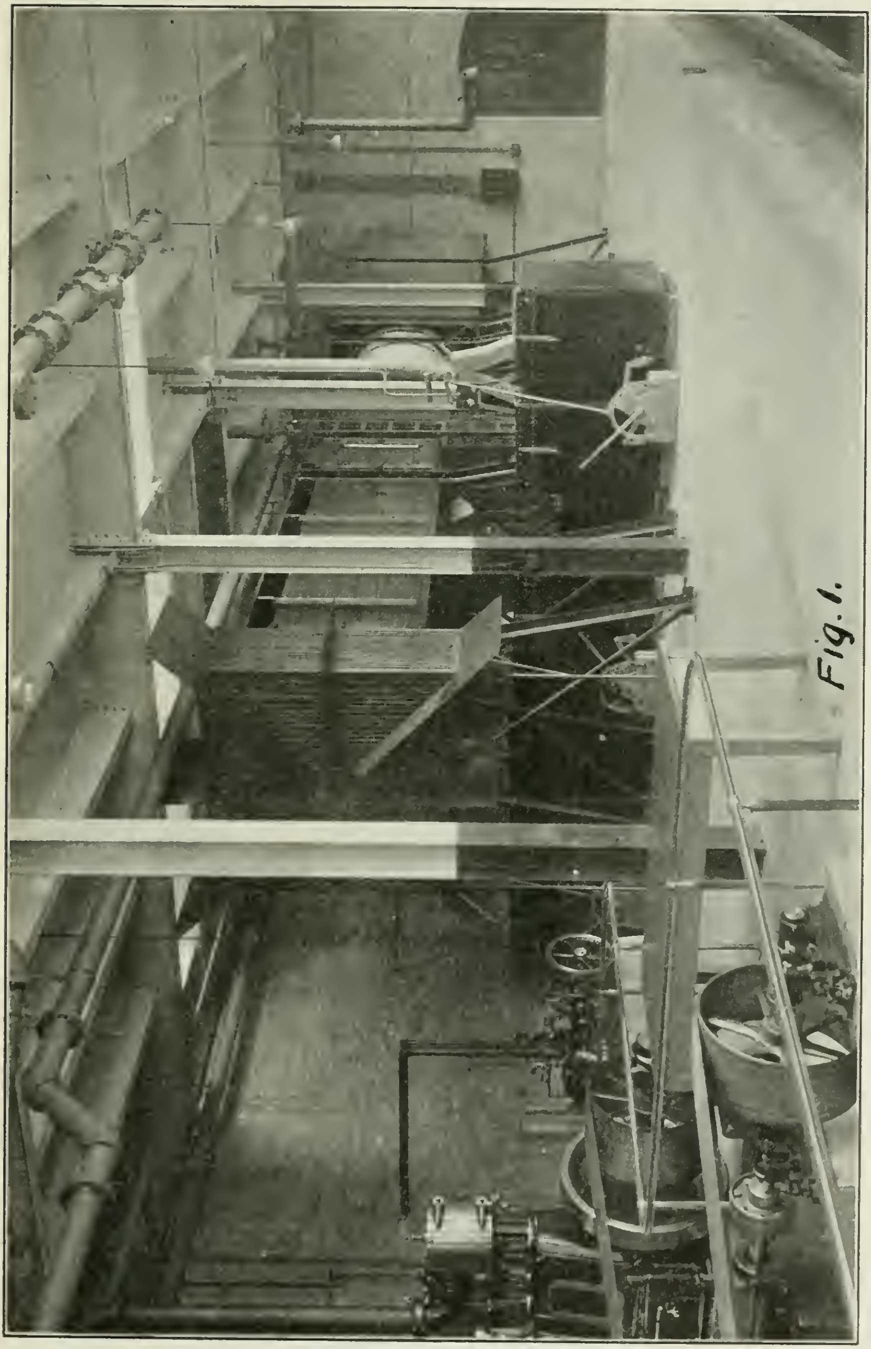


FIG. 1.—TORONTO WIND CHANNEL.

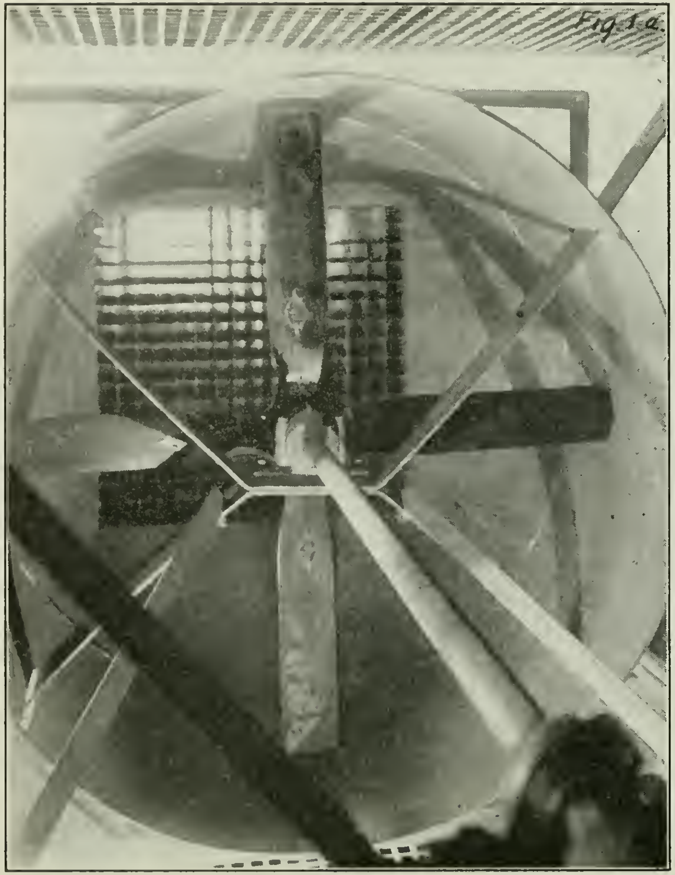


FIG. 1 (a).—PROPELLER AND MOUNTING.

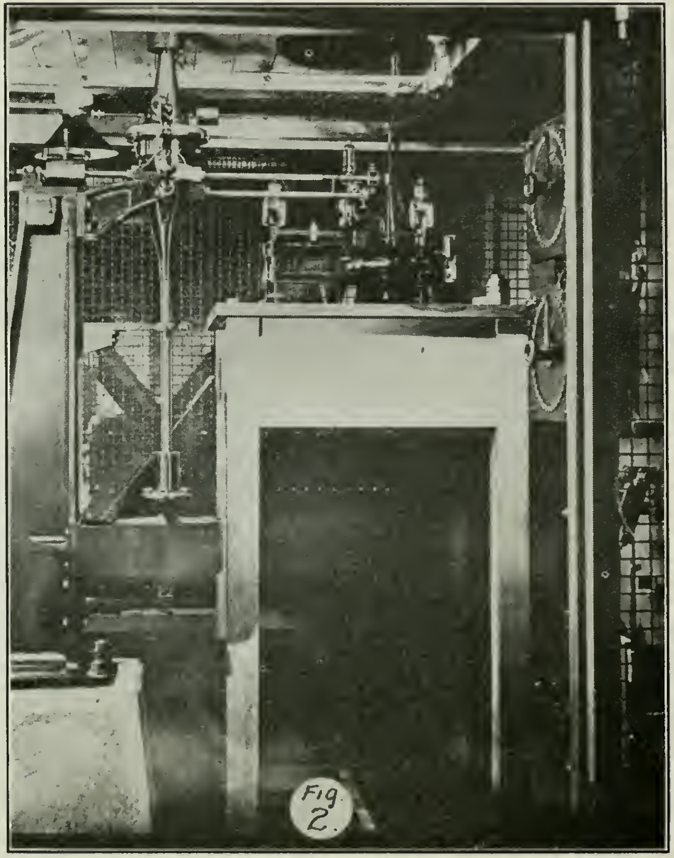


FIG. 2.—INSTRUMENT CABIN.

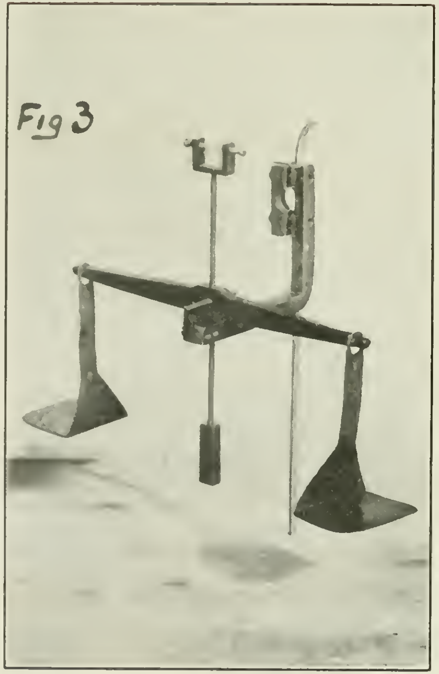
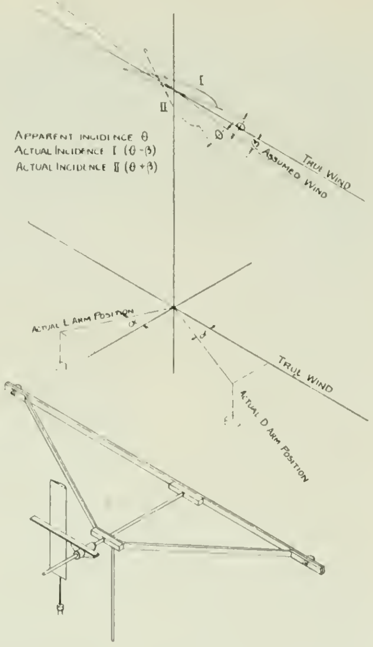


FIG. 3.—SPRING CALIBRATING BALANCE.



BALANCE AND MODEL ALIGNMENT
FIG 4

FIG (8.)
DIAGRAM OF ALIGNMENT APPARATUS

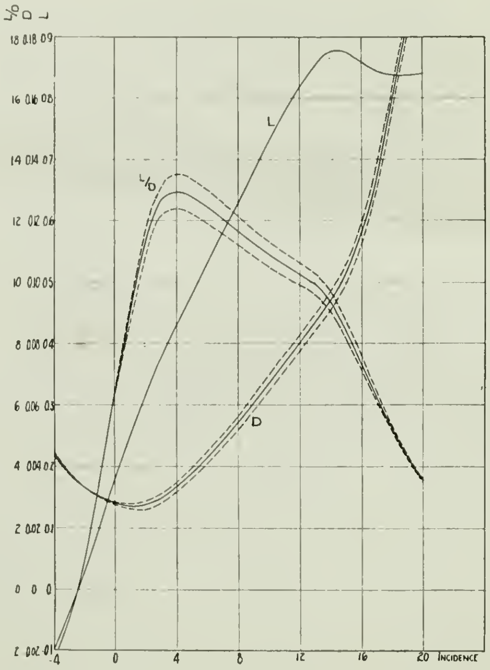


FIG 5.

EFFECT OF BALANCE MISALIGNMENT

CORRECT CURVES – FULL LINES
CURVES FOR α = ±0°.2 – DOTTED LINES

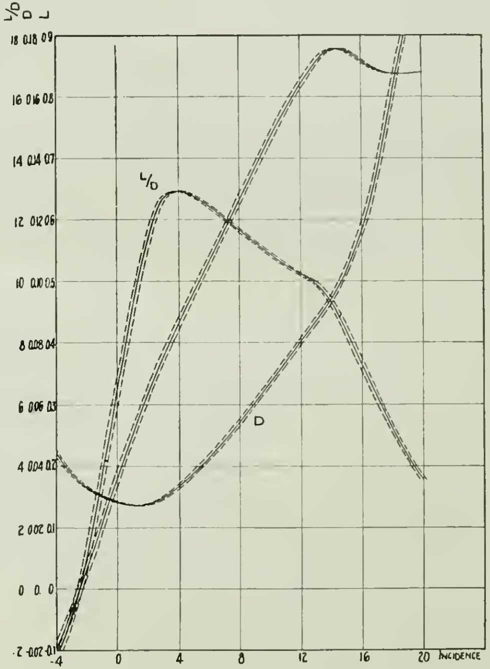


FIG 6

EFFECT OF MODEL MISALIGNMENT

CORRECT CURVES · FULL LINES
CURVES FOR β = ±0°.2 – DOTTED LINES

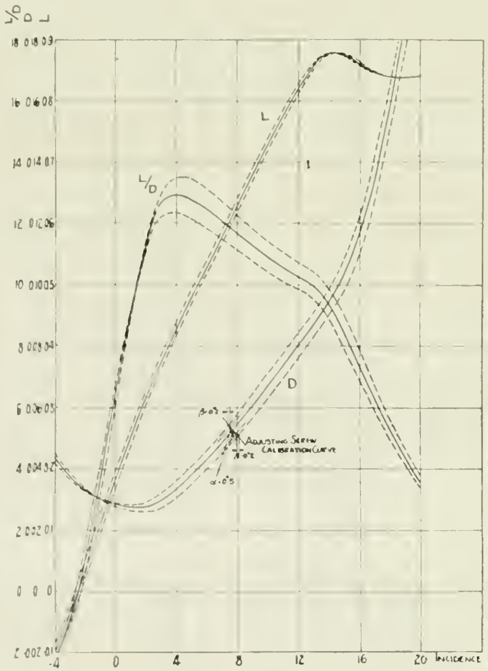


FIG 7

CUMULATIVE EFFECT OF BALANCE AND MODEL MISALIGNMENT

CORRECT CURVES · FULL LINES

CURVES FOR α ± 0°8 AND β ± 0°2 · DOTTED LINES

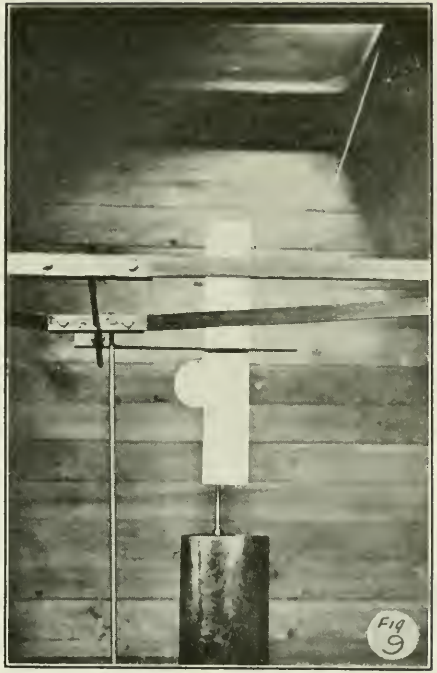


FIG. 9.—ALIGNMENT APPARATUS.

Aeronautic Research Paper No. 2.
September 1920.

AIR SPEED INDICATORS FOR WIND CHANNELS.

J. H. Parkin
Assistant Professor of Design

Introduction.

In wind channel work two sets of measurements are, in general necessary, air velocity and the forces on the model. A high degree of precision can be attained with the present aerodynamic balances in the measurement of the forces on the model, and these forces vary as the square of the velocity. An error of ½% on velocity thus results in 1% error in the measurement of the forces. Hence the accurate indication of the air speed in wind channel work is of vital importance.

An investigation has, therefore, been undertaken, partly experimental, partly of the literature of the subject with a view to determining the best type for use in the Toronto wind channel, bearing in mind the requirements of such an instrument.

Requirements.

The conditions to be fulfilled by a wind channel anemometer are briefly as follows:

Accuracy and Sensitivity. Accuracy is a most important requirement of a wind channel speed indicator, and the accuracy must be permanent, suffering no deterioration in service. Aside from the accuracy of the anemometer there are other factors which limit the precision of the reading. The velocity may vary across the channel ±1%, eddies pass down the channel and the voltage varies, all of which result in fluctuations in the air speed. The latter, and not the accuracy of the instrument generally, determine the precision of the readings. Ordinarily an accuracy of 0.2% is sufficient (see R. & M. 103, 1913-14, p. 53). This precision, coupled with the small magnitude of the force or other effect of the air used to determine the speed, necessitates considerable magnification in the anemometer.

Pulsations, Damping and Lag.—In all wind channels there are, as already noted, pulsations of velocity of greater or less extent depending on the excellence of the channel. If the anemometer has a damping action the effect of the pulsations will be more or less eliminated but a greater or less time lag will be introduced, the instrument will be sluggish. Sluggishness is detrimental in wind channel work where simultaneous, instantaneous readings of air velocity and the forces on the model are taken. Any lag of the velocity indicating system in excess of the balance, and the balance is very quick to respond, will result in errors. The degree of damping in the anemometer will ultimately depend on the magnitude and frequency of the pulsations. In present day channels the latter are small and little damping is required.

Obstruction in the Channel. Any obstruction in the channel between the entrance and the model is detrimental to accurate work (see R. & M. 67, 1912-13, p. 55). Hence the ideal anemometer is one offering no obstruction to the air flow, or which does not project into nor disturb the flow.

The position of the instrument in the channel is important for if it projects into the channel and is upstream from the model it will disturb the flow past the latter, and if placed downstream the model will disturb the flow past the anemometer, while, if placed in the same transverse plane as the model, it is also likely to be disturbed to an extent depending on its distance from the wall and model due to the well-known fact that the airstream is disturbed a considerable distance upstream from an object. If of the non-obstruction type the instrument is best placed slightly ahead of the model. In the N.P.L. 3 ft. channel, 2 ft. ahead of the model was found best (see R. & M. 103, 1913-14, p. 53).

Anemometer heads, which operate through the direct action of the airstream on them, should for best results be placed in such a position that the velocity of flow past them is a maximum. In wind channels, as in other passages, the velocity drops very rapidly near the walls, and the flow is unsteady and turbulent. A traverse made at Toronto showed a rapid increase in velocity up to a point 5″ from the wall. Hence in this case the anemometer head should be placed at least 5″ from the channel wall.

Calibration. Whether an anemometer is such that velocities can be calculated directly from its indication and its dimensions, or requires calibration, is not important provided a standard of comparison is available for calibrating.

Self-Indicating or Manual. Where velocities of varying magnitude are to be read it is important that the anemometer be self-indicating, requiring no manipulation to determine its reading. But where the velocity has merely to be held constant at some definite value there is no difference between the indicating instrument and the one that has to be adjusted for each reading.

Miscellaneous. Simplicity of design in the anemometer is also of importance. The anemometer system should be simple, direct and compact, involving few pieces of apparatus and direct connections. The connection between head and indicating device of the instrument should be such that the two parts may be separated some distance. Convenience is also of importance involving, as it does in this work, the question of fatigue. The anemometer should be easily read, without unnecessary eye strain. The velocity should preferably be indicated directly in feet per second or miles per hour.

CONSTRUCTION AND CLASSIFICATION.

Air speed indicators may be regarded as composed of two principal parts, one, the head or that part inserted in the airstream and on which the air acts in some manner proportional to a function of the velocity; and the other, some form of indicating device on which the magnitude of the air effect on the head is indicated or from which the velocity is determined.

PART I.—ANEMOMETERS.

There are two principal types of anemometer, the Electrical and the Pressure type. The former type depends for its action upon the dissipation of heat from electrically heated wires in an air stream, while the latter is actuated by the pressure of the air due to its velocity.

ELECTRIC ANEMOMETERS.

The two Electrical anemometers best known in America are the Thomas Meter and the King Linear Hot Wire Anemometer.

THOMAS METER.

In this meter an electrically heated grid of wires is used to heat the air through a definite range of temperature, the latter being controlled by two electrical resistance thermometers forming two arms of a Wheatstone Bridge. The quantity of heat supplied the grid being known from the electrical instruments in the heater circuit, and the fixed rise in temperature, the mass of air flowing may be found.

The meter has been proved very accurate under a great variety of conditions, the maximum error never exceeding ½% and usually being much less. The meter is bulky, relatively complex and indicates the average mass flow of air over the whole cross-section. It is hence unsuitable for wind channel work.

KING LINEAR HOT WIRE ANEMOMETER.

In this instrument use is made of the Kelven Double Bridge in which a minute platinum wire forms one of the arms or resistances. A wire 2½ to 3 mils in diameter is most suitable and is held in a specially designed fork which is inserted in the air current. The electric current through the platinum wire indicated by an ammeter is adjusted until the galvanometer indicates a balance. The resistance and, therefore, the temperature of the wire is then at a predetermined value, and the velocity of the air may be found from the ammeter and the heat dissipating properties of the wire. The latter should be determined for each wire used, by direct calibration on a whirling arm.

The instrument is very accurate. The fork may, if necessary, be made very small. Its disturbing effect will depend upon the nature of the work but is extremely small. If high wire temperatures are employed variations of atmospheric pressure, temperature and humidity are ordinarily negligible. From standpoint of ageing of the wire a low temperature is desirable and also 2½ mils diameter is the lower limit. The life of a wire depends upon the conditions of service.

The energy dissipated as heat from the wire when held at a constant temperature, *i.e.*, resistance, depends upon the mass flow of the air, density and velocity, hence no density correction is necessary. The instrument, however, is relatively complicated, involving a number of pieces of apparatus.

For the routine indicating of air velocity in the wind channels wherein it is a case of holding a constant air speed, the current once adjusted would require no further manipulation and the galvanometer only would require to be observed. The most advantageous use of this instrument, and one for which it is particularly well adapted and would prove invaluable, is, in the investigation of velocity conditions, close to the surface of models in positions inaccessible to the ordinary types of pressure anemometers.

PRESSURE ANEMOMETERS.

Anemometers belonging to this class are of two principal types: namely, Cup or Screw Anemometers and Pressure Tube Anemometers. In the former the force acting on the head is the air pressure on a cup or blade, and in the latter the pressure on the air in the tube. In both cases the pressure is practically proportional to the density and the square of the speed. Hence no density correction is necessary.

CUP OR SCREW ANEMOMETERS.

In this type of instrument the head consists of a light wheel carrying a number of cups or blades, windmill fashion, supported in frictionless bearings. The wheel rotates under the action of the air pressure on the cups or blades at a speed only approximately proportional to the air speed and the revolutions of the wheel are recorded by a suitable mechanism from which, in conjunction with a time observation, the velocity may be determined. For accurate work the exact factor connecting air speed and speed of rotation of wheel must be known.

The presence of the moving part, with its attendant friction, lubrication and inertia errors, is a disadvantage of this type. The friction error is most marked at low speeds. The inertia effect results, in a fluctuating current, in too high an indication. It may be largely overcome by light construction.

From the wind channel standpoint, besides the questionable accuracy due to the above causes, there is the difficulty of transmission between head and indicating device and the obstruction of the flow in the channel. Further, unless the instrument is of the tachometer type indicating velocities directly the necessity of taking the reading over an interval of time and making a time observation prohibits the use of this type of instrument since speed fluctuations cannot be followed. Even in the tachometer type the inertia of the moving parts prevents small rapid variations in velocity being followed.

PRESSURE TUBE ANEMOMETERS.

Anemometers of this type are exclusively used in aeronautical work, not only in laboratories but as air speed indicators on aircraft. They are simple, direct, and when properly made and used very accurate. In these instruments pressure heads or collectors of various forms are used connected to sensitive manometers or other pressure indicating device, on which the pressure or pressure difference is accurately indicated and from which the velocity may be calculated. Because of the great variety of micromanometers used for this work they are dealt with in a separate section of this paper.

The dynamic pressure of air at the velocities ordinarily used in wind channel work is small, at 40 ft. per sec. the velocity head is only 0.365 inches of water, and for this reason the head of anemometers of this type is frequently so made as to increase or magnify the velocity effect (*i.e.*, pressure difference). In the case of aircraft speed indicators, the greater the pressure difference transmitted to the indicating mechanism the more positive the action. Recording instruments would in some cases be advantageous in wind channel experiments and for actuating such instruments a large pressure difference and large flow are required.

The heads or collectors of Pressure Tube Anemometers are of the following forms:

(*a*) Pitot-static Tube.
(*b*) Double Pitot Tube or Pitot-aspirator.
(*c*) Aspirator.
(*d*) Side Plate.
(*e*) Venturi Tube.
(*f*) Pitot-Venturi.

Pitot-Static Tube. This form of pressure head is very extensively used in aeronautical work because of its simplicity and accuracy.

In its usual form the Pitot-static tube consists of two circular tubes terminating in a hollow conical nozzle at the forward end (see Fig. 8). The nozzle forms the Pitot or impact opening, the outer tube carrying the static openings. The tubes are generally of brass 5/16″ and 3/16″ O.D. At the rear or heel the tubes are either bent through 90° to form the shank on the other end of which are the pressure connection nipples, or enter a stream line shank and connect to two ducts which convey the pressures to the nipples.

It has been shown that while practically any form of opening will serve as an impact opening the static openings must be carefully designed, the approved form being 0.04 inch diameter holes carefully drilled in the outer tube just back of the conical nose, with all burrs removed. The conical nose should have a fine taper, not to improve the dynamic opening, but so that the air will flow smoothly past the static openings at all speeds thus permitting the true static pressure to be obtained. In the Ogilvie tube the nose is rounded and relatively blunt.

The Pitot or dynamic opening has frequently been used in combination with a piezometer ring for velocity measurements. The piezometer ring secures the true static pressure. In some types of British air speed indicators for aircraft the Pitot and Static tubes are quite separate, the latter having a closed pointed end as in the British mark IV A.

A point not often stressed but which may lead to serious error is the influence on the accuracy of the Pitot-static tube of bodies in its rear. It is well known that the air is disturbed a considerable distance in front of objects in motion relatively to it and for this reason if a Pitot-static tube is made with the portion bearing the static openings too short an object brought up close to the heel will disturb the stream at the static openings resulting in an incorrect static pressure being secured. A case of this nature occurred at Toronto where, in the course of making traverses of the wind channel, it became necessary to hold a Pitot-static tube (the left hand one in Fig. 8) in such a way that a 1″ dia. hemispherical head was close to the heel and $3\frac{3}{16}''$ behind the static openings.

Even this relatively small and more or less stream-line body resulted in a reading of velocity 1% low. The standard British and American tubes have the portions parallel to the wind 15⅛″ and 12⅝″ long respectively with the rearmost static openings 13″ and 8⅝″ ahead of the heel; and a tube largely used at Toronto (right hand one in Fig. 8) is similarly constructed.

If a tube be held too close to the wall of the channel or other object the static readings will also be incorrect.

The theory of the Pitot-static tube is well known. It is commonly developed on a basis of Bernoulli's theorem, considering the air incompressible, but more correctly from the principles of thermodynamics taking into account the compressibility of the air. In the latter case, for adiabatic compression, the pressure on the impact opening is

$$p = p_0 \left(1 + \frac{(\gamma - 1)\, \rho_0 v^2}{2\, \gamma\, p_0} \right)^{\frac{\gamma}{\gamma - 1}}$$

so that a Pitot-static tube transmitts to a differential gauge, the difference between this pressure and the normal static pressure or

$$p - p_0 = \frac{\rho_0 v^2}{2} + \frac{1}{\gamma - 1} \frac{\rho_0 v^2}{2} + + +$$

In this equation all the terms after $\frac{\rho_0 v^2}{2}$ are negligible to a precision of 1% for speeds below about 225 ft per sec. This means that up to a speed of 150 m.p.h. the compressibility of the air may be neglected without serious error and the above equation reduces to that developed on a basis of Bernoulli's theorem.

The Pitot-static tube, constructed as above described, therefore, requires no calibration, the equation $p = \frac{\rho_0 v^2}{2}$ applying with an accuracy of 1/10 % of the velocity up to 50 ft./sec., and 1% up to 225 ft. per sec. At 40 ft./sec. a water differential gauge would only indicate 0.365 inches of water which, as will be seen later, is small in comparison with that of other heads. Unless the tube is constructed properly the flow of air will be restricted in the annular space between the tubes, resulting in the throttling or a damping action and consequent lag. A tube tested at Toronto exhibited this defect, for in comparison with an open 3/16″ brass tube (aspirator) 20% more time was required for a slanting alcohol gauge to reach its reading. The disadvantage of the Pitot-static tube as a permanent air speed indicator in a wind channel is its disturbance of the flow. It is very useful as a movable exploring tube but it cannot be used in confined positions too close to surfaces or where there is great uncertainty in the direction of flow.

Double Pitot Tube (*Pitot-aspirator.*) Pitot tubes are frequently made with one opening upstream and the other down, the latter then acting as an aspirator. The Stauschiebe and Pneumometer are of this type. The R.A.F. head, in which a cone with open base downstream covers the pressure opening, the Clift tube of somewhat similar construction and other air craft speed indicators employ the same principle. The "yawmeter" used at the N.P.L. in aerodynamic research is also an example of this type, employing four dynamic openings and an aspirator opening in a hollow cone. Recently a small double-ended tube was used in a wind tunnel investigation in the United States (McCook Field).

No theory has been developed for this type of tube as it involves a consideration of the external flow about the tube and the negative pressure in the rear. The pressure difference may be 40% higher than given by the Pitot-static tube, and if the formula of the latter is used a coefficient must be found by calibration. Some typical coefficients for use in $h = c\ \frac{\rho v^2}{2}$ are: Clift 1.4, Stauschiebe and Pneumometer 1.37, McCook Field 1.36.

This construction thus secures a certain degree of magnification and freedom from direction effects as compared with the Pitot-static.

Aspirator. A downstream opening or aspirator has been used as an anemometer head. The aspirator is connected to one side of a differential gauge, the other side of which is open to the room. The gauge reading is then a function of the velocity and will depend as in the Double Pitot on the external flow. In tests at Toronto with a simple aspirator of 5/16″ O.D. tubing, bent at 90° to give a length of 3″ parallel to the wind, and finished off square a reading 47% greater than with a Pitot-static tube was obtained, the air flow was such that the movement of the liquid in a slanting alcohol manometer was not retarded over that of the free gauge, and the pulsations were 0.7 of those of the Pitot-static under similar conditions. The disturbing effect is equal to that of the Pitot-static.

Side Plate. In wind channels of the enclosed N.P.L. or Eiffel types there is a pressure drop through the channel due to losses in the entrance, honeycomb and in tube itself in the N.P.L. channel or the chamber in the Eiffel type. The pressure at the working section is, therefore, below room or atmospheric pressure.

On a basis of Bernoulli's theorem, considering the complete path of the air through channel and room, conditions may be expressed by

$$p_r + \frac{\rho v_r^2}{2} = p_c + \frac{\rho v_c^2}{2} + f\left(\frac{\rho v_c^2}{2}\right) = p_0$$

where the subscripts r and c refer to room and channel, p_0 is the static pressure where the velocity is zero and $f\left(\frac{\rho v_c^2}{2}\right)$ the friction loss in the channel up to the working section which is a function of $\left(\frac{\rho v_c^2}{2}\right)$ Since the area of the channel cross-section is 16 sq. ft. and of the free area of the room 560 sq. ft. v_c is $\frac{1}{35} v_r$ or $\frac{v_r^2}{v_c^2} = \frac{1}{1225}$ Hence v_r^2 may be neglected and

$$p_r = p_0 = p_c + \frac{\rho v_c^2}{2} + f\left(\frac{\rho v_c^2.}{2}\right)$$

A manometer connected between channel and room, reading differences in static pressure, reads then

$$p_r - p_c = \frac{\rho v_c^2}{2} + f\left(\frac{\rho v_c^2}{2}\right)$$

and its reading is thus proportional to a function of the velocity. The unknown, but small, friction loss prevents calculation of the velocity from the manometer reading with any accuracy but the side plate, if calibrated, can be used as an accurate air speed indicator.

This method was introduced by Eiffel (see Resistance of the Air and Aviation, Paris, 1912) and is used in both the Eiffel and the N.P.L. type channels (British Report for 1912-13, p. 61). While no difficulty is experienced in taking off the true static pressure from the chamber of the Eiffel channels, care must be exercised in the N.P.L. channels to secure the true static pressure. The method adopted in the latter case makes use of the Side Plate, which is essentially a simplification of the piezometer ring long recognized as the most accurate means of determining the true static pressure in pipes. A smooth flat plate or disk is fixed in the side of the channel with its face flush with the channel wall. The plate is carefully drilled with a single large hole (3/16″ dia.) or several fine ones (0″.02—0″.04 dia.) which communicate through a brass nipple soldered on the back of the plate, and rubber tubing with the gauge.

It will be evident from the above that the side plate will show a slightly greater reading than the Pitot-static tube, that is, will have a greater magnification. In the Eiffel type wind channel at Leland Stanford University the pressure difference between chamber and room is 1.142 times the velocity head. At Toronto the side plate showed a

34%† greater reading than the Pitot-static Tube (velocity head). The lag, even with only five $0''.02$ dia. holes, was no greater than the aspirator while the pulsations were 0.65 of those of the Pitot-static under similar conditions. The accuracy of the device is as great as that of the calibrating instruments.

The advantage of the side plate which renders it superior to all other types of head for wind channel use is the absence of obstruction and interference with the air flow in the channel.

Venturi Tube. The Venturi principle is in extensive use in measuring instruments of many kinds for liquids and gases. It is now employed in many pressure heads for air speed indicators for aircraft.

The theory of the Venturi tube, when placed in a pipe line to determine the flow of fluid, is well known. If the fluid is a liquid and incompressible the theory is developed on a basis of Bernoulli's theorem resulting in the equation:

$$v = \sqrt{\frac{1}{\left(\frac{A}{A_1}\right)^2 - 1}} \sqrt{2\frac{p - p_1}{\rho}}$$

If the fluid is a gas, and hence compressible isentropic flow is assumed and the equation is

$$v = \sqrt{\frac{2\left(\frac{\kappa}{\kappa - 1}\right)\frac{p}{\rho}\left[1 - \left(\frac{p}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right]}{\left(\frac{A}{A_1}\right)^2\left(\frac{p}{p_1}\right)^{2/\kappa} - 1}}$$

where v is the entrance velocity in each case and A_1 the throat area.

But in the Venturi tube when used in free air, in addition to the inner flow, the outer flow must be considered, or the flow around the outside of the tube (as in Double Pitot tube and aspirator). No theory has been developed to cover this outer flow. Experimental investigation indicates that the outer flow is important since the velocity of entry will depend upon the pressure (suction) at the rear of the exit cone which will be dependent on the outer flow conditions. The above equation takes no account of exit cone conditions and hence is inadequate when the Venturi tube is in the open air.

The equations, however, when transformed into

$$p - p_1 = \left[\left(\frac{A}{A_1}\right)^2 - 1\right]\frac{\rho v^2}{2}$$

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†This is larger than would be shown without an intake of the Toronto type. The intake renders the side plate more effective.

indicate the principal advantage of the Venturi tube for the purpose under discussion, namely, the greatly increased pressure difference obtainable by properly proportioning the entrance and throat areas. In this way greater magnification is secured and in the case of recording or mechanical indicating instruments a greater pressure difference with which to work.

An extensive investigation of Venturi tubes by Captains Toussaint and Lepère resulted in the following conclusions:

(1) The length and angle of the entry cone has little influence on the efficiency of the Venturi a length between 2 and 2.5 cms. (0.7874 and 0.9843 inches) and an angle of 20° are suggested.

(2) The suction is decreased with increase in the space between entrance and exit cones, a space of 0.7 mm. (.0276 inches) when throat diameter is 1 cm. is recommended. If reduced beyond this throttling is likely to result with consequent reduction in sensitiveness.

(3) The length of the exit cone is important, the efficiency drops as the cone is lengthened. About 160 mm. (6.299 inches) appears to be a satisfactory length with a cone angle of 4° 50′.

(4) There is no apparent advantage in using variable cone angles, *i.e.*, curved surfaces.

(5) The efficiency of the tubes drops with their size, other things being equal.

(6) Double Venturis were unusually good, suctions eight times that of the inner tube alone were secured.

Tests made by the Bureau of Construction & Repair, U.S. Navy, during the development of an air speed indicator and reported by Zahm confirm many of the above points. It was found that with a ½″ throat Venturi tube there was a marked gain in suction with lengthening of exit cone until a length of about 6″ was reached after which the gain was immaterial. An important point discovered was that flaring the base of the exit cone, trumpet fashion, adds considerably to the suction. Gains of as high as 50% were noted, due, doubtless, to the decreased pressure in rear of trumpet end causing an increase of velocity through the tube. Lengthening the entry cone beyond 1″ (for a ½″ throat) reduced the suction slightly as did flaring the entrance cone.

To further increase the suction Venturi tubes are frequently made with double throat, that is a small tube mounted in a large one so that the outlet of the exit cone of the small tube is at the throat of the larger tube. In this way the pressure at the throat of the smaller tube can be very much reduced. Toussaint and Lepère record suctions with double tubes eight times that of the inner tube alone and Zahm also records marked gains which were further increased by flaring the exits.

Care must be taken in using Venturi pressure heads just as in the case of Pitot static heads. Zahm reports that the Venturi must be kept about 25 diameters ahead of an obstruction, even a fairly stream-lined one, such as a strut, to prevent blanketing of the tube and loss of suction.

Pitot-Venturi. To further increase the pressure difference the Venturi tube is frequently combined with a Pitot, or impact, opening. Many aeroplane air speed indicators are of this type. The magnification is greater than a simple Venturi tube of the same dimensions but otherwise the same points apply as to the Pitot Tube and Venturi tube separately. For wind channel work the Pitot Venturi would offer slightly more obstruction than either Pitot or Venturi alone since both tubes would require to be placed away from the wall 5″ or if, as is generally done in aeroplane work, the Pitot opening was arranged on the standard supporting the Venturi, the latter would have to be more than 5″ away in order that the Pitot might be 5″ from the wall and in the region of steady flow.

PART II. MICROMANOMETERS.

INTRODUCTION.

Sensitive manometers form a very essential part of the equipment of an aerodynamic laboratory. Their principal use is in connection with a pressure head in the determination of air speed but they have different other uses such as in pressure distribution and direction of flow investigations. Many types have been developed, some more suitable for aerodynamical work than others. Brief descriptions are presented here of some of the more common micromanometers in use in aerodynamic testing, with points regarding their design, accuracy, limitations and manipulation, based largely on experience gained at Toronto in the use of these instruments. It is felt the information presented may be of service to those interested in such instruments,

The principal requirements of aerodynamic micromanometers may be summarized as follows:

RANGE AND PRECISION.

Micromanometers are sensitive pressure gauges of the differential type, adapted for the accurate measurement of very small pressure differences. A micromanometer may arbitrarily be defined as a manometer for the measurement of pressures up to 3″ of water. The measurement of similar pressure differences between larger pressures is not encountered in aerodynamic work, and the gauges used are not generally suited for higher pressures.

In aerodynamic work the air speed most commonly employed is 40 ft. per sec. for which the dynamic pressure is 0″.365 water. At 60 ft. per sec. the dynamic head is 0″.822 and a speed of 115 ft. per sec. is necessary before a pressure of 3″ of water is reached. A range of 3″ is thus ordinarily sufficient.

The degree of precision or sensitivity desirable is 0″.0005 of water although 0″.001 is frequently close enough. The precision of a velocity measurement is limited usually, not by the accuracy of the instruments but by the steadiness of the air current.

Gauges should be free in principle and construction from sources of indeterminate errors such as capillarity, viscosity. hysteresis and friction errors, which cannot be definitely evaluated.

Damping. The presence of inertia in a micromanometer and consequent lag is undesirable and should be reduced as much as possible. If lag is required in order to allow time for a gauge to be adjusted to the pressure, or to render a reading possible with a fluctuating pressure, it is better to have the lag or damping applied in a definitely known manner rather than as an uncertain value in the gauge itself.

Damping may be secured in various ways. In many gauges employing a liquid a stop cock is provided which, by partly closing, will give any degree of damping. The same effect can be secured by inserting, either in the gauge itself or its connections, short lengths of fine bore tubing, choosing a suitable bore for the desired damping effect. This method has been successfully used at Toronto with a slanting manometer. Stanton used this method in some of his researches, arranging two long capillary tubes in the connections to the gauge. These tubes were bypassed by two ordinary bore tubes with stop-cocks. While adjusting the gauge the pressure was applied slowly through the fine tubes, but when making the final readings through the larger tubes and cocks.

Convenience. The gauge should be arranged for convenience and ease of reading. Thus in liquid gauges coloured liquids are an aid. The reading of only one meniscus in such a gauge is also desirable especially with fluctuating pressures where simultaneous accurate reading of two moving meniscuses is practically impossible. Arrangements so that the zero reading is zero eliminating the necessity of subtracting a zero correction also facilitates the work. The provision of scales on the gauge reading directly in the desired units, or of large size calibration curves for the conversion of readings, is a great help.

Standards. Micromanometers may be divided into two classes, those in which the pressure may be determined from the gauge itself and those which require calibration against an instrument of the first class. The former may be termed absolute or self standards and at least one of this type should be in every laboratory for calibration purposes. They are frequently less convenient to use than the second type, and the latter are, therefore, after calibration against the standard more generally used in laboratory work.

TYPES.

Micromanometers may be classified according to their principle of operation as follows:

1. *Mechanical Type.* The movement of a diaphragm or other device under the pressure is mechanically magnified. While such instruments have been devised and used, of great sensitivity, ordinarily they are not sufficiently delicate for aerodynamic work. Instruments of this type are generally less simple in principle and more complicated in construction than those of the next class. In addition the pressures cannot be calculated directly from their indications as the forces involved are too complex. Calibration against a standard or by means of a definite physical phenomena must be used. Such a calibration of extremely sensitive gauges is difficult. Less sensitive gauges of this type are almost exclusively used with aircraft speed indicators.

2. *Gravity Type.* In this class of instrument the liquid filled U tube construction is employed and various devices employed to magnify the displacement or movement of the liquid or measure the small movement accurately. Instruments of this type may be self-indicating or require manual adjustment.

Self-Indicating. (*a*) Magnification of the displacement is secured by using the equivalent of a very light liquid, that is two non-miscible liquids of nearly the same specific gravity so employed that their difference in specific gravity is the operative factor. This is the so-called Two Liquid Differential Gauge.

(*b*) Increase in sensitivity is obtained by sloping one leg of the U tube thus increasing the movement of the meniscus for a given vertical displacement, as in the Krell or slanting alcohol manometer.

Manual Adjustment. (*c*) The position of the surfaces or one of the surfaces of the liquid in the gauge is indicated by sharp needle points attached to micrometer heads, or by microscope fitted with crosshairs and mounted on a micrometer screw, or some similar device, as in the Screw or Threlfall Micromanometer.

(*d*) The movement of liquid from one leg to the other of the U tube under the applied pressure is prevented by tilting the gauge or raising one leg to an extent just sufficient to compensate for the pressure applied, means being provided for accurately noting that no movement of liquid has taken place. The Chattock, Direct Lift and Toepler manometers are of this type.

3. *Weighing Type.* In manometers of this type, the pressures whose difference is to be measured are introduced into two chambers or inverted cups, sealed in liquid and suspended from a balance beam, the difference in weight resulting from the pressures within the bells is then a measure of the pressures. This type of gauge is used at the Leland Stanford University laboratory and in some German laboratories.

MECHANICAL TYPES.

FRY MICROMANOMETER.

This instrument is one of the most, if not the most, sensitive yet devised, having ten times the sensitivity of the most commonly used aerodynamic gauge, the Chattock. It is claimed that the sensitivity is 0.001 dyne/sq. cm. or 0.0000004 inches of water.

The pressures are applied on the two sides of a thin, tightly stretched circular membrane. A mirror is hung by means of a quartz bifilar suspension, one point of support being the centre of the membrane and the other a point in the frame. A deflection of the membrane rotates the mirror and the pressure is indicated by the spot of light from the mirror on a scale. The sensitiveness may be varied by changing the horizontal distance between the points of support of the mirror by levelling screws. An animal membrane (Badische) of thickness 0.001 mm. was found best. Rubber and goldbeaters skin was not suitable and cyclostyle paper lacked sensitiveness. Copper was suggested for higher pressures.

N.P.L. PHOTOGRAPHIC PULSATIONS GAUGE (BOOTH AND EDEN).

This gauge is very similar to that of Fry. A thin metal diaphragm is used, or parchment in some cases. A Perry twisted strip is connected at one end to the centre of the diaphragm and at the other to the frame and carries at the centre a light mirror. Deflections of the diaphragm produce large rotations in the strip and movements in the mirror. The latter throws a spot of light on a moving photographic plate. The natural oscillations of the gauge are damped electromagnetically.

OGILVIE AIRCRAFT GAUGE.

A flexible rubber diaphragm is employed in this gauge, to the centre of which a silk cord is attached. The cord takes one turn around a roller with axis parallel to the diaphragm and two around a roller with axis perpendicular to the diaphragm. The latter roller carries the needle. The rollers are carried in the frame and zero is maintained by means of a spiral spring acting on the second roller. This gauge gives an indication of 300° on the dial for about 2.25 inches of water (100 ft./sec. velocity on a Pitot-static Tube). The rubber diaphragm apparently stands up well as some have been in use a year without signs of deterioration. Generally rubber diaphragms age resulting in a change in calibration of the instrument and an increase in sensitivity.

CLIFT AIRCRAFT GAUGE.

In this gauge the deflection of a flexible diaphragm is transmitted through a light disk to a plunger at the centre of the latter. The plunger acts against a flat cantilever spring the movement of which is transmitted through a lever to the needle by means of a mechanism similar to that used in aneroids.

GRAVITY TYPES—SELF-INDICATING.

TWO LIQUID DIFFERENTIAL GAUGE.

This gauge has been described in detail by J. A. Smith in a paper before the Victorian Institute of Engineers. It consists of a U tube in which the upper parts of the legs are enlarged into cisterns as shown in Fig. 1. The tubes recommended are from $\frac{1}{8}''$ to $\frac{1}{4}''$ bore, and the length 30 times the desired range in water. If a larger bore is used the meniscus is indeterminate and irregular, if smaller the liquid column breaks. The shorter the tubes the less will be the inertia and the more rapid the movement of the gauge. With $12'' \times \frac{1}{4}''$ tubes there is no perceptible lag while with $48'' \times \frac{1}{8}''$ there is a lag of one minute. The cisterns are arbitrarily made 100 times the tube area and about 2″ deep.

The liquids recommended are alcohol and kerosene the former coloured with analine dye. The specific gravity of the alcohol can be adjusted to be as close to that of the kerosene as desired by the addition of water. If the specific gravities are too nearly equal (less than 0.05 difference) the manipulation of the gauge is difficult. Water or methylated spirits may also be used with the kerosene. If the former, the magnification, is reduced, and care must be taken with the latter that there are no constituents in the methylated spirits soluble in kerosene This can be assured by keeping both in the same container for some time before using. Methyl alcohol and gasoline have also been used although volatile and somewhat soluble in each other. Amyl ether and water are less volatile.

The formula for the gauge can be readily determined as

$$P_1 - P_2 = R(S_2 - S_1) + \frac{A_t}{A_c} R \, . \, (S_2 + S_1)$$

where R is the movement of the surface of separation and the other quantities as shown in Fig. 1(*a*). From this it is seen that with alcohol and oil of specific gravities 0.834 and 0.79 respectively, the magnification secured will be 17 – 18 while, if water and oil are used, the magnification is about 6.

If the cisterns are infinite in area the equation reduces to $P_1 - P_2 = R(S_2 - S_1)$ and if no cisterns are employed to $P_1 - P_2 = 2RS_2$.

When the gauge is arranged with oil on top of both legs as in Fig. 1(*b*) two meniscuses must be read and the equation becomes

$$P_1 - P_2 = R(S_2 - S_1) + \frac{A_t}{A_c} . R . S_1$$

and a slightly greater magnification is secured, with oil and alcohol as above the magnification would be about 23, but the gauge is less convenient to use.

This type of gauge is subject to errors due to capillarity, viscosity imperfections in tubing and cannot be used as an Absolute Standard.

A recording gauge of this type has been used in connection with an R.A.F. pressure head in an instrument used in a gust research. The legs were 3/16″ bore tubes with 1″ dia. cisterns, the whole being $11\frac{1}{2}$″ overall. Benzine and methyl alcohol were used.

To avoid change of zero with tilting a concentric gauge of this type has been devised at the R.A.E. and used successfully on aircraft, see Fig. 1(*c*).

A two liquid differential gauge was designed and made up at Toronto to have a range of about $1\frac{1}{2}$″ at a magnification of 20.1. A movable scale (77 mm. long) and zero were improvements facilitating the taking of readings, and a movable vernier on transparent celluloid was provided for more accurate readings. Both zero and vernier marks were on the back of the celluloid to reduce parallax errors.

Different liquid combinations were tried, ethyl alcohol and kerosene, methylated spirits and kerosene, benzol and water. The specific gravities of these were accurately determined at a number of temperatures and are shown in Fig. 16.

With the first two combinations using an analine dye, scarlet B, difficulty was experienced through the alcohol and oil clinging to the tubes even after the latter were thoroughly cleaned. A distinct film of colour remained on the tube after the alcohol receded. This film gradually left the tubes but the accuracy of the gauge was adversely affected. Calibration against a Chattock gauge showed the precision to be very poor, the difference between readings for increasing and decreasing pressures being large. Varying the magnification by changing the density of the alcohol did not eliminate the trouble.

The benzol-water combination using dye proved unsatisfactory owing to the high rate of evaporation of the benzol. The precision was still low.

Different sizes of tubes, from 3/16″ up to ⅜″ bore, were unsuccessfully tried in an endeavour to overcome the difficulty although no trouble was experienced with the meniscuses.

The inertia of the gauge at Toronto was large, resulting in excessive lag.

This type of gauge, as constructed at Toronto, has been found very unsatisfactory and quite unsuited for aerodynamical laboratory purposes. Better results might be secured with shorter legs and smaller magnification.

SLANTING ALCOHOL GAUGE (KRELL).

The slanting gauge (see Fig. 2) consists of an inclined tube, usually set at an inclination of 3° or 6° to the horizontal giving magnifications of 20 and 10 respectively, forming one leg of the U tube, and a reservoir having an area several hundred times that of the tube forming the other leg. The use of the latter renders it unnecessary to consider the movement of the liquid level in the reservoir and only one meniscus, that in the inclined tube, need be read.

This gauge has been investigated at the Massachusetts Institute of Technology. It was found that the inclined tube is very important. It must be free from all flaws, uniform in diameter and perfectly straight. For use with alcohol 0″.22 bore is recommended. The precision when velocities are calculated from dimensions is 1.5% on the velocity. It is thus not suitable for an absolute standard but the gauge responds to changes in velocity of ¼% and hence if calibrated against an accurate standard may be as precise as the standard.

A slanting alcohol gauge has been used extensively at Toronto (see Fig. 9). The movable bed consists of a flat piece of steel ½″×2½″×14″ long (this might better be 18−20″ long) with a triangular slot carefully milled in it to take the glass tube and fitted with two steel brackets 1¼″ wide, one finished off at 3° and the other at 6° to the bed. The platforms carry machinists levels. The bed is pivotted as shown and arranged with swivelling cone-pointed adjusting screw. The glass tube is held in the slot by means of transparent celluloid clips thus permitting the meniscus to be observed at all points. A strip of white paper behind the glass improved the visibility greatly. The tube was marked with a file mark and a corresponding mark made on one of the celluloid clips so that if the tube is accidentally shifted when making a connection it can be be returned to exactly the same position as that in which it was calibrated.

The scale is bevelled off to a knife edge and projects half over the tube facilitating reading and reducing errors due to parallax. Two scales are used, one graduated in millimeters, the other in feet per second air velocity.

The reservoir is of spun copper 5″ dia. × 3″ deep and is mounted on a tilting brass platform with elevating screw for adjusting the zero. Alcohol (S.G. 0.8153) coloured with analine dye is used as the liquid.

The gauge has been carefully calibrated against a Chattock manometer and used extensively in aerodynamic research. It has given every satisfaction. It is precise, consistent, very quick to respond and convenient to read. It will be evident that when calibrated the gauge is independent of errors in setting angle between tube and levels, of position of base, etc; all that is necessary in setting up is that the spirit level be properly levelled and zero adjusted.

An interesting point in connection with the calibration of this gauge was the necessity for lagging the copper reservoir against temperature effects. The presence of the observer reading the gauge caused marked changes in the reading due to the expansion of the air in the closed system between the slanting gauge and standard Chattock.

GRAVITY TYPES—MANUAL ADJUSTMENT.

SCREW MICROMANOMETER (THRELFALL).

The screw micromanometer was probably first used and described by Threlfall. The instrument consists of two reservoirs of large cross-section connected at the bottom, forming the U tube (see Fig. 3). The positions of the liquid surfaces in the reservoirs are accurately indicated by needle points forming the ends of carefully threaded spindles fitted with micrometer heads. The points are screwed up or down to the surface until the point and its reflection on the liquid surface just touch and the reading taken on the micrometer head. Sometimes only one indicating point is used, when the head equals twice the movement of the point.

The screws may enter the reservoirs from either top or bottom, but as a difficulty with this gauge is that of maintaining the stuffing boxes tight, it is preferable from this standpoint to have the screws enter from the bottom, so that any leak is at once made apparent by the liquid.

Threlfall, however, found that in bad light a point meeting the liquid surface underneath was less accurate than one meeting it from above. He suggested that use of ball tips as being better for contact below the surface. Coating the points with paraffin was found fairly successful in preventing them carrying liquid when meeting the surface from above. Threlfall employed coloured water as the liquid.

The screw gauge made and used at Toronto is shown in Fig. 10. 4″ lengths of $2\frac{1}{2}$″ bore glass tubing, with ends ground square are clamped between heads, using rubber gaskets. A single nut on a central standard with bridge and two points was found most satisfactory for tightening up. The heads, then, have freedom to adjust themselves to the gaskets. The indicating spindles are cut with 20 threads per inch, and have bevelled micrometer heads or discs divided into 100 divisions. The central standard is graduated in 1/20's of an inch so that the micrometer heads read to 0″.0005.

It has been found in use that the gauge, while sluggish, is comparatively accurate and can be read with certainty to 0″.001 (0.03 mm.) of water. It is free from errors due to capillarity irregularities in refraction through the glass tube, etc. Its accuracy depends on the precision with which the threads are cut.

CHATTOCK MICROMANOMETER.

The Chattock gauge is probably the most extensively used micromanometer for aerodynamic work. It was originally devised and used by Prof. Chattock for the measurement of very minute pressures and was employed by Stanton in his researches on pressures on a flat plate.

The gauge (see Figs. 4 and 11) is in the form of a broad U tube of glass, the legs being 13″ between centres and enlarged to form two bulbs or cups, 2″ in dia. into the tops of which are fitted corks carrying the connection tubes. The horizontal connecting limb of the U has at the centre a device for indicating the movement of liquid from leg to leg. The device consists of a third smaller indicator bulb to which the horizontal tube from one cup is connected at the side while that from the other cup enters at the bottom and is continued concentrically about half way up. The top of the central tube is ground to a knife edge. The central bulb is divided in a ground joint for filling and cleaning purposes and at the top is connected through a stop-cock with an oil reservoir. The dimension recommended for the central tube is 0″.22 bore and for the enclosing central bulb about 1″.4. The horizontal limb is provided with a 2 mm. stop-cock for throttling or to render the instrument portable.

The glass U tube is carried in the upper moving part of a metal frame into which it is fitted by means of eccentrically bored corks. The upper part of the frame is carried on the lower by three hardened pivots forming the three points of a T, $6\frac{1}{2}''$ across the head and 10″ long. Two pivots at the head of the T form a hinge about which the upper part of the frame may be tilted by means of an elevating screw, carrying the third pivot, and micrometer head. A flat or helical spring at one end and a guide at the other maintain the two parts in place.

The gauge is filled with water the upper part of the central bulb containing oil admitted from the reservoir above.

When in zero position the water has risen in the central tube until a water bubble is formed in the oil on top of the tube. When a pressure is applied to one leg of the U there is a tendency for the water to move from one leg to the other, which tends to cause an enlargement or contraction of the bubble. This tendency is counteracted by changing the elevation of one leg of the U until the difference in elevation is just equal to the applied pressure. The bubble is thus an indicator showing when a balance (no flow) is effected. The position of the bubble is observed by means of a microscope fitted with cross lines, a mirror being used to illuminate the bubble.

The difference in elevation of the bulbs forming the legs of the U can then be calculated from the dimensions of the gauge and turns of the elevating screw and the pressure thus determined.

Evidently such a gauge is free from capillarity, viscosity and other errors arising from a movement of liquid in a tube. The gauge is extremely sensitive and an absolute standard. It is a very satisfactory laboratory standard.

The range of the gauge as described above is $\frac{3}{4}''$ of water. Where a greater range is desired it may be constructed with the bulbs 26″ apart (see left hand gauge in Fig. 11) giving a range of $1\frac{1}{2}''$ of water, and for smaller pressures a smaller centre distance.

Other forms of glassware, as shown in Fig. 12, are provided for differend purposes. The upper one, filled with water or mercury, is used for approximate determinations or unsteady pressures, the level in one leg being observed by the microscope. The lower piece, filled with mercury and oil, is used for water pressures.

The sensitivity of the gauge could be improved by making the bulbs or cups larger in diameter relatively to the bubble tube.

Liquids. The liquids to be used in the Chattock gauge has received some attention at Toronto. The preferable liquid is, of course, water, but the kind of oil used in the indicator bulb must be considered. The oil must be transparent to permit observation of the bubble, viscous to enable a proper bubble to form, and inert in water or whatever liquid is used. The oil must not cloud or form a skin at the surface of separation. The density of the oil is unimportant as it has no effect of the reading.

Castor oil has been extensively used in Chattock gauges but possesses the drawback that it clouds in the presence of water rendering the bubble invisible and effecting also a reduction in the sensitivity. This is overcome by dissolving a little table salt in the water. At Toronto two strengths of brine were used, one 1:10 prepared by dissolving 1 gram of salt in 10 c.c. of distilled water, the other 1:20. The specific gravities of these two brines at different temperatures were accurately determined and curves plotted (see Fig. 16). The curves are very useful for determining densities at any temperature and in addition serve as a check on individual specific gravity determinations.

The necessity for using brine with castor oil is cited as a disadvantage of the Chattock gauge, and if brine is used it does involve some care in its preparation and determination of its S.G. As a result of tests at Toronto, however, it has been found that a medicinal paraffin oil, prepared by the Standard Oil Co. and sold under the trade name Nujol, is a very satisfactory oil for use in Chattock gauges. The oil is water white, inert in distilled water, and sufficiently viscous to permit the formation of an excellent bubble.

Comparative tests were made with castor oil, Nujol, benzol and kerosene over water, 1:10 and 1:20 brine, in test tubes for long periods. The Nujol was found to be quite inert in water there being no evidence of clouding or skin formation after 4 months. The castor oil over water clouded at the end of a few days in the immediate neighbourhood of the bubble, and remained clouded although a temperature effect was evident, the cloud being more dense on extremely cold days. Over the brines the castor oil was quite unclouded after 4 months. Benzol clouds over distilled water just as the castor oil, and forms a skin over brine more pronounced in the case of the 1:10 brine (after 2 months). Kerosene is apparently inert in all three after 2 months. Both benzol and kerosene however are lacking in viscosity for use in Chattock gauges. Castor oil with 1:10 and 1:20 brine and Nujol and water have all been extensively used at Toronto. The castor oil is found to get sluggish after a time and the brine increases slightly (0.1% after 120 days) in density owing to evaporation of the water. The distilled water-Nujol combination is the one now exclusively used, and gives every satisfaction.

It has been stated that coating the outside of the top of the bubble tube with paraffin wax permits of a larger displacement of the bubble without rupture. This did not prove very successful at Toronto as in castor oil after a few days the wax flakes off, while in Nujol the wax dissolves.

Toronto Gauges. Two Chattock gauges have been used at Toronto, one made by the Cambridge Scientific Instrument Co. (right hand Fig. 11), the other made up at the University (left hand Fig. 11). On the former the bulbs are 13″ apart, in the latter 26″. The range of the latter, $1\frac{1}{2}''$ water for 15 turns of wheel, is thus twice that of the former, $\frac{3}{4}''$ for 30 turns of wheel. The Cambridge elevating screw has 50 threads per inch, with the micrometer wheel divided into 100 divisions and when filled with water 1 turn of the wheel corresponds to 0″.024886 water. A movement of less than one division on the wheel, or that is 0″.00025 water, can be detected in the bubble. The instrument made at Toronto has an elevating screw with 20 threads per inch and the wheel carries

100 graduations so that when filled with water one turn corresponds to 0.12728 water. The gauge is sensitive to 0.2 division or 0″.00025 water.

Large scale calibration curves have been drawn for these gauges, as shown in Fig. 15. The curves are such that 0.01 turn on the Cambridge or 0.002 turn on the Toronto can be easily read off and the pressures determined to 0″.0002 water which is represented by 1 mm.

These two gauges have been very thoroughly tested together and have been found to agree with an average error of 0.1% and an extreme error of 0.5% at very low pressures, the latter corresponding to a very small absolute error.

Technique. The method of setting up the Chattock gauge is as follows: The stop-cocks and ground joints are first greased with Nujol. Water is poured into the two main bulbs until about half-full. The stop-cock in the horizontal tube is then closed and, that from the empty oil reservoir being open, the water is blown up into the central bulb expelling the air. The oil cock is then closed and oil poured into the oil reservoir after which, the cock is opened allowing the oil to descend into the central tube until the top of the bubble tube is immersed in oil. The glassware is now placed in the metal frame, using eccentrically bored corks to make up for slight differences between centre to centre distances of shanks below glass bulbs and the holes in the frame. The micrometer head is set at zero and the gauge adjusted by means of the levelling screws until the bubble is on the microscope cross line. The gauge is now ready for use but the zero should be frequently checked.

A too wide excursion of the bubble should not be permitted owing to its causing a slight change in zero reading. If a large excursion does accidentally occur, whether rupture of bubble occurs or not, the gauge should be returned to zero and allowed to stand open for some time to permit it assuming its proper position again.

A well-rounded bubble is superior to a flat one from a sensitivity standpoint. A height of bubble of about $\frac{1}{2}$ the bore of the bubble tube has been found best. Mathematically it has been shown that the best height is equal to $\frac{\sqrt{3}}{2}d$.

Sources of Error—(a) Measurements. It is of extreme importance in determining the constants of the gauge, that the necessary measurements be accurately made. The centre to centre distance of bulbs and pivots must be found to within 0″.001, while the pitch of the elevating screw should be known to 0″.0001. Provision for reducing wear in the screw or of taking it up is thus of great importance. To render errors in the pitch of the elevating screw of less importance the centre distance between pivots might be increased to say 20″.

(*b*) *Tilt.* Another possible source of error in this gauge is that due to the angle of tilt. A given movement of the elevating screw causes less elevation of the bulb when angle of tilt is small then when large. This error in the Cambridge instrument amounts to less than 0″.0005 or 0.06% for the extreme tilt of the gauge, which is negligible.

(*c*) *Temperature.* Temperature change may also introduce errors in the reading of so sensitive a gauge. For an extreme range of temperature from 55 to 80°F. the error with 1:20 brine is 0″.00086 for full range of standard instrument and .0015 for 1:10 brine. For the ordinary temperature changes likely to take place during an experiment the temperature errors may, therefore, be neglected. It will be found, however, that the gauge at times does not return to zero after having been in use. This is due to a change in temperature and the remedy is to frequently shut off the gauge and check the zero during an experiment.

A useful device employed at Toronto for the rough estimation of variations in pressure and hence velocity in a fluctuating air current is a graduated scale in the microscope. The scale enables the range of fluctuations of the bubble to be measured and by means of a calibration the pressures corresponding to a given movement of the bubble can be ascertained. This is obviously only useful for rough estimating purposes as it destroys the original principle of the gauge based on no movement of liquid in tubes.

The chief disadvantage of the Chattock gauge for wind channel work is the eye strain involved in long continued observation through the microscope. This has been overcome in England by using an optical projector by which the image of the bubble is thrown on a screen. This arrangement is good since besides relieving the eyes, it permits both lift and drag observers to watch the velocity thus eliminating considerable time lag in the readings.

The Chattock gauges are used as laboratory standards at Toronto, and one in connection with the side plate for routine velocity measurement.

DIRECT LIFT GAUGE.

A gauge employing the same principle as the Chattock gauge, but somewhat simpler and less delicate, is the Direct Lift Gauge which finds considerable favour in laboratories in the United States. It comprises features of both Chattock and Krell gauges.

In principle it consists (see Fig. 5) of a U tube one leg of which is a reservoir so large in area that the change in level in it for any movement of liquid in the other leg, of glass and rubber tubing; may be neglected. The second leg is a slanting glass tube, whose slope may be varied, carried on a nut that is raised or lowered vertically by means of a micrometer screw.

The zero is first established with the alcohol in the glass tube at a fixed mark, then when a pressure is applied the nut bearing the slanting tube is raised until the alcohol is again at the mark. The pressure is then read off directly from the micrometer screw. The slope at which the tube is inclined governs the sensitivity, the meniscus in the tube acting as an indicator just as the bubble in the Chattock.

Since the meniscus is always maintained at the same place in the tube there is theoretically no movement of liquid in the gauge and hence no corrections are necessary as in the Krell slanting manometer for variations in glass tubing, capillarity, viscosity, etc., and a reservoir is not essential. Practically, however, since reservoir and tube are connected by a flexible rubber tube there may be slight variations in its volume as the tube is raised and a large reservoir minimizes the effects of such variations.

A direct lift gauge has been designed and used at Toronto and as it possesses some new features it will be described.

The base (see Fig. 13) is a three legged iron casting fitted with levelling screws and a level. It carries at the centre a vertical standard of 1″ square cold rolled steel, fitted at one side with a $\frac{1}{4}$″ square key, and in front with a scale graduated in hundredths of an inch and clips for a glass tube. To the left of the standard is an adjustable socket which carries a spindle soldered into the reservoir. The latter is of spun brass 8″ dia.×2$\frac{1}{2}$″ deep. It is thus 1000 times the cross-sectional area of a $\frac{1}{4}$″ bore tube. The adjustable socket permits of the adjustment of the zero of the gauge so that it comes exactly at the zero of the scale, thus eliminating corrections.

To the right of the standard, in the base, is an adjustable steel bushing in which rotates the lower end of an accurately cut $\frac{3}{4}$″ dia. 10 thread per inch screw. The upper end of the latter rotates in a head carried by the top of the standard. There are steel washers at each end of the screw and the adjustable bushing permits the taking up of any play. The top of the screw carries a 3″ diameter micrometer wheel divided into one-hundredths and reading by vernier to one-thousandths of a turn or that is 0″.00001 vertical displacement.

A block of steel 1″×2″ deep is accurately tapped a snug fit on the screw and is cut with a keyway to run on the key of the standard. The block carries a rotatable tube plate fitted with tube clips, which can be locked at any desired angle by a thumb screw. The tube clips have a 90° notch so that any size of glass tubing can be taken.

A feature that renders the gauge more convenient is the vertical glass tube up the front of the standard beside the scale. Both this tube and the slanting one are connected through a T to the reservoir. In practice, when uncertainty exists as to the pressure to be measured, the slanting tube is shut off by stop-cock until the pressure is indicated approximately on the vertical tube. The slanting tube then brought to this elevation, the cock opened and the pressure read accurately.

The gauge has been constructed to measure pressures up to 10″ of water in order to have a laboratory standard capable of taking pressures in excess of the Chattock gauges.

The construction is such that the screw can be accurately measured by means of micrometers between the head of the standard and the elevating block, or the vertical lift thus measured by micrometers. It is therefore an absolute standard.

Ethyl alcohol coloured with analine dye is the liquid used.

The Toronto vertical lift gauge has been thoroughly tested and found satisfactory. The average of a number of careful tests by two independent observers indicated that the pitch of the thread was 0″.10007. The accuracy with which the gauge could be read was determined by displacing it from zero and redetermining the zero. It was found possible to read the gauge on an average to within 0″.0002 with an extreme error of 0″.0005.

Different bores and slopes of glass tube were tried. An excellent meniscus was obtained with 0″.22 bore tube at a slope of 1° 40′, but the gauge was very sluggish. With this tube a 3°.0 slope was found to give best results as to sensitivity. The movement of the meniscus at this slope being about 20 times the pressure in inches of water and the corresponding magnification 22.5, smaller angles resulted in no appreciable increase in sensitivity and considerable increase in sluggishness.

For use with the side plate in the wind channel the gauge was fitted with a 0″.16 bore tube to reduce sluggishness.

TOEPLER MICROMANOMETER.

A gauge closely resembling the Chattock gauge is that of Toepler (see Fig. 6(*a*)). The U tube in this gauge is in the form of a flat V of glass tubing of about $\frac{1}{8}$″ bore, the angle of the V being very close to 180°. The bent tube filled with coloured liquid is mounted in front of a finely graduated scale. One menicsus of the fluid is under accurate observation by means of a microscope fitted with crosshairs. The tube, scale and microscope are mounted as in the Chattock on the upper tilting part of a metal frame, which can be tilted about two fixed conical pivots by means of a micrometer screw.

The sensitiveness of the gauge depends on the angle of the bent tube since it will be clear that if the liquid be allowed to move, instead of tilting the gauge, that the movement δl of the liquid along the tube for a given pressure δp will be equal to $\frac{\delta p}{2\sigma \sin \alpha}$. Hence the smaller α (see Fig. 6(*b*)) the greater the magnification. The gauge is sometimes used this way, that is as a simple slanting manometer and is then subject to the defects of the latter.

The gauge should be set up approximately level and so that the shapes and positions of the two meniscuses are practically the same. The cross hair of the microscope is focused on the portion of the right hand meniscus farthest to the left. The distance between the two meniscuses l_1 and the pivots l_2 should be measured to the nearest $0''.005$ or 0.1 mm.

When a pressure is applied the tube is tilted until the difference in elevation of the two meniscuses equals the applied pressure. This is accomplished without movement of the liquid in the tube by keeping the right hand meniscus always on the crosshair of the microscope. Then,

$$p = n.t.\frac{l_1}{l_2}.\sigma = n \times \text{constant}$$

where n is number of turns of the micrometer screw of pitch t and σ the S.G. of the liquid.

Toepler used the gauge in connection with a switching arrangement, by which the pressure was applied first to left leg then to the right in rapid succession, in which case the equation becomes

$$p = nt\frac{l_1}{l_2}\frac{\sigma}{2}$$

where n is now the turns between the two extreme positions.

There is little inertia in this gauge, but there is a possibility of slight error due to variation in form of the meniscus as the slant increases. The gauge is an absolute standard. It is subject to none of the errors due to capillarity, viscosity, etc., arising out of moving the liquid.

WEIGHING TYPE.

WEIGHING MANOMETER.

A sensitive manometer differing from the usual types is that used at the Leland Stanford University and some Germany laboratories (see Fig. 7). It consists of two manometric cells or inverted cups suspended on the opposite ends of the beam of a sensitive balance. The lower ends of the cups are sealed under some suitable liquid, in this case kerosene. The pressure connections are carried up through the liquid into the cups. The difference in pressure inside the two cells disturbs the equilibrium of the balance to restore which weights must be added to one beam or the other. The magnitude of the added weights, together with the area of the cells, then give the pressure.

The gauge is sensitive to only 0″.002 and evidently possesses considerable inertia. It requires manipulation and is less convenient than some of the more common manometers, even with weights of such magnitude that the reading is given directly in the desired unit.

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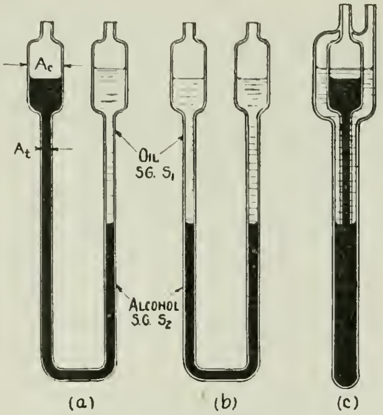

FIG. 1. TWO LIQUID DIFFERENTIAL GAUGE

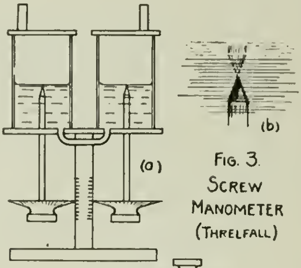

FIG. 3. SCREW MANOMETER (THRELFALL)

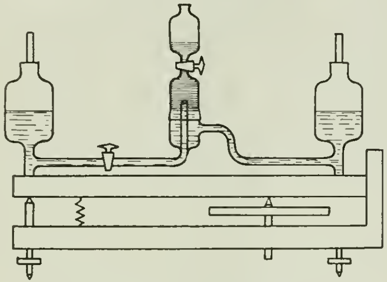

FIG. 4 CHATTOCK MANOMETER

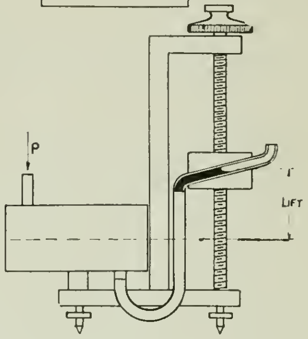

FIG. 5 DIRECT LIFT GAUGE

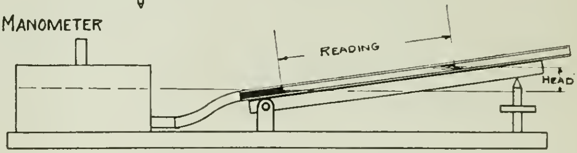

FIG 2 SLANTING MANOMETER (KRELL)

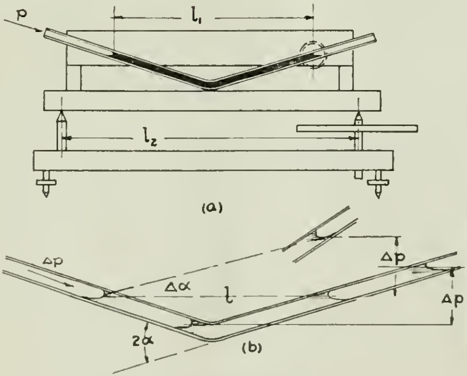

FIG. 6 TOEPLER MANOMETER

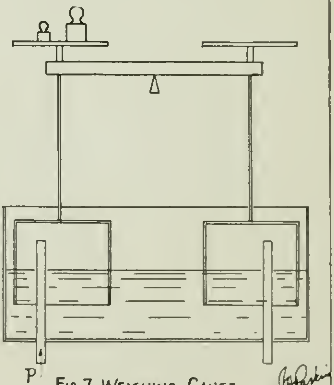

FIG. 7 WEIGHING GAUGE

MICROMANOMETERS.

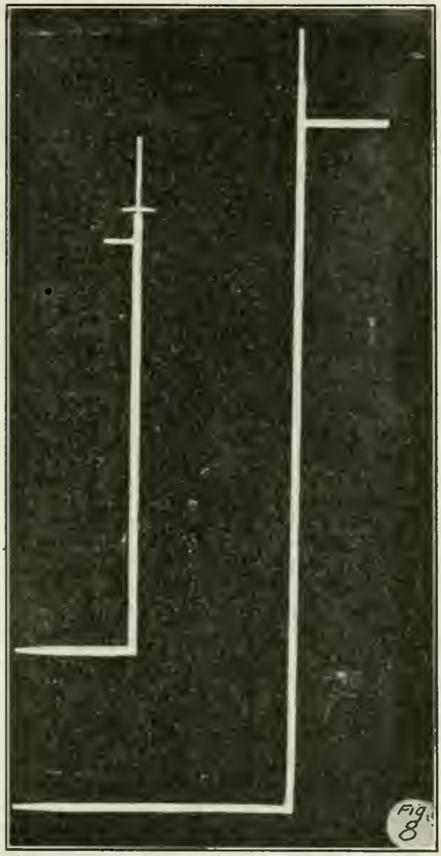

FIG. 8.—PITOT-STATIC TUBES

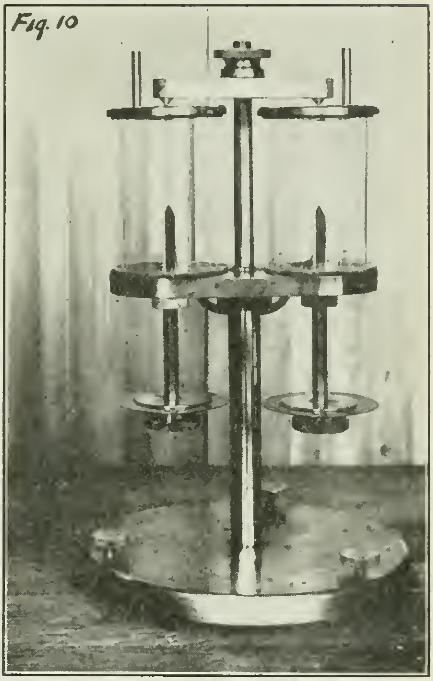

FIG. 10.—SCREW MANOMETER (THRELFALL).

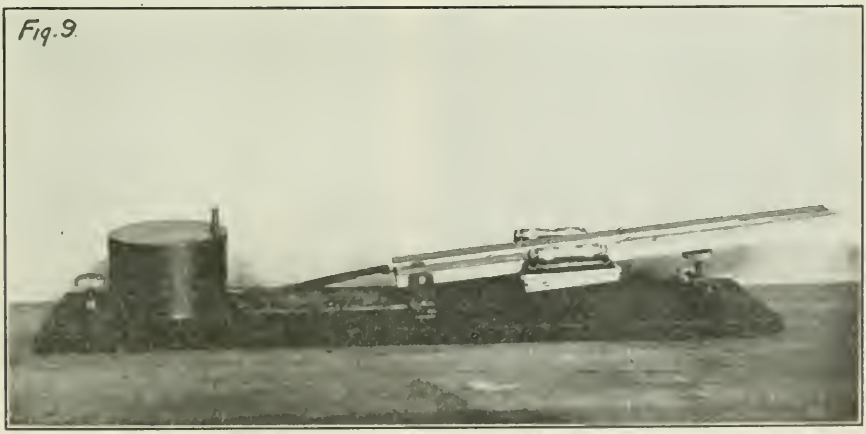

FIG. 9.—SLANTING ALCOHOL MANOMETER (KRELL).

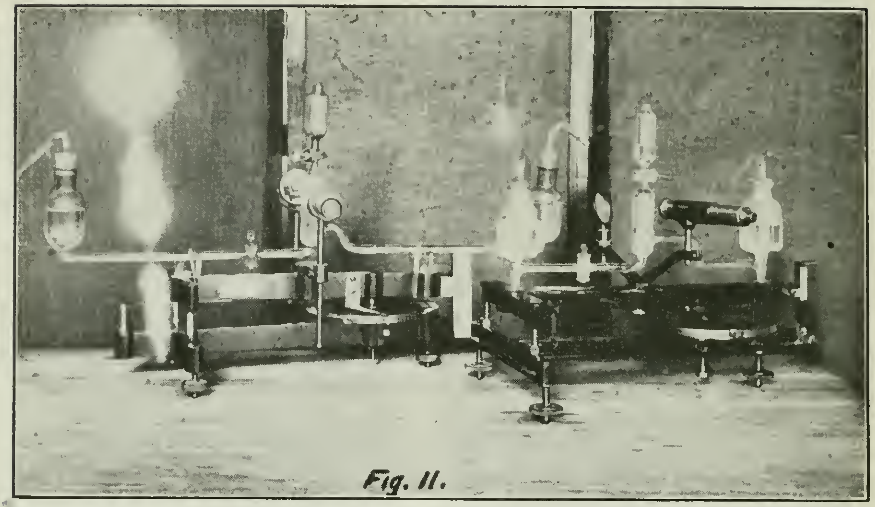

FIG 11.—CHATTOCK MANOMETERS.

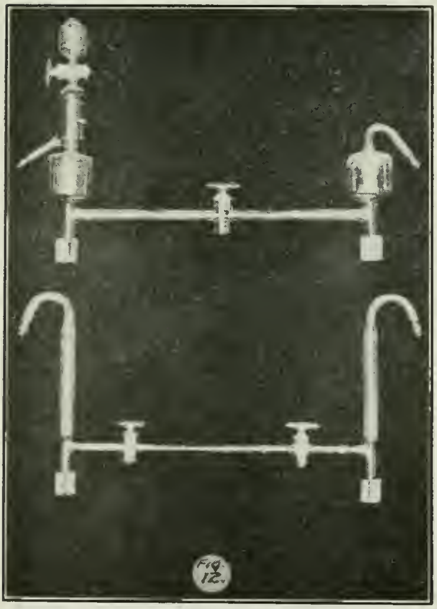

FIG. 12.—GLASSWARE FOR CHATTOCK MANOMETERS.

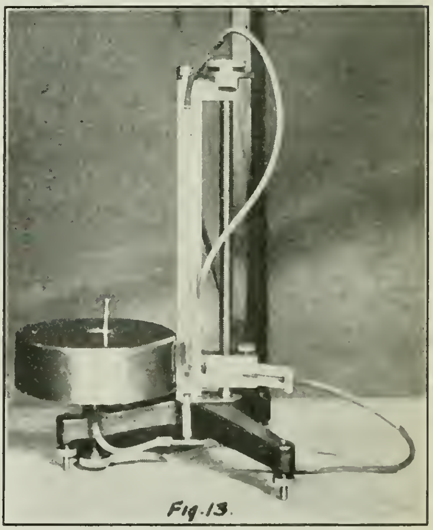

FIG. 13.—DIRECT LIFT GAUGE.

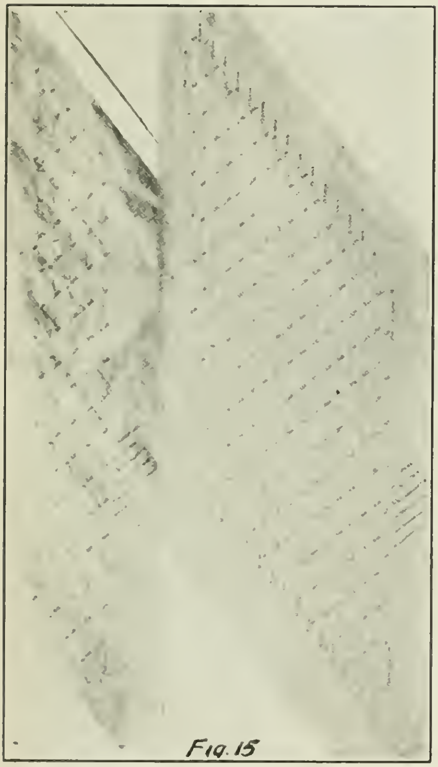

FIG. 15.—CHATTOCK CALIBRATION CURVE.

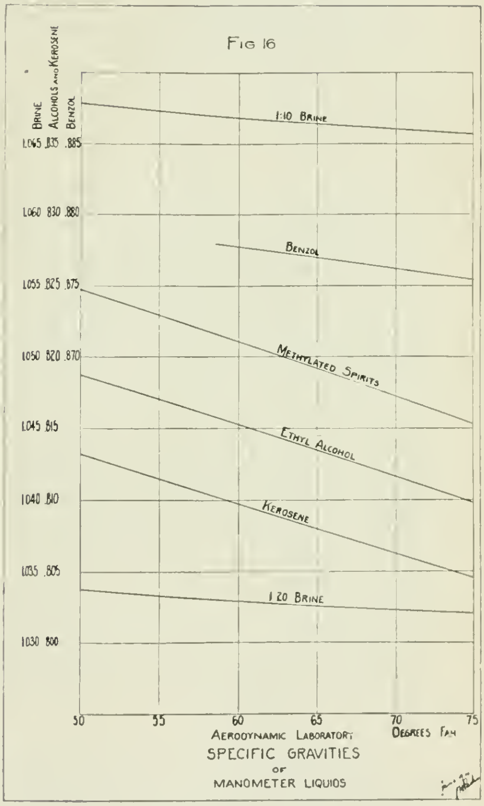

FIG. 16.—SPECIFIC GRAVITIES OF MANOMETER LIQUIDS.

Aeronautical Research Paper No. 3.
Sept. 1920.

STUDIES WITH A FULL SCALE FOUR FOOT N.P.L. TYPE WIND CHANNEL OF THE PROPER FORM OF INTAKE TO ELIMINATE DISTURBANCES DUE TO ITS UNSYMMETRICAL POSITION AND THE PRESENCE OF LARGE OBSTRUCTIONS IN THE BUILDING.

J. H. Parkin, and H. C. Crane

Introduction.

Wind channels of the N.P.L. type secure a large degree of freedom from pulsations with uniformity of velocity over the working cross-section of the channel, provided the channel is placed symmetrically in a sufficiently large building which contains no bulky objects. The investigation resulting in the N.P.L. design (see R. & M. 67, 1912-13, p. 48) showed that a wind channel should be housed in a building to protect it from external winds which have an appreciable effect on the steadiness of flow and that the size of this building is important, the larger the building relatively to the channel the smaller the pulsations, the steadiest flow being secured in an "infinite" building. With a channel as finally designed it was found possible to reduce the size of the building to 6×6×15 diameters without seriously increasing the fluctuations. It was further shown that the presence of other apparatus in the building had a serious detrimental effect on the steadiness, the effect for a given object varying inversely with the size of the building.

Toronto Installation.

The wind channel at Toronto, of the standard 4 ft. N.P.L. type, is installed in the Lower Hydraulic Laboratory which is 40′×113′×17′ 6″ high (15′ 6″ from floor to under side of deepest ceiling beams). This is well above the limiting size of building for this channel, which requires a 24′×24′×60′ building. The laboratory, however, contains a large amount of apparatus and bulky equipment such as engines, pumps and tanks, while a line of columns down the centre necessitated placing the wind channel off centre. It was realized that the presence of this other equipment and the eccentric position would give trouble but it was felt that by means of a suitable form of intake the effects could be largely nullified.

The channel was installed in the laboratory as close to the centre line as possible, the side of the distributor touches a column, with its centre line 9′ 6″ above the floor in order to secure headroom under the distributor to permit of other work being carried on in the laboratory. The channel is thus close to the south side of the laboratory and very close to the ceiling. The latter is supported on deep I beams perpendicular to the axis of the channel.

The likely causes of unsteadiness include a row of six tanks, each 4′×3′×4′ 6″ high, extending across the laboratory under the channel 10′ from the bellmouth. These tanks with a 24″I beam supporting the ceiling immediately above them, together with a column, piping, etc., reduce the cross-sectional area of the room 20.7%, leaving a net area for the return flow of the air of 559 sq. ft. (see Plate 1).

There is an opening 5′ 6″×27′ 0″ in the ceiling between the Lower and Upper Hydraulic Laboratories, 4′ back of the bellmouth, a steam engine roughly 8′×4′×6′ high situated 10′ in front of and 4′ south of the bellmouth, a 6″ pipe running parallel to the channel 9″ north of the centre line and 12″ above the upper edge of the bellmouth, and other small tanks along the south wall. Besides the well opening in the ceiling there are three doorways, two in the south and one in the west wall of the laboratory.

As a result of all these undesirable features the flow of air in the room was very much disturbed, as was shown by a smoke investigation (see Plate 2), and the flow of air in the channel was characterized by marked pulsations and an uneven distribution of velocity over the working section.

Object of Research.

An investigation was, therefore, undertaken to determine the best form of intake to reduce the pulsations and secure a uniform distribution of velocity over the working cross-section.

Summary of Results.

The studies have resulted in the development of a very satisfactory intake. It reduces the pulsations to a degree permitting very precise work, and secures a velocity distribution uniform within 1% either way over the central 30″ square which is equal to those published for the M.I.T. and N.P.L. The effects of open doors to other parts of the building are practically nullified. The intake is symmetrical in form, independent of eccentricity and objects in the room, simple in construction and readily applied to standard N.P.L. type channels.

Method of Investigation.

Velocity traverses of the channel at the working section taking readings every 6″ and in some cases every 3″ were made with various types and arrangements of intakes. For this purpose a reference Pitot-Static Tube of the standard N.P.L. type was mounted in the chuck of the balance and connected to a Chattock micromanometer. This tube was employed to hold the velocity constant while a second tube was mounted on a standard so that it could be moved, from without the channel, parallel to itself in vertical paths across the channel. Adjustments for position across the channel had to be made from within. The traversing Pitot-Static tube had the Pitot opening $9\frac{5}{8}''$ ahead of the shank and the static openings $5\frac{3}{4}''$, the dimensions otherwise being standard. The latter tube was connected to a slanting alcohol manometer of the Krell type, previously calibrated against a Chattock gauge and fitted with scales reading in feet per second. A side plate connected to a second Chattock gauge was also used to indicate the pressure in the channel at the working section, and later as the reference velocity indicator.

In work of this kind where fluctuating velocities are encountered and simultaneous readings of two gauges have to be made, considerable errors are likely unless care is taken that the readings are simultaneous. The method frequently employed of using two observers, each reading one gauge on signal, was abandoned in favour of one observer reading both gauges. For this purpose the microscope of the reference Chattock gauge was arranged directly over the tube of the slanting gauge and at about the position of the meniscus in the latter. Observing the Chattock, and properly regulating the speed by means of rheostats, when the bubble of the Chattock was on zero the observer had only to drop his eyes to the alcohol gauge immediately below to read the speed at the position of the traversing Pitot tube. In practice this method was found to involve less time lag and error than the two observer method.

The relative steadiness of the channel with different intakes was roughly estimated by means of the movement of the bubble in the Chattock gauge. To facilitate this a scale was mounted in the microscope in order to measure the excursions of the bubble.

In some tests long fine silk threads were used to determine the directions of flow inside the intakes and bellmouth, and smoke was also employed to investigate the air movements in the room in the neighbourhood of the entrance.

Final traverses of the channel were carefully checked by two independent observers.

The velocity traverses were then plotted to show the velocity gradient for each vertical path and from these gradients a velocity profile was prepared showing the distribution of velocity over the whole cross-section in terms of the average velocity in the centre 30″ square. Owing to the fact that the traverses were taken on the 3″ lines (3″, 9″, 15″, etc., from the side) the average velocity used is really for a 36″ square since the boundary readings of the 30″ square were used.

On the profiles have been plotted lines of equal velocity +5, +2, +1, +½, −½, −1 and −2 per cent. from the average. Within the central 30″ square areas within ½% either way are left plain, from ½ to 1% high the hatching is in one direction, while from ½ to 1% low the hatching is in the opposite direction; cross hatching indicates more than 1% variation from the average. The relative values of different arrangements may thus be seen at a glance.

The velocity profiles proved very useful in searching for the causes of disturbances and deciding on the modifications necessary in the intake to correct them.

Discussion of the Intakes and Results.

The intake arrangements tested were of many different types and variations of each type (see Plate 3) as the traverses suggested alterations. They may conveniently be arranged and discussed in different series or groups, as follows, each having particular characteristics.

Series 1. Usual bellmouth, with no intake.
Series 2. Baffles perpendicular to the channel axis.
Series 3. Baffles parallel to the channel axis.
Series 4. Blanking off parts of the distributor.
Series 5. Wire screen.
Series 6. Latticed intakes.
  (*a*) Slots perpendicular to the axis.
  (*b*) Portions blanked off.
  (*c*) Bevelled corners.
  (*d*) Checkerwork (portions of the distributor).
  (e_1) Slots parallel to the axis.
  (e_2) Slats parallel to the axis.
  (e_3) Blades parallel to the axis.
  (*f*) Shortening the intake.
  (*g*) Radial blades on the front.
  (*h*) Radial and parallel blades.
  (*h*) Tests of the final form of intake.

Series 1.—*Usual Bellmouth, without intake of any kind.*

Preliminary traverses with no intake other than the bellmouth indicated the presence of a region of high velocities on the south side of the channel and of low and very unsteady velocities over the north side (see Plate 8, Traverse 19). A thread study showed that this was caused by the large flow of air back along the relatively unobstructed north side of the room, entering the bellmouth and sweeping across the channel from north to south before entering the honeycomb, thus piling up on the south side and giving rise to the high velocities on that side.

Series 2.—Solid Baffles Perpendicular to the Channel Axis.

Solid plates baffles projecting out from the bellmouth perpendicular to the channel axis (Plate 3, Series 2, *a*, *b* and *c*) naturally suggested themselves, to obstruct the air flow in the room along the north side and below the channel and in this way to equalize to some extent the flow on all four sides. Different sizes and positions of these were tried and, while comparatively good velocity distribution was obtained by some of them, the unsteadiness was made much worse.

Series 3.—Solid Baffles Parallel to the Channel Axis.

These baffles were similar to the last series, but were placed parallel to the channel axis (Plate 3, Series 3, *a*, *b* and *c*) in an attempt to overcome the unsteadiness found with baffles in the perpendicular position. The pulsations were found to be equally bad and the traverses poorer.

Series 4.—Blanking off Parts of the Distributor.

The effect of blanking off portions of the north wall of the distributor was tried in an effort to reduce the flow back along the north side of the room and increase that on the south, thus equalizing the flow from all sides into the bellmouth. From the standpoint of velocity distribution the results were very poor.

Series 5.—Wire Screen (Plate 3, Series 5).

The effect of the distributor in an N.P.L. type channel in breaking up the violent propeller slipstream and returning it to the room without serious disturbance suggested the trying out of screens in the channel entrance. A screen of "expanded metal lath" was tried in different positions. The pulsations were very considerably reduced. The difference in steadiness between the two flows, with and without the screen, could easily be detected when the channel was entered to change the setting of the Pitot tube. With the screen the flow felt smooth and even while without the screen it was bumpy and jerky. The effect on velocity distribution was not good.

An interesting observation in connection with this series was the effect of permitting the screen to sag into the channel under the drag of the air, or of drawing it outward with wires. A sag of only 4″ downstream, as compared with an equal amount upstream, had a marked effect on the velocity distribution and on the pressure drop through the screen, resulting in an 8% greater reading on the side plate. The distribution of velocity was very sensitive to the position of the screen between bellmouth and honeycomb.

Series 6.—Latticed Intakes.

The smoothness of the air flow with the screen in place seemed to foreshadow good results by breaking up the air stream at the entrance. To accomplish this it was decided to build a latticed box or distributor in front of the bellmouth. It was constructed of 7/8″×7/8″ stock, of the same cross-sectional size as the bellmouth and projected 4′ in front of it (Plate 3, Series 6*a*). The slats were spaced various distances apart. At first it was attempted to so grade the area of the slots as to correct the unevenness of velocity distribution in the channel. This was in some cases carried to the extent of completely blanking off portions of the intake sides in effect thus combining the latticed intake with the baffles of Series 3. Grading on the whole proved unsatisfactory.

The effects of various alterations in the latticed intake may best be studied by reference to the profiles plotted on Plates 4 to 8.

Plate 4.

Profile 41 was obtained when sections taken from the distributor were placed on the top, bottom and sides (Plate 3, Series 6*d*) in the expectation that the orifice action of the apertures, by breaking up the cross currents which were known to exist (see Plate 2) might result in a better velocity distribution. The results, as shown, were not good. In the light of later work it appears this may have been due to the proportions of the intake being too large.

An attempt to break up the cross currents at the top alone, where they were most serious, by placing the slats parallel to the axis (Plate 3, Series $6e_1$), resulted in profile 37.

Profile 38 (Series 6*b*) resulted from completely covering the top and bottom. Here the air rushed in from the sides to the centre causing a rectangular area of high velocities at the centre.

Blanking off 1 ft. adjacent to the channel on the north side produced poor results as shown in profile 34.

In tests 39 and 40 an attempt was made to approach the hemispherical intake by cutting off the corners (Plate 3, Series 6*c*). The difference in profiles is due to 40 having the north, top and bottom spacing reduced with respect to 39.

Plate 5.

This plate is arranged to illustrate the different effect which the reducing of the spacing on one side has in the case of intakes four and two feet long. Both profiles, No. 29 and No. 31, were obtained with a 4 ft. intake the only difference being that the free space between the slots on the north side was less in No. 31 than in No. 29. The effect of this was to reduce the velocity on the south, opposite, side in the channel.

With the intake shortened to 2 ft. the effect of reducing the spacing was the reverse, namely, to reduce the velocity on the adjacent side. This is illustrated by comparing No 54 with No 51 and No 53 with No. 52. In each case the right hand profile shows the effect of reducing the spacing on the south (left side), which effect was to increase the velocity on the opposite side. Note the effect of changing from type e_3 to type h_1, that is the addition of the guide vanes to the front (see Plate 3) This accounts for the change in profile in the transition from No. 52 to No. 51 and No. 53 to No. 54.

Plate 6.

The persistence in all the traverses of the low velocities around the side led to the conclusion that the intake was too long thus permitting too great a flow of air in the sides in proportion to the front resulting in a piling up of the air in the centre. A series of tests was, therefore, run with the intake shortened various amounts. Plate 6 shows the effect of this alteration. Profiles No. 36 and No. 44 are for an intake 4 ft. long, No. 45 and No. 46 for 3 ft. and No. 47 and No. 48 for 2 ft. As the length is decreased the region of high velocities at the centre is changed to one of low velocities in the centre. The point of transition was found to be at a length of about 2 ft. 6 in., which length was, therefore, adopted in the final intake.

The effect of placing blades on the sides, transition from $6a$ to $6e_3$ (see Plate 3) was to change profiles No. 36, No. 45 and No. 47, on the left, to No. 44, No. 46 and No. 48, on the right. Although the blades have seemingly other influences probably the chief result of their use is the breaking up of the high and low velocity regions into smaller ones. This agrees with the change effected by the addition of star guide vanes to the front (see 51, 52; 54, 53, Plate 5).

Plate 7.

Profile No. 57 was obtained with a 2 ft. type $6h_2$ intake and shows the persistence of the low region in the centre. This was wiped out on increasing the length to 2 ft. 6 in. (see Profile No. 58, Plate 8).

The effects of various slight modifications in the intake as finally adopted, type $6h_2$, were investigated. Widening the corner blades 6″ had practically no effect. Fairing or streamlining the backs of the bellmouth, as is the present practice with N.P.L. type channels, was detrimental to the velocity distribution as shown by profile No. 64. The streamlining may have been advantageous with an intake of greater length, but as satisfactory distribution of velocity had been secured, this was not tried. As first constructed the radial blades on the front were not joined at the centre, a 12″ diameter space being left free. Traverses showed a region of unsymmetrical velocity in the centre which was largely corrected by carrying the vertical and horizontal blades to the centre.

The effect of opening doors from the laboratory to other parts of the building was also determined. Opening the door between the Hydraulic and Thermodynamic Laboratories had little effect (see No. 62). This door is slightly in front of the entrance to the channel, about 12 ft. south and on the same level as the channel. When all three doors were opened a small region of low velocities at the centre resulted (see No. 63). The effect on the steadiness of the channel was not noticeable.

Plate 8.

This plate is self-explanatory. The improvement in velocity distribution effected by the intake is shown (see Profiles No. 58 and No. 19). The final intake is seen to reduce the velocity variation from the mean over the working section to about $\frac{1}{2}\%$.

From the results published by the M.I.T. (see *Smith. Misc. Col.*, vol. 62, No. 4, p. 38) and the N.P.L. (see *R. & M.* No. 68, 1912-13, Fig. 5) profiles have been plotted for these channels for purposes of comparison. These profiles may, however, be somewhat in error as the data was rather incomplete.

Traverses were run with mean velocities of 33, 40 and 53 ft. per sec. to determine if the performance of the intake was affected by a change in velocity, but the profiles plotted do not show any such effect.

Description of the Toronto Intake (see Plates 9 and 10).

The intake, as finally adopted, is in the form of a box, equal in cross-section to the outside of the bellmouth, 6 ft. 5 ins. square, and projecting 30 ins. from the face of the latter. The sides and front are made up of latticed gratings $\frac{7}{8} \times \frac{7}{8}$ dressed stock, spaced one diameter apart in order to secure a directive effect on the air. The gratings are of the simpler slot construction as used in the later N.P.L. distributors instead of the checkerwork of the first channel. The five gratings are screwed together and to the bellmouth to form a latticed box. The weight is carried on two legs of 1″ W.I. pipe.

Projecting from the corners and the mid points of the sides of the intake are eight guides or blades parallel to the channel axis. They are of $\frac{1}{4}$ in. thick, three ply laminated wood supported on light iron brackets. The corner blades are 18 ins. and the side ones 12 ins. wide and 30 ins. long. The eight guides are continued across the front of the intake in a star formation.

The slot area in the intake for the flow of air is 47.9 sq. ft. so that the velocity through the slots is 13.76 ft. per sec. or only 33% of the channel velocity.

Advantages of the Toronto Intake.

The investigation shows that the intake secures a satisfactory air stream under conditions very adverse from an aerodynamic point of view as at Toronto where the building contained many kinds of disturbing elements. Non-uniform velocity distribution due to eccentric position, bulky objects, open doors, etc., is corrected, and pulsations due to these causes reduced to a degree permitting precise work. While these results were secured at Toronto with a free area for the return of the air approximating that laid down in the N.P.L. report (6×6 diameters) it is believed that results practically as good could be secured in a smaller building.

The velocity distribution at 40 ft. per sec. over the central 30 ins. square is uniform within 1% either way (practically within ½% either way) which compares very favourably with the distribution secured under ideal building conditions. The velocity distribution is practically unaffected by changes in speed.

The low pressure region in the centre of the channel cross-section seems to be present in all channels of this type (see profiles for N.P.L., M.I.T. and Toronto No. 58) and is considered to be due to the comparative ineffectiveness of the central portion of the propeller (*R. & M.* 564, p. 8).

The pulsations are reduced to such an extent that the effect on a Chattock gauge is hardly noticeable and measurements on the balance easily made. Tests have been run with a standard RAF6A aerofoil without oil in any of the balance dashpots.

The intake is of simple and inexpensive construction, relatively small and easily applied to the standard N.P.L. type channel. It is symmetrical in form and independent of eccentricity and other disturbing elements.

The advantages above enumerated are gained at the expense of a very small increase in friction loss and consequent power consumption. The loss in the intake, including bellmouth, is only 0.0077 ins. of water as compared with 0.089 ins. in the honeycomb. The power consumed in friction in the intake is only 0.047 H.P. and up to the working section 0.71 H.P. The total power to bring the air to the model is 2.92 H.P. at 40 ft. per sec.

Since the development of the Toronto intake the 1918-19 Report of the British Advisory Committee for Aeronautics has been received in which it is stated (p. 27) that "the introduction of a cellular wall or diaphragm across the room materially increases the steadiness and enabled the length of the distributor end and, consequently, the length of the building in which the channel was housed, to be considerably reduced." In effect this is what has been done in the Toronto intake except that the wall or diaphragm has been disposed around the entrance to the channel instead of across the room. It is, therefore, probable that the same advantages would accrue from the use of the intake as from that of the wall, namely, possible shortening of the distributor and reduction in size of building or room where housed. Further, the intake, while specially developed to overcome adverse building conditions, might be used with advantage in the case of ideal building conditions because of the greater precision made possible by the reduction in pulsations.

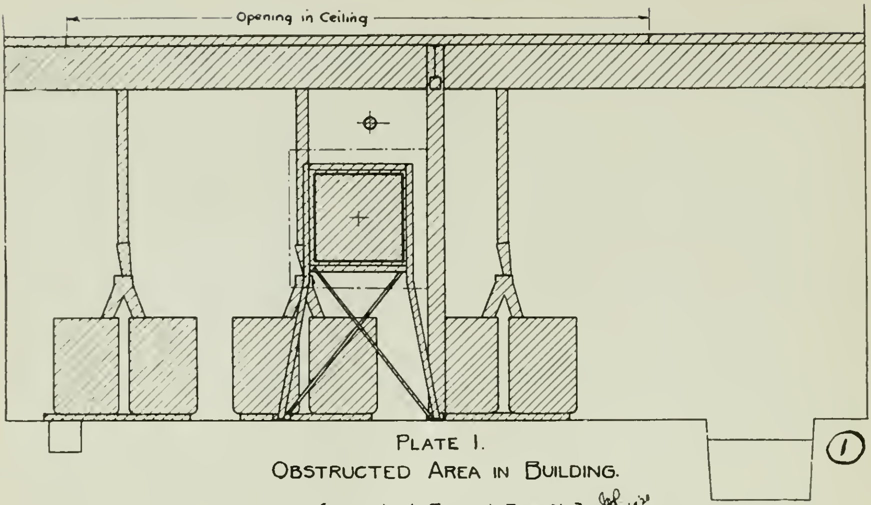

PLATE 1.
OBSTRUCTED AREA IN BUILDING.

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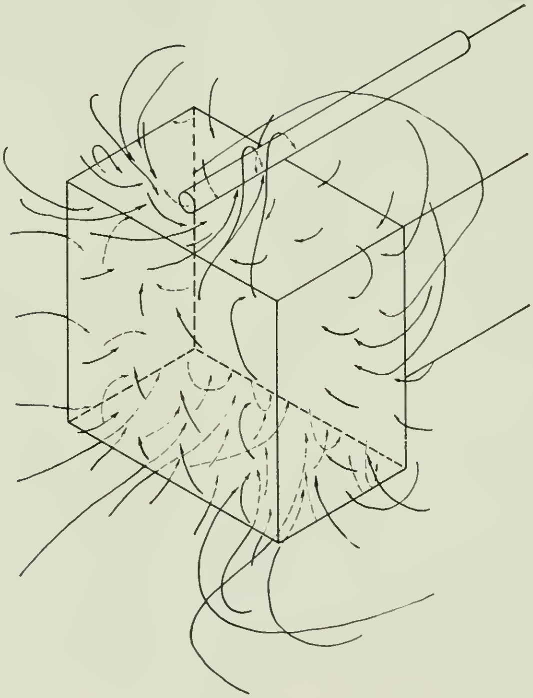

PLATE 2.
SMOKE INVESTIGATION OF AIR CURRENTS ABOUT INTAKE.

VIEW FROM TOP-NORTHEAST CORNER

Aeronautical Research Paper No 3
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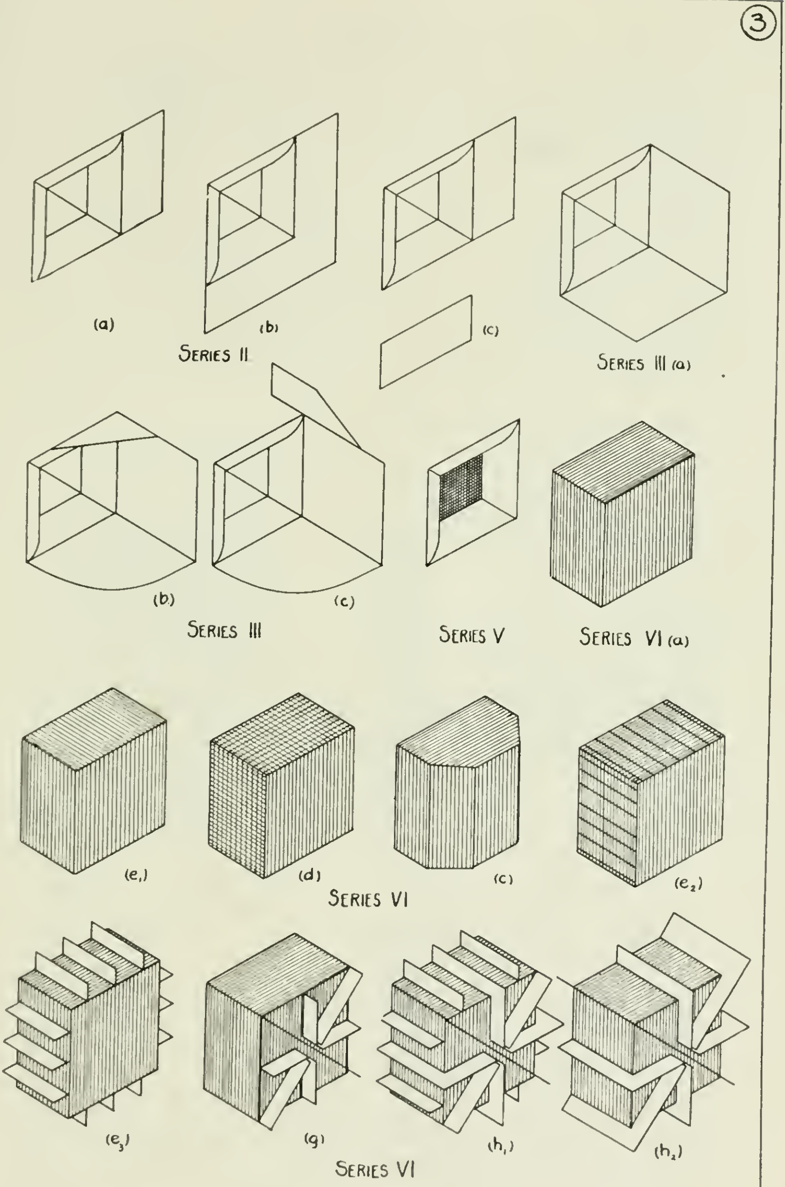

PLATE 3.
INTAKE ARRANGEMENTS
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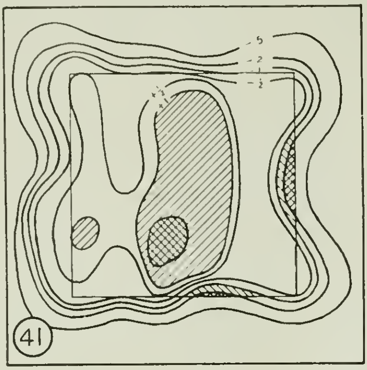
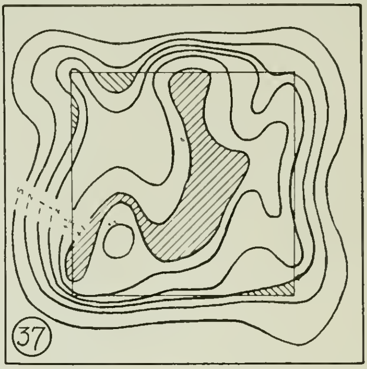
④

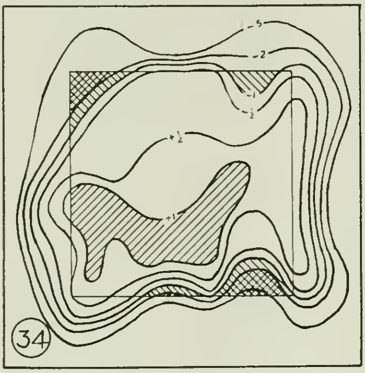
PLATE 4



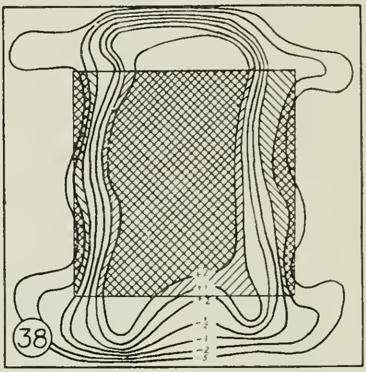
㊶ SECTIONS FROM DISTRIBUTOR SERIES VI d



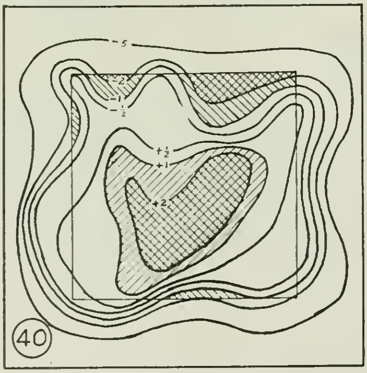
㊲ SLOTS PARALLEL TO AXIS SERIES VI e_1



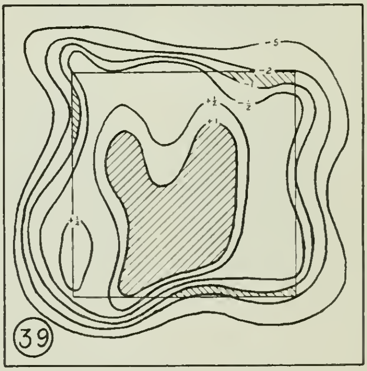
㉞ 1FT OF N SIDE BLANKED OFF SERIES VI b



㊳ TOP AND BOTTOM BLANKED OFF SERIES VI b



㊵ BEVELLED CORNERS SERIES VI c



㊴ BEVELLED CORNERS SERIES VI c

㊴ Spacing reduced on top, bottom and N side.

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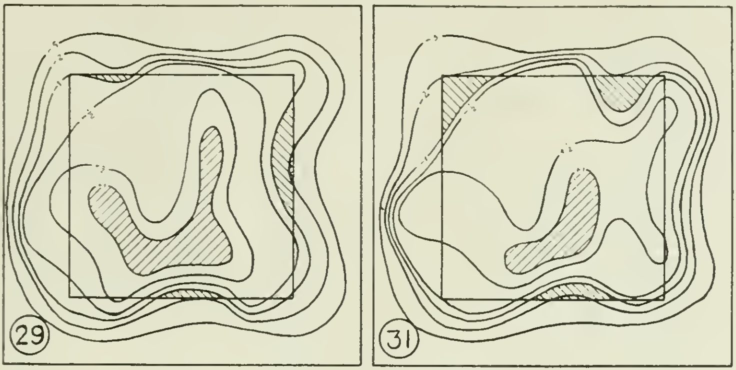
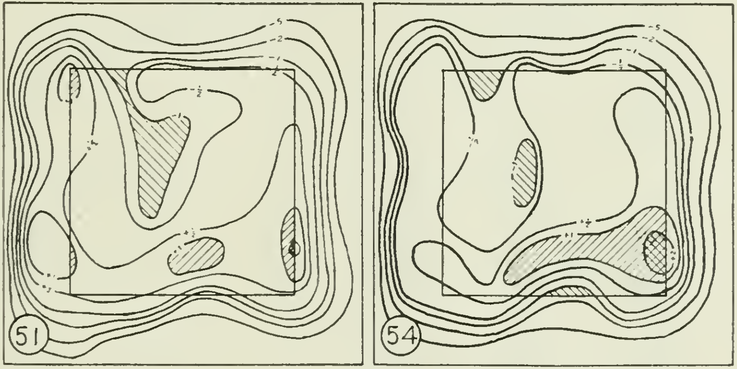
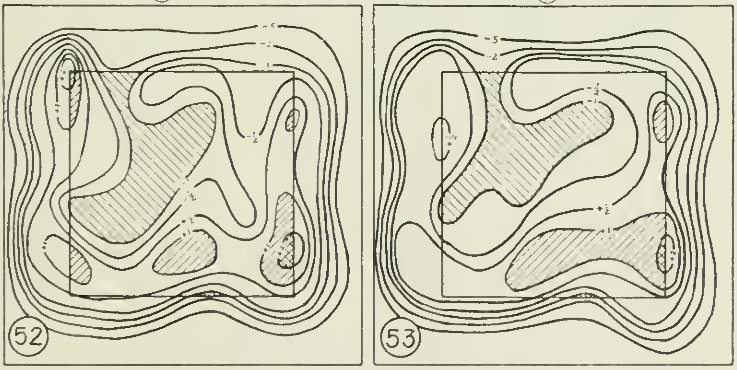
PLATE 5 ⑤



4 FT LATTICED INTAKE SERIES VI a
㉛ Spacing on N side reduced compared with ㉙



2 FT LATTICED INTAKE, BLADES ON FRONT AND SIDES SERIES VI h$_1$
(54) Spacing reduced on S side as compared with (51)



2 FT LATTICED INTAKE, BLADES ON SIDES ONLY SERIES VI e$_3$ and f
(53) Spacing reduced on S side as compared with (52)

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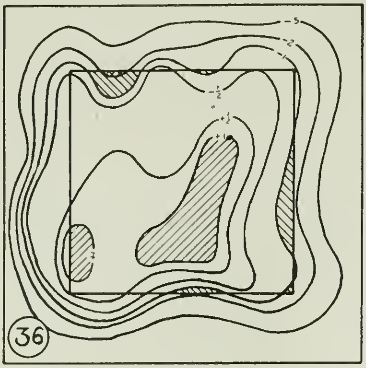
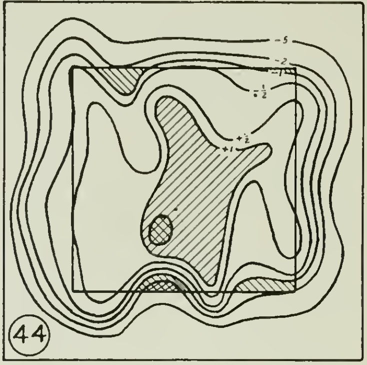
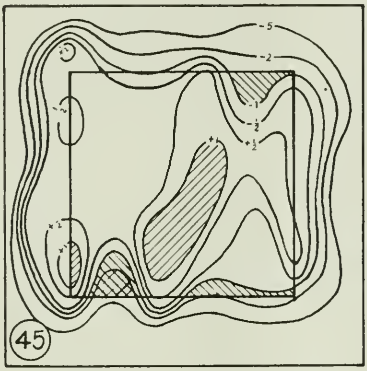
PLATE 6. ⑥



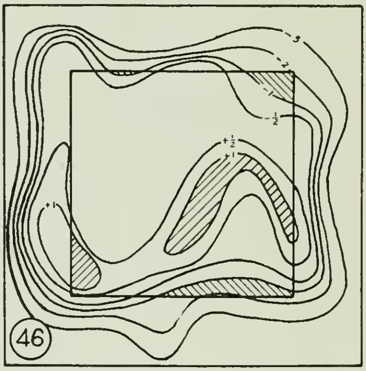
4FT INTAKE, NO BLADES SERIES VI a



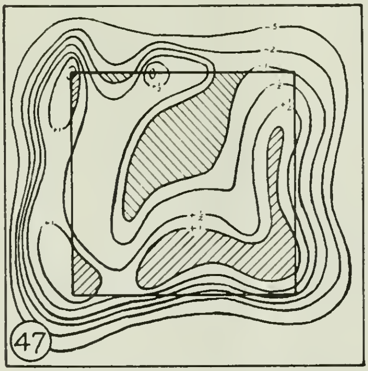
4FT INTAKE, BLADES ON SIDES SERIES VI e_3



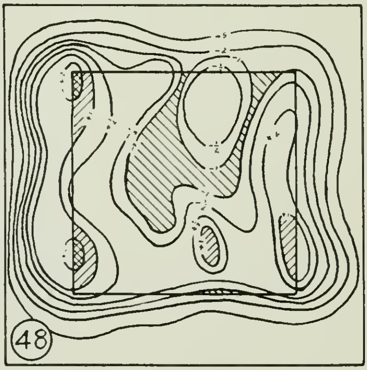
3FT INTAKE NO BLADES SERIES VI f



3FT INTAKE, BLADES ON SIDES SERIES VI e_3&f



2FT INTAKE, NO BLADES SERIES VI f



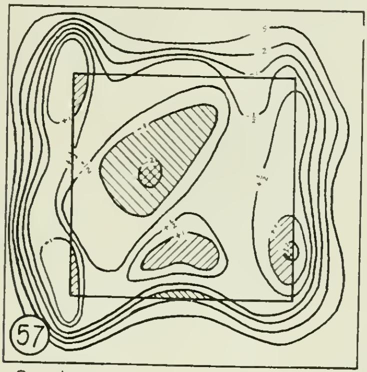
2FT INTAKE BLADES ON SIDES SERIES VI e_3&f

In all except ㊱ the spacing of the slots was $1\frac{1}{4}''$ on N side and 1 diameter on the remaining sides. In ㊱ the spacing was 1 diameter throughout.

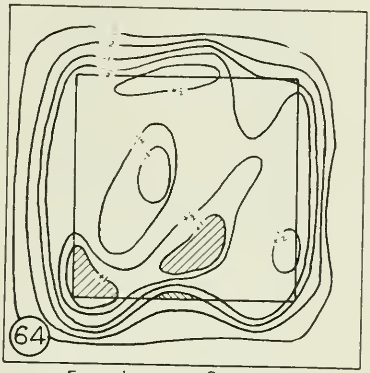
Aeronautic Research Paper No 3
UNIVERSITY OF TORONTO

⑦

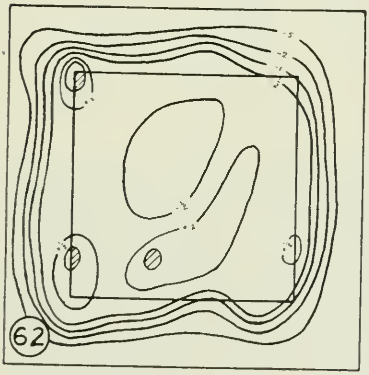
PLATE 7



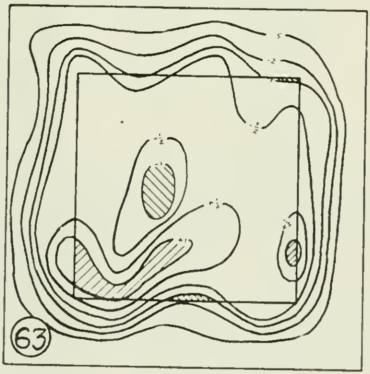
(57) 2FT INTAKE. ALL BLADES SERIES VI f & h_1



(64) FINAL INTAKE SERIES VI h_2
Outside of bellmouth streamlined



(62) FINAL INTAKE SERIES VI h_2
South door open



(63) FINAL INTAKE SERIES VI h_2
All doors open.

Aeronautic Research Paper No. 3
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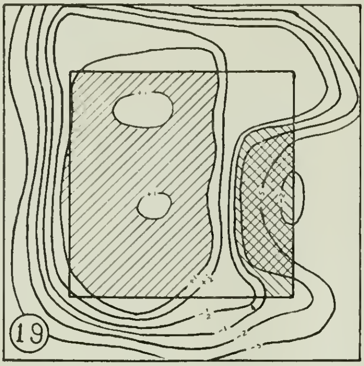
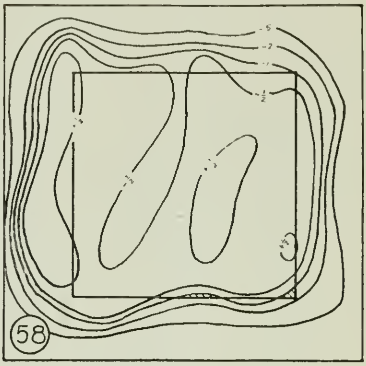
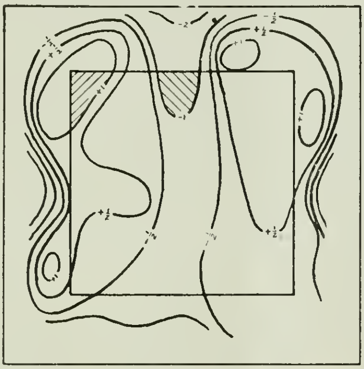
PLATE 8



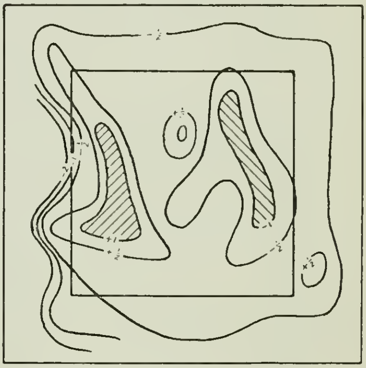
NO INTAKE SERIES I



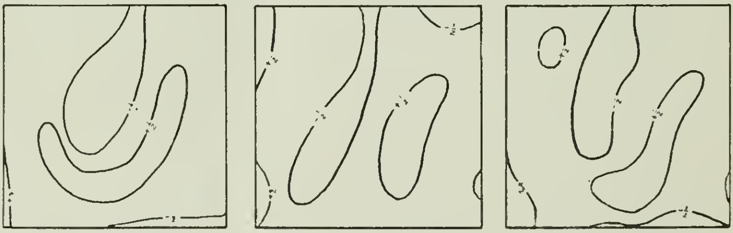
FINAL INTAKE SERIES VI h_2



M·I·T·



N.P. L.



VEL 33 FPS VEL 40 FPS VEL. 53 FPS

FINAL INTAKE - SPEED EFFECT

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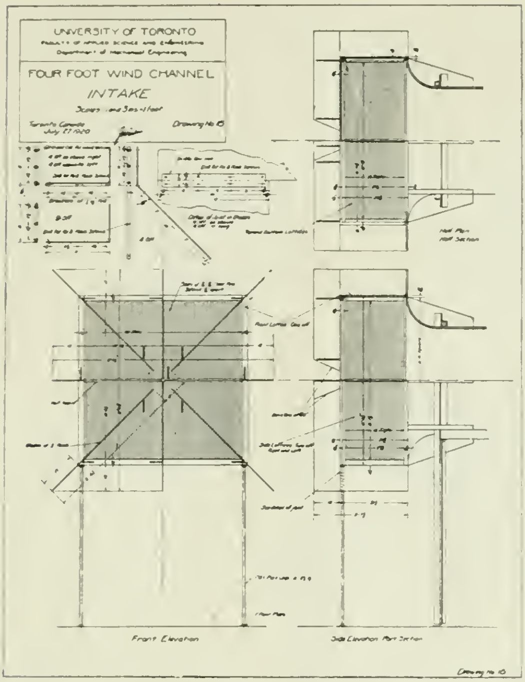

FIG. 9.—TORONTO INTAKE.

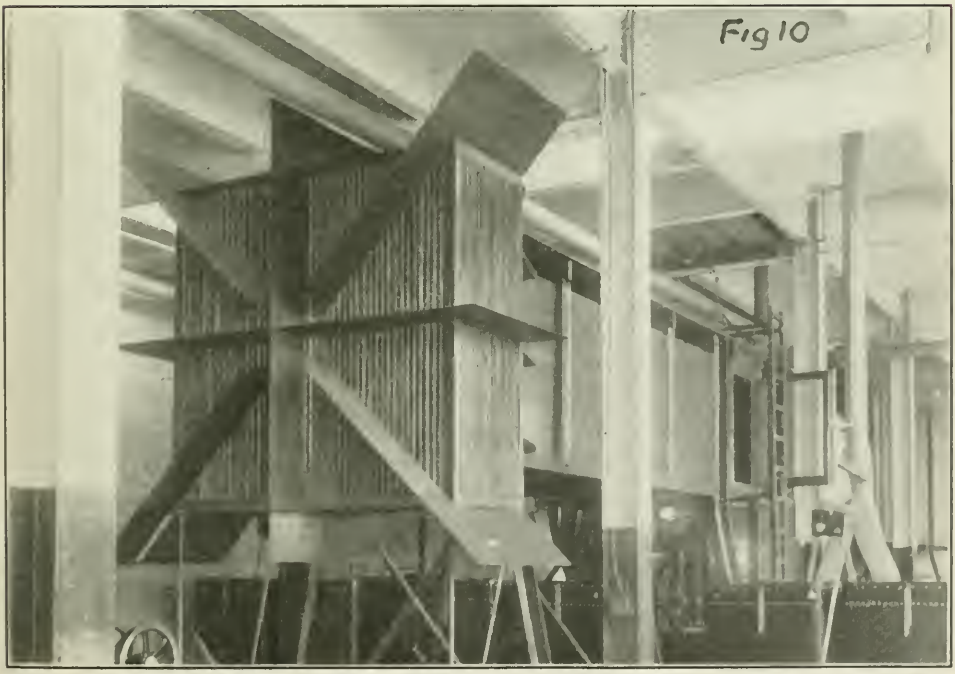

FIG. 10.—INTAKE ON TORONTO CHANNEL.

Aeronautical Research Paper No. 4.
Dec. 1920.

WING TIP RESEARCH*

PART I.—EFFECT OF OUTLINE AND WASHOUT OF CAMBER ON THE AERODYNAMIC EFFICIENCY.

J. H. PARKIN, H. C. CRANE, AND S. L. GALBRAITH

SUMMARY.

This paper deals with the preliminary research in what is intended to be a comprehensive investigation of the aerodynamic properties of wing tips. The influence of the wing tip on the performance of the aerofoil, the relative aerodynamic efficiencies of different tip outlines and the variation in the efficiencies as each tip is progressively varied from the square tip, were investigated, as well as the effect of progressively varying the washout of upper and lower camber.

Five series of tips were tested, namely, Elliptical, Straight Raked, Curved Raked, Washout of Upper Camber and Washout of Lower Camber, each consisting of three tips based on the square tip, in all twenty-three tips. The RAF6A aerofoil was used with an average aspect ratio of 6. The models were of aluminum. Tests were made at a wind speed of 40 ft. per sec.

The investigation indicates that the square tips are aerodynamically inefficient; all the tips tested were effective in increasing the L/D between $-4°$ and the critical angle (14°), but the general form of the characteristic curves is little affected.

The most effective type of tip tested was the positive straight rake tip which for 45° rake showed 17% gain in maximum L/D and 10% for 15° rake. The curved rake is less effective than the straight rake.

Camber washout should be effected in the lower, less effective surface of the aerofoil and tips should be maintained of good section throughout.

The results confirm mathematical theory which indicates that the tip vortex drag disappears at the angle of zero lift.

The tip embodying all the recommendations resulting from this research is one with a positive raked plan form, washout of under camber and washout of incidence to that of no lift for normal flight angle.

**Read before the Mechanical Section, Engineering Institute of Canada*

OBJECT AND SCOPE OF RESEARCH.

Aeroplanes are constructed at the present time with wing tips of many different forms, and there is considerable difference of opinion regarding the influence of tip form on the aerodynamic properties of the aerofoil as well as the relative merits of the different shapes used. While the merit of a wing tip is judged with respect to several factors in addition to the aerodynamic efficiency there is a lack of correlated data regarding even this factor. A certain amount of experimental data is available (see list of references appended) but it is more or less disconnected, and in some cases contradictory, having been determined, generally for odd types or a single series, under different conditions and in various laboratories. Direct comparison of types is thus difficult.

The present research has, therefore, been undertaken with the object of supplying information regarding the influence of tip form and the relative efficiencies of common wing tip forms under similar conditions and further to indicate the way in which the efficiencies vary as the tips of each type are progressively varied in form.

The range of tips tested while small is sufficiently comprehensive to embrace most types in present day use and it is hoped the results may serve as a guide in the design or selection of wing tips.

SOURCES OF LOSS IN WING TIPS.

The sources of loss in wing tips, in addition to the skin friction loss which is largely independent of the particular form, are:

(*a*) *Eddy Losses* (Plate 1, Fig. *a*). In square or poorly shaped tips the abrupt changes in contour disturb the air flow creating eddies and whirls with consequent loss. This loss is probably relatively small.

(*b*) *End Flow* (Plate 1, Figs. *b* and *c*). There is a flow of air around the tip from the under high pressure side to the upper low pressure region. This flow has been detected experimentally (R. & M. 197). The escape of air from under to upper side results in reduction in pressure on the under side and loss of suction on the upper side (Fig. *b*) and hence of lift, in addition to an increase in drag. Devices, such as vertical blades on the tips to prevent the end flow, are unsatisfactory from the stability standpoint.

(*c*) *Tip Vortices.* The vortex theory of sustentation developed by Lanchester is based on a cyclic element of flow about the aerofoil or a vortex surrounding the aerofoil (Plate 1, Figs. *g*, *h* and *i*) and trailing off to the rear at the tips. The presence of this vortex about the aerofoil has been indicated experimentally (R. & M. 197) and the trailing vortices have frequently been observed and more recently photographed, Plate 1, Fig. *f*. (N.A.C.A. Report 83, Part 2)). The tip vortices exert a drag upon the aerofoil.

It has been shown mathematically by de Bothezat (N.A.C.A. Report 28, p. 43) that the tip vortices disappear at zero lift and an expression deduced for the theoretical drag due to vortices indicates that the drag decreases as the aspect ratio increases and hence disappears for infinite span. It is further stated that when the aspect ratio is sufficiently large the influence of the tip form on the air resistance is negligible.

It would appear from this paper that tapering the wing tips in plan form, either with positive or negative rake, elliptical tips or some other means, should be effective in reducing the tip vortex drag through increasing, as far as the tip vortices are concerned, the "apparent" aspect ratio. Similarly washout of camber, resulting in reduction of lift, or washout of incidence, of tips to that corresponding to zero lift at normal angle of flight, should reduce drag due to this cause.

Birds' wings and aeroplanes of the Taube and arrow type probably gain considerable advantage from these modifications.

EXTENT OF PROBLEM.

A comprehensive investigation of the aerodynamic properties of wing tips would include studies of the effect of the following factors on the aerodynamic efficiency.

(*a*) Plan form.

(*b*) Plan form in conjunction with aspect ratio.

(*c*) Washout or grading of camber, upper, lower and together.

(*d*) Washout of incidence. Tests at N.P.L. (R. & M. 557, p. 4) and mathematical theory indicate the probability that washout of incidence may possess advantages.

(*e*) Plan form with different typical aerofoil sections, high speed, high lift, thick sections, etc. to determine whether the effect of a given change is the same for all sections.

(*f*) Plan form on biplane combinations with equal and unequal span of the two planes.

(*g*) Scale effect. It is believed that the relatively small aerodynamic advantage shown by some tips in model tests may be much greater in the full scale machine. This requires investigation.

In practice, in addition to the aerodynamic efficiency, there are certain other factors which exert an important influence on the selection of a wing tip. These factors, which frequently operate against each other, include stability, ailerons and control, and construction. Tips of good aerodynamic efficiency may have undesirable effects on the stability or affect the lightness of control (R. & M. 550, p. 4) and the tips are, therefore, of importance in connection with lateral control. From the constructional point of view tips of good aerodynamic characteristics frequently present costly and troublesome structural difficulties and render the fitting of the ailerons difficult.

An extension of the investigation is, therefore, desirable to study the tip forms of best efficiency from the standpoints of stability, ailerons and control.

Tip form is also of interest in connection with tail planes and longitudinal stability and control (R. & M. 532).

Although the conditions of relative air flow are radically different in the propeller than those obtaining in the main planes the tip form has an important bearing on propeller blade action.[1] It is doubtful, however, due to the different conditions, whether the effect of changes in form are the same in propeller blades as in aerofoils. This point requires investigation.

It will be evident, therefore, that the complete study of the aerodynamic properties of wing tips will be very extensive in scope. The present investigation is of a preliminary nature only dealing with the effect of plan form variation and to a very limited extent with washout of camber. It is hoped to proceed with the investigation of the other points from time to time as opportunity offers.

TIP FORMS TESTED.

A difficulty in a research of this kind is the selection of representative types to be studied, such that the results obtained may be broadly applicable. An examination of various machines indicated that the tips may be broadly divided into the following principal types:

(*a*) Square tips.
(*b*) Square tips with slightly rounded corners.
(*c*) Elliptical tips.
(*d*) Straight raked tips, both positive and negative.
(*e*) Tips of irregular curvature in which the extreme tip is either forward of or in the rear of midchord.

The basic tips selected for this research were, therefore, of simple geometrical outlines approximating the above, namely:

(*a*) Elliptical.
(*b*) Straight raked, positive and negative.
(*c*) Curved raked, positive and negative.

The square tips formed the basis of comparison. Slightly rounded square tips were not tested since they differ little from the square tips. The curved raked tips are raked by means of a circular arc of radius equal to some multiple of the chord with centre on leading or trailing edge, and was chosen as most closely approximating the more irregular curved tips.

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[1] As an illustration of this the case is noted of a propeller designed for certain conditions, with square tipped blades, which on trial rotated at too low a speed. In an attempt to increase the speed the propeller blade tips were rounded. This alteration instead of having the desired effect resulted in a lower speed through increasing the lift.

(*a*) *Elliptical Series* (Plate 2, Tips 14-16). There are two ways in which the elliptical tip outline may be varied, one by variation of the length of the axis in the direction of the span (transverse axis) the other by distorting the ellipse by moving the transverse axis forward or backward of mid chord. The former method of variation was used in tests at the N.P.L. (R. & M. 152) and was adopted in this research, with the difference that average aspect ratio was here maintained constant while in the British tests the extreme span was constant. Two tips, 14 and 16, correspond to the N.P.L. tips 2 and 3, the third, 15, semicircular, being an intermediate form. The transverse semi-axes were $\frac{3}{8}$, $\frac{1}{2}$ and $\frac{3}{4}$ of the wing chord or 1/16, 1/12 and $\frac{1}{8}$ of the span.

(*b*) *Raked Series* (Plate 2, Tips 2-7). Both positive and negative rakes of 15°, 30° and 45° were tested.

(*c*) *Curved Raked Series* (Plate 2, Tips 8-13). Tips were tested with radii of rake equal to chord, twice chord and four times chord, and both positive and negative rake.

Camber Washout. The camber of the aerofoil section is washed out or faired to a greater or less extent in all the tips tested except the square tips. This results from maintaining the RAF6A section throughout. The washout here referred to is one involving a change in the aerofoil section independent of the outline. Two series were tested:

(*d*) *Washout of Upper Camber* (Plate 2, Tips 17-19). With the square tipped outline, the thickness or camber of the section at the extreme tip was successively reduced to 0.7, 0.4 and 0.1 of the original camber, all ordinates being reduced in the same proportion. Lines joining points on the profile of the section at the inner end of the tip with the corresponding points on the reduced section at the extreme tip were thus straight.

(*e*) *Washout of Under Camber* (Plate 2, Tips 20-22). Since the under face of the RAF6A section is flat the thickness of the extreme tip section was reduced in the same was as above, but from the bottom, the flat under side being sloped upward at different angles for the different degrees of washout. This involved a change in tip outline as well as section.

(*f*) *Streamline Body on Tip* (Plate 2, Tips 23). The objection from the standpoint of stability to the use of vertical flat plates or blades on the wing tips to reduce end flow does not apply to a streamline body, although the latter is less effective in preventing end flow. The bodies used were of a form generated by the rotation of the RAF6 section about its chord. The bodies were made of brass and secured by screws to the square wing tips.

The different tips were made detachable from the main aerofoil. By this arrangement the tips only were changed for successive tests, all other conditions remaining constant, and imperfections of contour or surface, deflection of balance or spindle and spindle effects were present equally in all the tests. The relative efficiencies of the tips were thus accurately indicated. This has been further facilitated by designing the tips in accordance with the following principles:

1. Constant aspect ratio of 6 for the aerofoil with the tips. The aspect ratio is the average span divided by the chord, or the area divided by the square of the chord. Hence the total area of the aerofoil and tips was with two exceptions (Tips 10 and 13, Plate 2) the same throughout.

2. The tips were limited, with the same two exceptions, to an average length of 2″ that is 66% of the chord or 11% of the mean span, leaving the centre 14″ or 77.7% of mean span of uniform section. This principle was adopted to facilitate practical full scale construction through eliminating many special ribs.

3. All corners were rounded, those with a contained angle exceeding 90° having a radius 0.2 chord, and those with a contained angle less than 90° having a radius of 0.1 chord.

4. Sections of tips tested for form effect were everywhere maintained of RAF6A section parallel to the wind. Those tested for camber washout effect varied in section.

5. The square tip constituted the first or basis of each series made up of three formed tips (see Plate 2).

Construction of Models. The models were of cast aluminum alloy containing a small percentage of zinc to render the metal free cutting, thoroughly annealed to remove warping tendencies. Most of the material for the tips was machined to RAF6A section in the usual manner, by first rough shaping in a milling machine and afterwards scraping to template and polishing. Later a milling cutter was prepared (Plate 4) and used to mill the sections which were afterwards polished.

The sections were cast and finished in 19″ lengths, from which the various tips were cut. The tips were first shaped to the desired contour and then scraped down to proper section, which was tested by templates of different sizes (Plate 4).

The tips were held in place on the main aerofoil by two steel dowels in each end of the aerofoil (Plate 3), holes being accurately drilled for the dowels in the tips by means of a jig.

The aerofoil with square tips was of standard size for this channel, namely, 3″ chord by 18″ span.

SUPPORT OF MODEL.

The nature of the research precluded the use of the standard end spindle for the support of the model since this would have interfered with the phenomena being studied. A cranked spindle was therefore adopted, attached to the underface of the aerofoil, at mid span and 1″ from the leading edge (Plate 24). The point of attachment is thus at the thickest section of the aerofoil permitting a strong fastening (screw and dowel) and the arrangement involves least interference, for this type of spindle, with the air-flow, being on the under, low lift, side and away from the leading edge.

Further the angles of incidence can be accurately and conveniently set as with the ordinary end spindle mounting.

The disadvantages of the cranked spindle are its very great relative drag and torque, and possibly slight interference with the flow about the lower wing tip.

Careful determinations of the resistance of the cranked spindle, and also of the end spindle, were made and the method and results are given in Appendix I.

CONDITIONS OF TEST

Each pair of tips was tested for a range of incidences from −6° to +20° at a wind speed of 40 ft. per sec., in a 4 ft. N.P.L. type wind channel fitted with Toronto intake (Aero. Res. Paper No. 3). Readings of lift and drag only were made as the presence of the cranked spindle with its large torque coupled with the very small variation in the position of the centre of pressure with the different tips rendered the torque readings of little value. The models were accurately lined up in the channel by means of the Alignment Apparatus (Aero. Res. Paper No. 1) and speeds were measured by side plate and Chattock micromanometer. The same two observers made the readings for all the tips.

The conditions of the test were standard, namely, temperature 60°F, pressure 29.921 inches, air density 0.07635 lbs. per cu. ft. (0.002373 slugs) and gravity at Toronto 32.174 ft. per sec. per sec.

RESULTS.

The lift and drag coefficients have been expressed in the usual non-dimensional or absolute coefficients C_y and C_x in the equations:

$$L = C_y \rho A V^2$$
$$D = C_x \rho A V^2$$

The results have been given in the form of curves of C_y, C_x and L/D plotted on angle of incidence as base, each series being plotted on one sheet to permit of easy comparison of the different tips. To better exhibit the general trend of the effect due to progressively varying one type of tip curves of L/D have in some cases been plotted on the tip dimension as base (Plates 9, 14 and 17).

DISCUSSION OF RESULTS.

Raked Series.

Curve 5. Positive rake results in an increase of lift which is most marked between 2° and 12°, and for rakes of 30° and 45°, the increase being negligible for 15° rake and increasing with the angle of rake.

Curve 7. Negative rake results in slight and erratic increase in lift.

Curve 5. Positive rake reduces the drag up to 12-14° incidence by a more or less constant amount. The connection between reduction in drag and angle of rake is not apparent. The reduction is most marked for an angle of −2°, the angle of no lift. Above 14° the tips everywhere result in an increase of drag.

Curve 7. Negative rake effects a reduction in drag for all angles of incidence tested which increases with the angle of rake. The reductions are very marked between 6° and 16° of incidence.

Curves 5 *and* 7. A peculiarity of the drag curves of both positive and negative raked tips is the hump in the curves occurring at 0° incidence. The explanation of this is not clear.

Curve 6. The L/D or aerodynamic efficiency of the aerofoi is increased between −3° and 14° by positive raked tips. The increase generally follows the angle of rake, but is most marked between 0° and 15° rake. The increase in maximum L/D is appreciable being for 15° rake about 10%, for 30° rake 15% and for 45° rake about 17%.

Curve 8. The increasse in L/D due to negative rake are generally less than for positive rake up to the angle of maximum L/D, between 3 and 4°, but thereafter are much greater and do not become negative above the critical angle as in the case of positive rake. The gain in maximum L/D is 1%(?) for 15°, 9% for 30° and 11% for 45° rake.

Curve 9. Curves of L/D on angle of rake for this series show that up to 6° incidence the effects of positive rake and of negative rake are fairly well balanced, but from 6 to 14° the negative rake is more effective, and above 14° there is no advantage in shaped tips. The most marked increase in L/D occurs for the first 15° of rake either way.

These curves agree very closely with those obtained at the M.I.T.

CURVED RAKE SERIES.

Curve 10. Positive curved rake results in no gain in lift except for rake where radius =c which is about equivalent to 45° straight rake.

Curve 12. The lift is reduced slightly by the negative curved rake except for a rake of radius =c which results in a slight gain in lift.

Curve 10. The effects of positive curved rake on drag are similar to those for the straight rake, a reduction in drag up to 16° incidence most marked for negative incidence of −2°, the connection with the radius of curvature being indefinite.

Curve 12. Negative curved rake effects on drag are also similar to corresponding straight rake, a decrease in drag over the whole range of incidences, most marked between 8° and 16° and the decrease varying inversely with the radius of curvature.

Curves 10 *and* 12. For this series also the drag curves exhibit the peculiar rise at 0° incidence shown by the straight raked series and further a marked reduction at −2°, the angle of zero lift.

Curve 11. An increase in L/D results from the use of curved rake tips for incidences between −4° and 15° which varies inversely as the radius of curvature, being greatest or a radius equal to the chord. The gain in maximum efficiency for radius =4C is 3%, for radius =2C is 8% and for radius =C is 15%. The gain is, therefore, slightly less than with the straight raked tips tested.

Curve 13. Negative curved rake results in an increase in L/D for all angles above −3°. Below the angle of maximum L/D the advantage s less than for positive curved rake and the connection with the radius of curvature is uncertain. The gain in maximum L/D is less than with straight being 8%(?) for radius =4C, 6% for radius =2C and 12% for radius =C. Above maximum L/D the advantage is greater than for positive curved rake.

Curve 14. The curves of L/D on rake for this series show generally the same characteristics as the straight raked series. Up to an incidence of 4° effects of positive and negative rake are equal, between 6° and 16° the negative tips snow a slight advantage, while above 16° and below 0° incidence both rakes are ineffective.

ELLIPTICAL SERIES.

Curve 15. A slight gain in lift is effected by the elliptical tips, which is practically constant for all tips up to 4° above which the semi-circular seems to exceed both of the other tips in lift.

Curve 15. The drag is everywhere decreased with elliptical tips, the reduction being generally the greater, the greater the axis of the ellipse in the direction of the span. Thus the ellipse of semi axis $=\frac{3}{4}$C has least drag. The greatest drag reduction occurs between 8° and 14° incidence.

Curves 16 *and* 17. The L/D curves are interesting in that they seem to show that both elliptical tips are superior to the semi-circular for ordinary flying angles, 0° to 8°. The gain in maximum L/D for both elliptical tips is approximately 8% and for the semi-circular 5%.

These results differ from those obtained at the N.P.L. One reason for the difference is probably the difference in the aspect ratio, the average aspect ratio being constant in these tests as explained, while in the N.P.L. tests the extreme span was constant.

Streamline Body on Tip.

Curve 18. The lift is not affected up to 0° and thereafter the lift is decreased an amount which increases with the incidence.

Curve 18. The drag is everywhere reduced by the addition of the body.

Curve 19. The L/D is increased for incidences above −2°, the maximum L/D being increased about 2% over the square tips. Between −2° and 4° the increase averages 10%.

Upper Camber Washout Series.

Curve 20. There is a general slight increase in lift for negative angles and decrease for positive angles of incidence but the effect is very slight and erratic.

Curve 20. The drag is decreased up to the critical angle. The reduction is most marked for negative angles and particularly −2°.

Curve 21. From −4° to 14° the L/D's are increased slightly. There is some indication that the best washout (up to maximum L/D) is about 0.5 camber and a very definite indication that a washout of 0.9 camber is excessive.

Under Camber Washout.

Curve 22. There is here a gain in lift up to the critical angle which is most marked between 8° and 12°. This differs from upper washout effect.

Curve 22. Up to the critical angle a general reduction in drag results, of about the same extent as with upper washout, and as in latter most marked for negative angles and particularly the angle of no lift.

Curve 23. The gain in L/D up to the angle of maximum L/D are greater than for upper washout. The curves seem to indicate that washout of under camber can be carried farther than for upper camber. In the neighbourhood of zero incidence a washout of 0.9 camber has the greatest L/D.

General Conclusions.

1. Square tips are aerodynamically inefficient, all the tips tested showing a greater L/D.

2. The tips tested are effective in increasing the L/D only for incidences between −3° to −4° and the critical angle. Below and above

this range the advantage of shaped tips disappears and the square tips possess greater L/D.

3. The series showing the greatest increase in L/D was the positive straight raked tip series, the maximum increase noted being 17% for a straight positive rake of 45°. The advantage for small rakes is considerable, 10% for 15° rake.

4. The positive straight raked tip should possess advantages from the control point of view since it enables the ailerons to be placed at a maximum distance from the axis of the machine.

5. Straight rake is superior to the curved rake.

6. It would appear that the extreme tip of the wing should be ahead of or behind the centre of the chord. In other words greater efficiency would result if the elliptical tips were distorted by moving the transverse axis forward or backward.

7. Washout of camber should be made in the lower, less effective surface maintaining the form of the upper more effective surface, thus affecting to a less extent the lift. The sections of the tip should be maintained of good aerodynamic form (R. & M. 575, p. 4).

8. The tips tested result in no appreciable change in the form of the characteristic curves.

9 The results seem to agree with the mathematical theory as given by de Bothezat in that the drag curves show marked reductions at the angle of zero lift ($-2°$) thus indicating the disappearance of the tip vortex drag at zero lift as shown by the theory.

10. This research indicates that for best aerodynamic efficiency the wing tip should have a positive rake in plan form, the under camber should be washed out maintaining the upper profile of good form and the incidence should be decreased or washed out at the tip to that of zero lift at normal flight angle. For instance, for this aerofoil, taking the normal flight angle as 4°, at this angle the wing tips should be at an angle of $-2°$ (no lift), that is with regard to normal wing chord the tips have an angle of $-6°$.

A tip embodying these recommendations is being prepared for testing at Toronto.

APPENDIX I.—DETERMINATION OF SPINDLE CORRECTION.

The spindle effect on the characteristics of an aerofoil is made up of two elements, one, ordinarily the smaller, its own direct resistance, and the other, its interference with and alternation of the normal air flow about the aerofoil. From the latter standpoint the end spindle used in conjunction with square tips has proved less troublesome and is recognized as the standard method of support for aerofoils with balances of the N.P.L. type.

The following methods were employed to determine the spindle corrections in this research. That used for the end spindle is a combination of those described in R. & M. s 148 and 198.

The aerofoil, with square tips, was mounted in the standard way (Plate 24, set up (*a*)) using tapered end spindle tapped into one tip 1″ from leading edge. Lifts and drags were measured over the full range of incidences.

A duplicate spindle fitted with dummy balance nut was next tapped into the other tip and a dummy streamline baffle mounted above the aerofoil in the same relative position to the upper spindle as the lower streamline bears to the supporting spindle (Plate 24, set up (*b*)). Lift and drag readings were made with this setting for the different incidences.

Both these sets of readings were corrected for spindle deflection at high incidences.

The differences between the corrected readings with two spindles and with one spindle gave the effect of the upper spindle. In order to determine from this the lower spindle effect and thus the true characteristics of the aerofoil with no spindle, the centre of pressure of the spindle had to be found.

To make this determination a straight spindle was fastened to the underside of the aerofoil at the centre and 1″ from the leading edge, and the aerofoil mounted in the channel in its natural flight position (Plate 24, set up (*c*)). Readings of drag and twisting moment about the balance axis were taken. Similar readings were made with one end spindle, carrying dummy nut, screwed into one tip of the aerofoil and the dummy streamline in position.

By subtraction the drag effect and the twisting moment of the dummy spindle and nut were found, and dividing the latter by the former the moment arm of the spindle drag was determined and so the centre of pressure of the spindle. The measurements and results were as follows:

Drag with end spindle and nut..................	0.30149 lbs.
Drag without end spindle and nut.................	0.29000 lbs.
Drag of end spindle and nut.....................	0.01149 lbs.
Moment reading with spindle and nut.............	9.243 turns
Moment reading without spindle and nut..........	8.668 turns
Moment reading due to spindle and nut............	0.575 turns

Moment due to spindle and nut $=0.575\times 0.2154=0.1239$ in. lbs.

Distance from axis to C.P. of spindle and nut $=\frac{0.1239}{0.01149}=10.78$ ins.

Hence C.P. is $10.78-9.00=1.78''$ from the end of the aerofoil.

Although no attempt was made to set the aerofoil at a definite angle of incidence in the last set up, if it be taken as 0° as it was approximately, the spindle resistance here can be compared with that found above between two and one spindle set ups. Thus the difference in drags at 0° was 0.0147 lbs. and correcting this to the centre of the channel, $0.0147 \times \frac{36}{36-10.78} = 0.01131$ which checks very well the above value of 0.01149 considering the possible difference in incidence.

Multiplying the spindle effect of the upper end spindle found as outlined above by the ratio of the distance between pivot of balance and C.P. of lower spindle $(36-10.78=25.22)$ and distance between pivot and C.P. of upper spindle $(36+10.78=46.78)$, namely, 0.5391 gave the lower spindle effect or the "end spindle correction."

The lift and drag readings with the lower spindle only, standard mounting, were then corrected for the spindle effect, resulting in the true lifts and drags for the aerofoil without spindle.

Readings were now taken of lift and drag with the aerofoil with square tips mounted on the cranked spindle. By subtracting from these readings the true no spindle lift and drag readings the "cranked spindle corrections" were obtained.

The readings of the different tests were all plotted to a very large scale on cross-section paper and the average curves were drawn (Plate 25). By employing this method rather than arithmetical averages points obviously in error were properly discounted.

From these curves the spindle effects were scaled, multiplied by the proper ratio in the case of the end spindle, and the corresponding spindle correction curves plotted (Plates 26 and 27).

DISCUSSION OF RESULTS.

It will be observed that the spindle corrections on lift for both spindles are more or less erratic. This is undoubtedly due to the fact that they are found by subtracting two relatively large quantities (lifts). The spindle effect, even of the cranked spindle, is very slight, as is shown by the lift curves on Plate 25. The uncertainty in the corrections is, therefore, relatively unimportant and the correction curves on Plate 27 can be taken as sufficiently accurate where it is deemed necessary to apply spindle corrections on lift. In the case of the end spindle used in this investigation a constant correction of -0.003 can be safely used. Spindle corrections on lift have been applied in working up the results of this research.

The corrections on drag are reasonably uniform and consistent for both spindles. In the case of the end spindle the corrections decrease slightly with increase in incidence up to 2° afterwards increasing up to 14° and above this angle becoming uncertain and erratic. For the cranked spindle the correction decreases practically uniformly to 14°

becoming very erratic above this angle. These results are in agreement with the results given in the N.A.C.A. Technical Note No. 37, where it is shown that with a U.S.A. 15 wing section the corrections on drag are nearly constant up to 10° becoming erratic for greater angles.

This effect is to be expected since the greater portion of the spindle correction is due to interference with the air flow about the model and only a small portion to the direct resistance. Hence when the flow becomes unsteady at and beyond the critical angle the interference effect of the spindle is reduced.

The advantage of this method of determining the cranked spindle correction is that it automatically compensates for all disturbing effects of the spindle, such as deflection and interference, and the correction is found for the spindle in the exact position in which it is to be used.

The disadvantage of the method is that it depends on differences and in this case on small differences of relatively large quantities. However, by exercising great care in making the observations and taking many check readings by independent observers at different times it is believed that the corrections have been determined with considerable accuracy.

APPENDIX II.—ACCURACY OF PROFILE OF MODEL AEROFOIL.

After running several tests with the square tipped aerofoil and comparing the characteristics so obtained with the standard characteristics for this section (R. & M. 195, Part 1) considerable disagreement was found, notably the reversal in the lift coefficient curve above the critical angle (Plate 28). Repeated tests confirmed the results obtained and indicated the probability of inaccuracies in the model.

The templates and models were made in an outside machine shop not connected with the University, supposedly experienced in precision machine work, but unfamiliar with this class of work. Facilities for checking up the contours were not available until recently the apparatus shown (Plate 30) was designed and made for this purpose.

The dimensions of the RAF6 aerofoil were first published in R. & M 72, Part III, p. 89 (1912-13). These dimensions were used in preparing the drawings from which the aerofoils for this research were cut. Subsequently dimensions were given in R. & M. 110, Part II, p. 98 (1913-14) and in R. & M. 195, Part I, p. 78 (1915-16). The figures in these latter reports differed considerably from the original figures of R. & M. 72, and the reports became available in Toronto after the aerofoils had been completed. The dimensions of R. & M. 195, being the most recent have been taken as standard in this research.

The profile of the main aerofoil used in this research has been carefully measured, by means of the apparatus referred to above, at four different sections. A graphical comparison of the mean of the four profiles thus determined, with those of R. & M. 72 and R. & M. 195, has been made on Plate 29, and the dimensions of the Toronto aerofoil and from the different R. & M.'s have also been tabulated.

The differences between the various published dimensions is appreciable.

With regard to the accuracy of the Toronto aerofoil, while it differs considerably from the dimensions to which it was cut, the maximum departure of 0″.006 occurring in the front portion of the aerofoil, it is practically as close to the standard dimensions differing a maximum of 0″.007, but in the rear portion of the profile. The accuracy of cutting should have been considerably better than this.

The effect on the characteristics of the errors in profile are appreciable (Plate 28). The errors, however, do not affect the comparative results for the different tips nor the conclusions drawn from the research.

There is another probable cause of the differences exhibited by the characteristics obtained at the N.P.L. and Toronto, namely, the channel itself. The nature of the air flow, that is the characteristics of the particular channel, are known to affect to some extent the results obtained. In this case the presence of the intake on the Toronto channel probably results in the breaking up of the air stream into an infinite number of small vortices which may be partially responsible for the differences shown in the characteristics of the RAF6A aerofoil. This point can only be cleared up by tests in the Toronto and N.P.L. channels on the same aerofoil.

REFERENCES ON WING TIPS AND AILERONS.

R. & M. 53, 1911-12. Notes on the properties of aerofoils as deduced from the results of various aeronautical laboratories.

R. & M. 152, 1914-15. Experiments on models of aeroplane wings.
  Part I sec. 4. Effect of shape of wing tip (Elliptical).
  Part II sec. 3. Effect of wing flaps or ailerons (Square tips).
  Part II sec. 4. Tests on some special forms of wing flaps (Rounded tips and backward extending flaps).

R. & M. 197, 1915-16. The flow of air around a wing tip.

R. & M. 330, 1917-18. The effect of rounding the wing tips of an aerofoil.

R. & M. 550, 1918-19. An investigation of the aerodynamic properties of wing ailerons.

R. & M. 557, 1918-19. Comparative tests of a biplane of RAF15 wing section when fitted with wing tips of four different types.

R. & M. 575, 1918-19. The effect of aspect ratio and shape of wing tip on aerofoil characteristics.

R. & M. 289, 1916-17, Vol. II. The velocity of the wind in front of the wing tips of a biplane in flight.

R. & M. 615, 1918-19. An investigation of the aerodynamic properties of wing ailerons.

N.A.C.A. Report 28. An introduction to the study of the laws of air resistance.

N.A.C.A. Report 83. Wind tunnel studies in aerodynamic phenomena at high speeds.

U.S. Tech. Orders No. 5, Feb. 1919, p. 28. Investigation of the effect of raked wing tips.

U.S. McCook Field Report, Serial No. 1018. A note on the effect of adding streamlined forms on wing tips.

U.S. Navy Dept., Bureau of Construction and Repair. Aircraft Technical Note No. 106, Wing Tip Forms, Oct. 10, 1918; No. 126. Corrections for Plan Form of Wing Tips, Nov. 21, 1918.

Resistance of the Air and Aviation—Eiffel. (Translated by Hunsaker.)

Nouvelles Recherches sur la Resistance de l'Air et l'Aviation—Eiffel.

Résumé des Principaux Travaux exécutés pendant la Guerre, 1915-18. Eiffel, pp. 31-45.

REFERENCES ON SPINDLE CORRECTIONS.

R. & M. 148, 1914-15. Experiments on the variation of forces and moments on an aerofoil as speed changes.

Appendix. Note on the absolute value of the drag coefficients as affected by the determination of spindle resistance.

R. & M. 198, 1915-16. Models of complete aeroplanes.

Part III, p. 150. Tests on complete Handley-Page biplane.

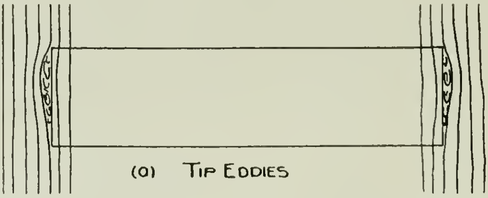
R. & M. 244, 1916-17, Vol. I, p. 36. Methods of support of models.

N.A.C.A. Tech. Note No. 37. The determination of the effective resistance of a spindle supporting a model aerofoil.

U.S. Tech. Orders No. 12, p. 36. Wind tunnel experimentation.

①

WING TIP LOSSES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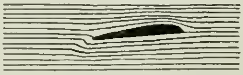
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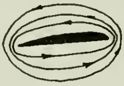
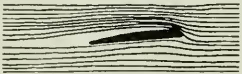
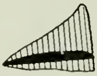

(h.) CYCLIC.

High Velocity, Low Pressure.

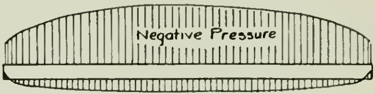


Low Velocity. High Pressure.

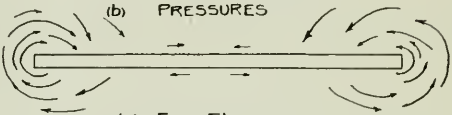
(i) COMBINED



(j) PRESSURE DISTRIBUTION



(b) PRESSURES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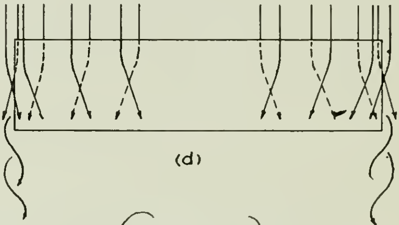
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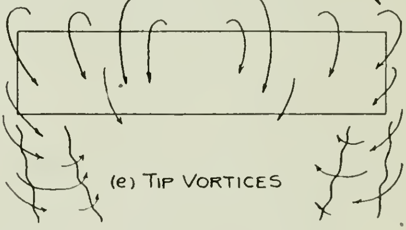
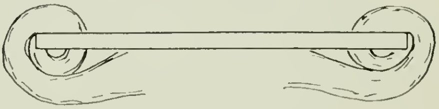
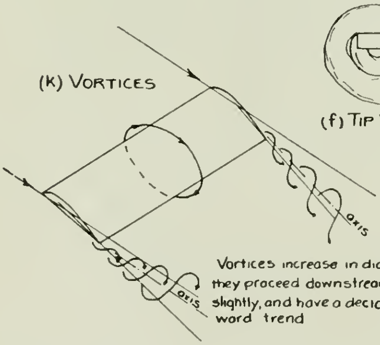

(e) TIP VORTICES


(f) TIP VORTICES PHOTOGRAPHED IN HIGH SPEED WIND CHANNEL - DAYTON

(K) VORTICES



Vortices increase in diameter as they proceed downstream, converge slightly, and have a decided downward trend

PLATE NO
WING TIP RESEARCH
Aeronautical Research Paper No 4
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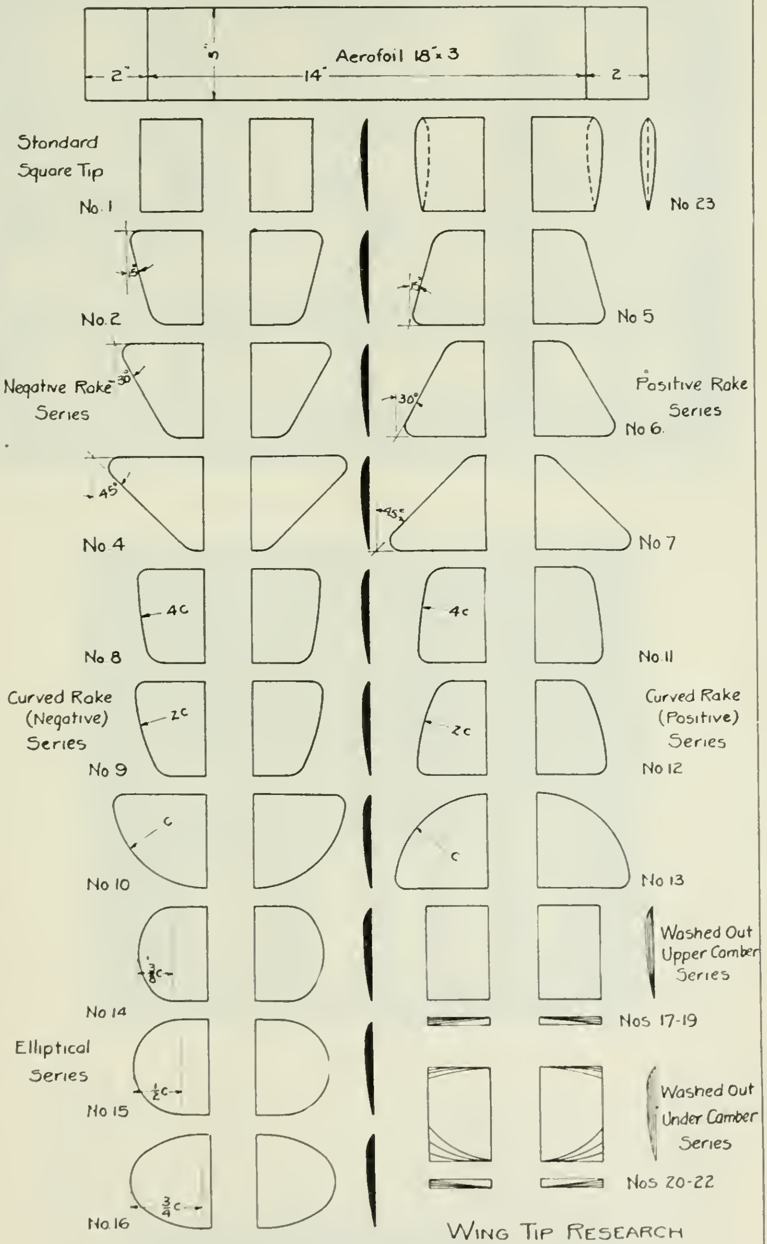
WING TIPS

②



WING TIP RESEARCH

Aeronautical Research Paper No 4

UNIVERSITY OF TORONTO

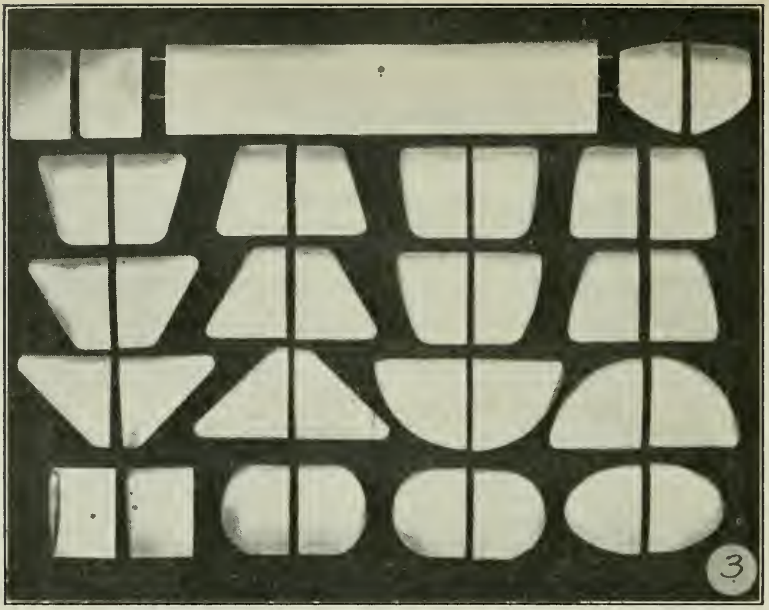


PLATE 3.—AEREOFOIL AND TIPS.

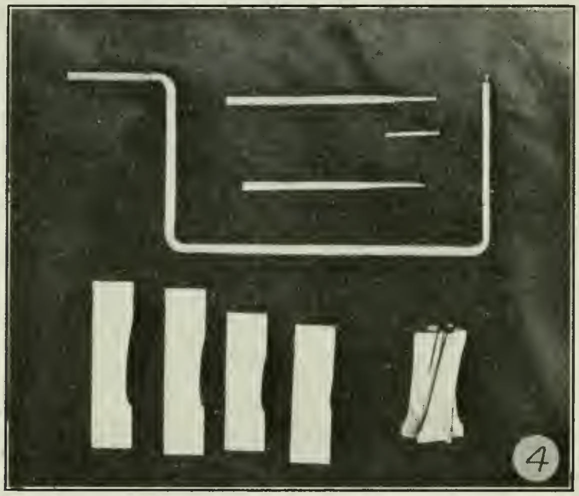


PLATE 4.—TEMPLATES SPINDLES AND CUTTER.

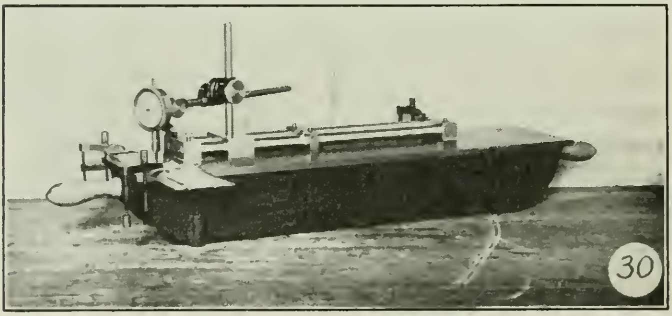
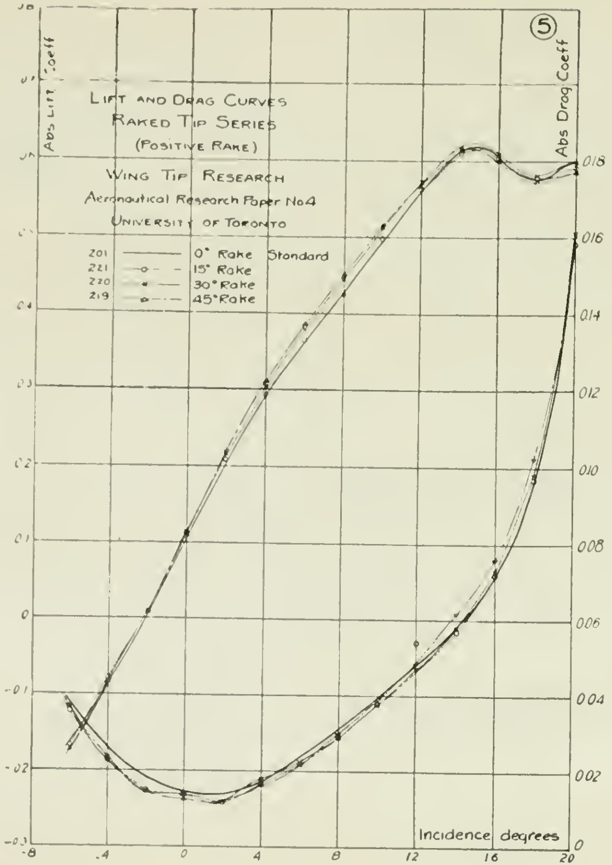


PLATE 30.—PROFILING APPARATUS.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Lift and Drag Curves
Raked Tip Series
(Positive Rake)
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
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221 15° Rake
220 30° Rake
219 45° Rake
Abs Lift Coeff
Abs Drag Coeff
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.18
0.16
0.14
0.12
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0.08
0.06
0.04
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-8
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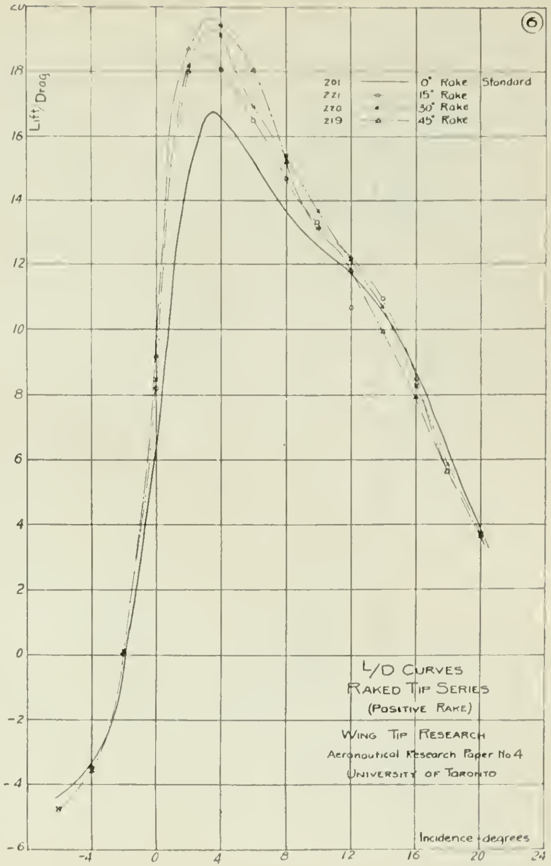
16
20
Incidence degrees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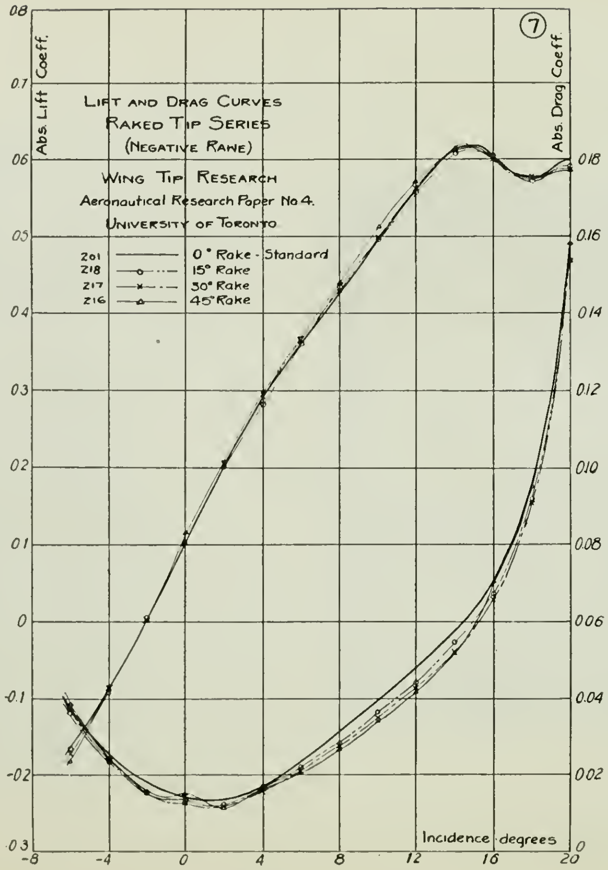
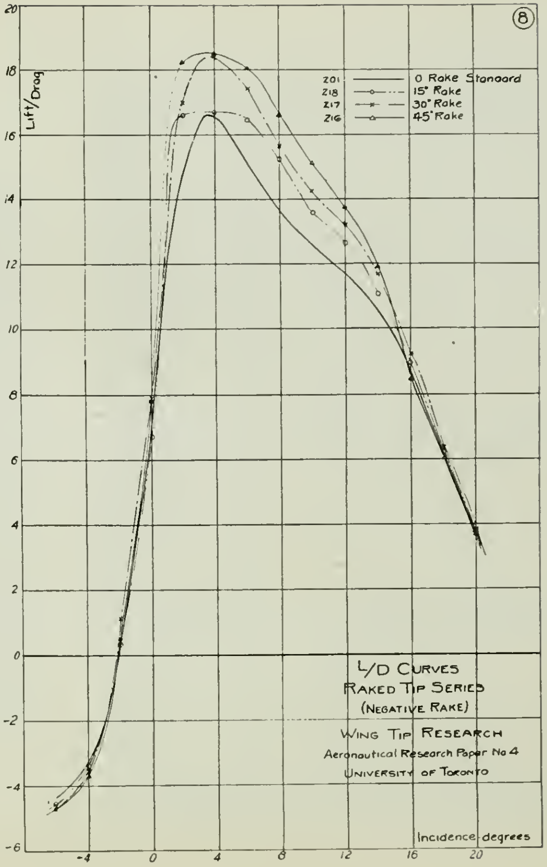
201 0° Rake Standard
221 15° Rake
220 30° Rake
219 45° Rake
Lift/Drag
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-4
0
4
8
12
16
20
24
Incidence degrees
L/D Curves
Raked Tip Series
(Positive Rake)
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto


⑦
Abs. Lift Coeff.
Abs. Drag Coeff.
Lift and Drag Curves
Raked Tip Series
(Negative Rane)
Wing Tip Research
Aeronautical Research Paper No 4.
University of Toronto
201 0° Rake - Standard
218 15° Rake
217 30° Rake
216 45° Rake
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
Incidence degrees
-8
-4
0
4
8
12
16
20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Lift/Drag
201 0 Rake Standard
218 15° Rake
217 30° Rake
216 45° Rake
L/D Curves
Raked Tip Series
(Negative Rake)
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
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16
14
12
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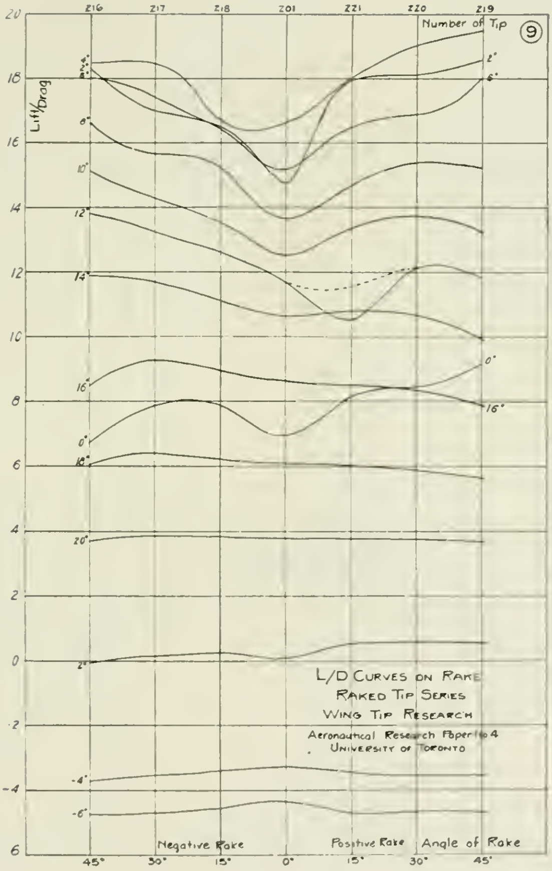
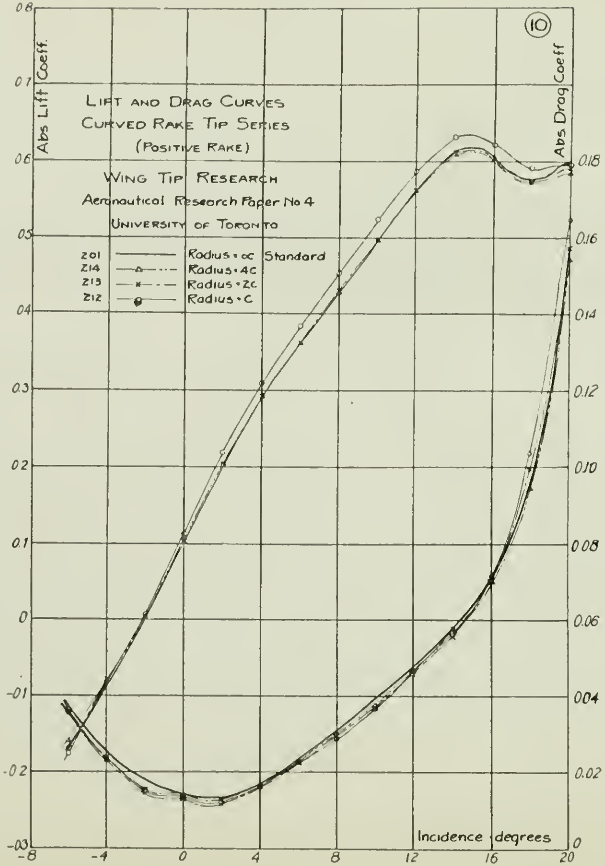
-2
-4
-6
Incidence degrees
-4
0
4
8
12
16
20



L/D Curves on Rake
Raked Tip Series
Wing Tip Research

Aeronautical Research Paper No 4
University of Toronto


(10)
LIFT AND DRAG CURVES
CURVED RAKE TIP SERIES
(POSITIVE RAKE)
WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO
Z01 Radius = ∞ Standard
Z14 Radius = 4c
Z13 Radius = 2c
Z12 Radius = c
Abs Lift Coeff.
Abs Drag Coeff
Incidence degrees
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.18
0.16
0.14
0.12
0.10
0.08
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0.04
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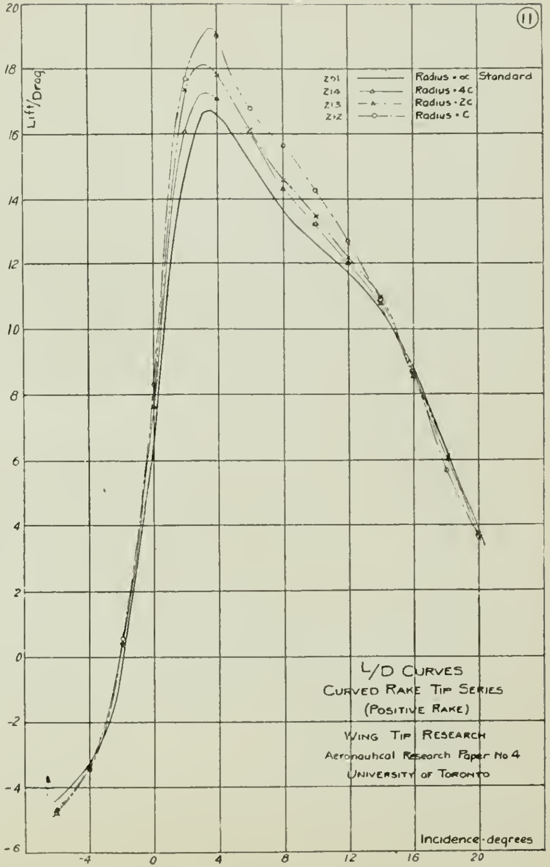
(11)
Z01 Radius = ∞ Standard
Z14 Radius = 4c
Z13 Radius = 2c
Z12 Radius = C
L/D CURVES
CURVED RAKE TIP SERIES
(POSITIVE RAKE)
WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO
Lift/Drag
Incidence degrees
20
18
16
14
12
10
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-6
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0
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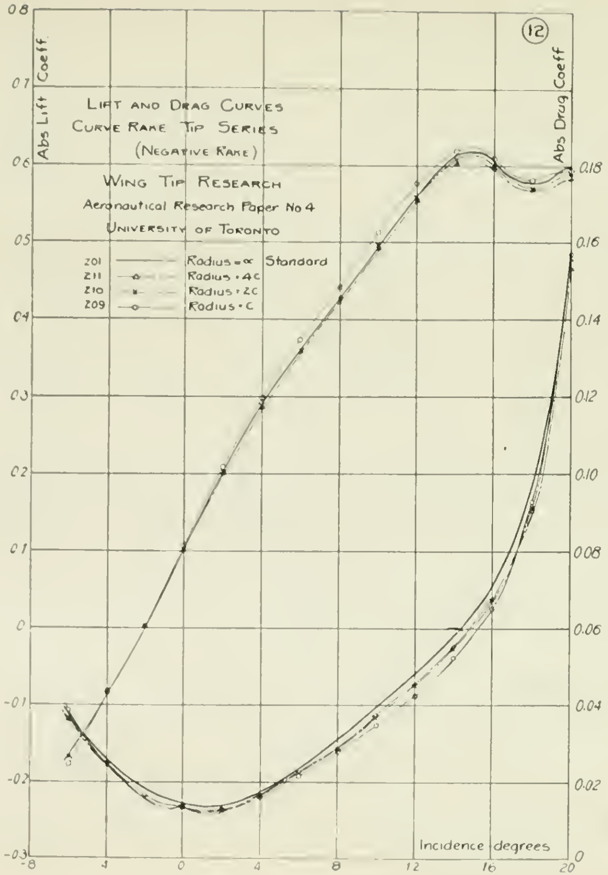
⑫
Lift and Drag Curves
Curve Rake Tip Series
(Negative Rake)
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
201 Radius = ∝ Standard
211 Radius • 4c
210 Radius • 2c
209 Radius • c
Abs Lift Coeff.
Abs Drag Coeff
Incidence degrees
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.18
0.16
0.14
0.12
0.10
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8
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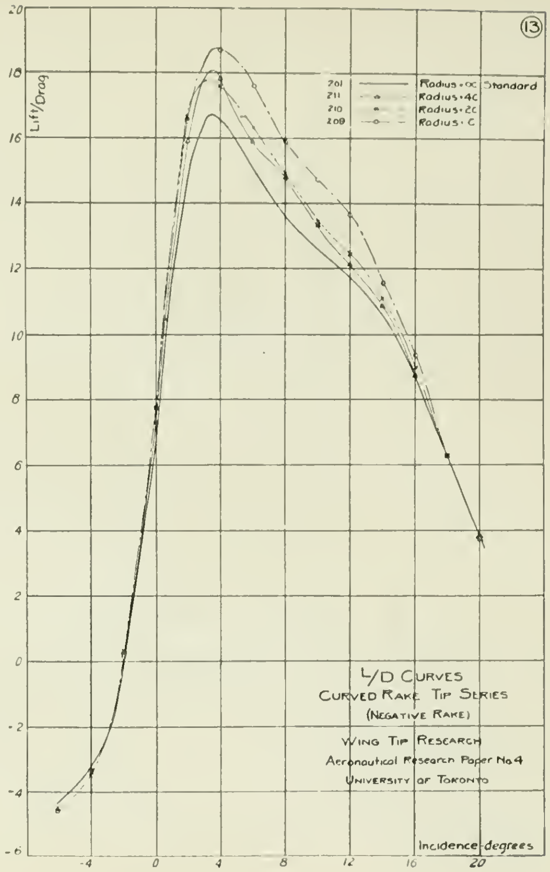
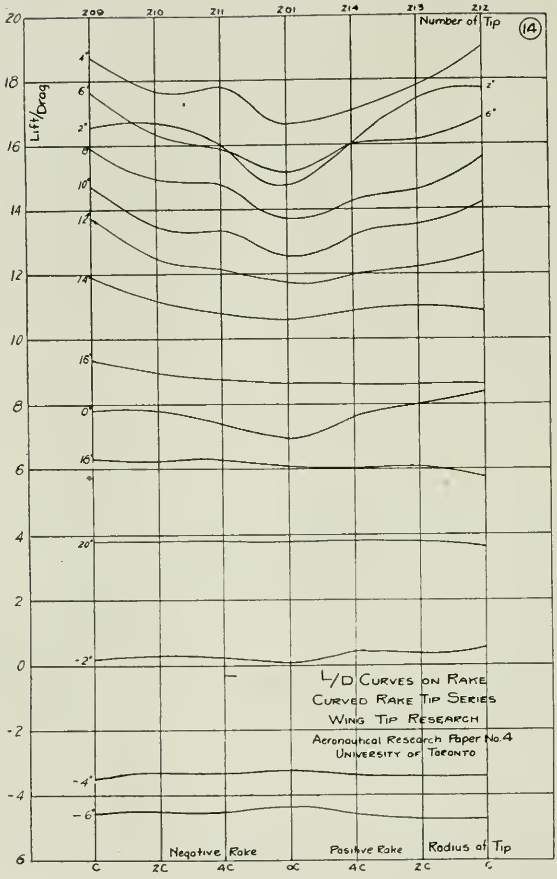
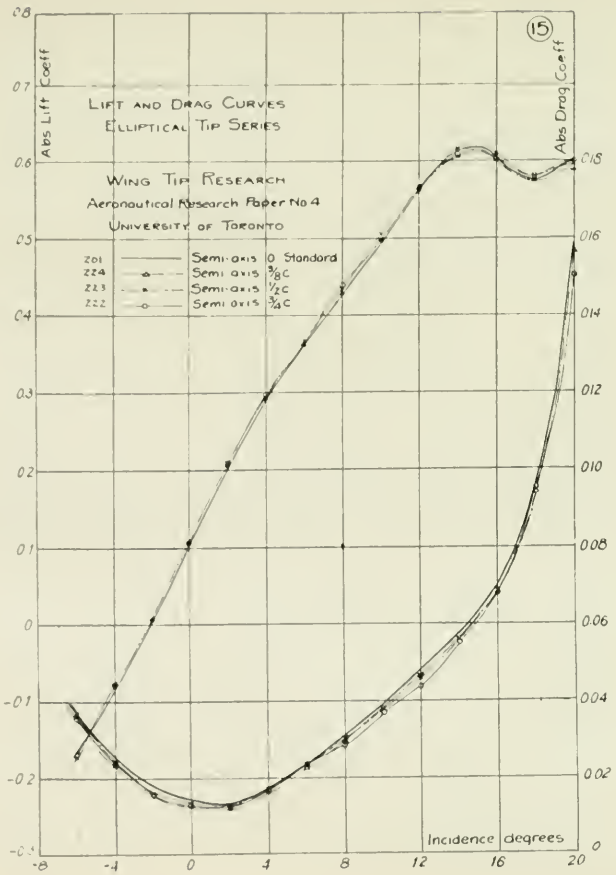

⑬
201 Radius • ∝ Standard
211 Radius • 4c
210 Radius • 2c
209 Radius • c
Lift/Drag
L/D Curves
Curved Rake Tip Series
(Negative Rake)
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
Incidence degrees
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-4
0
4
8
12
16
20



L/D CURVES ON RAKE
CURVED RAKE TIP SERIES
WING TIP RESEARCH
Aeronautical Research Paper No. 4
UNIVERSITY OF TORONTO



15
Lift and Drag Curves
Elliptical Tip Series
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
201 Semi-axis 0 Standard
224 Semi axis 3/8 C
223 Semi-axis 1/2 C
222 Semi axis 3/4 C
Abs Lift Coeff
Abs Drag Coeff
Incidence degrees
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
018
016
014
012
010
0.08
0.06
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0
4
8
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16
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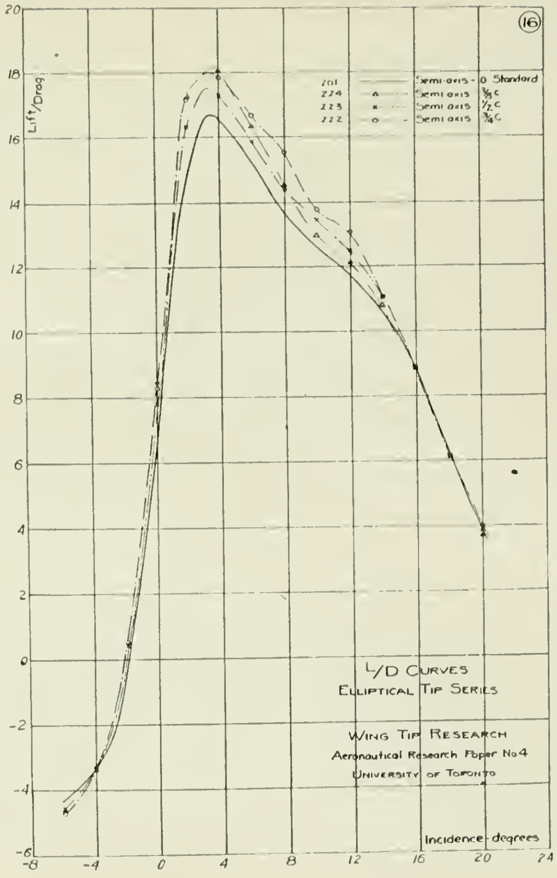
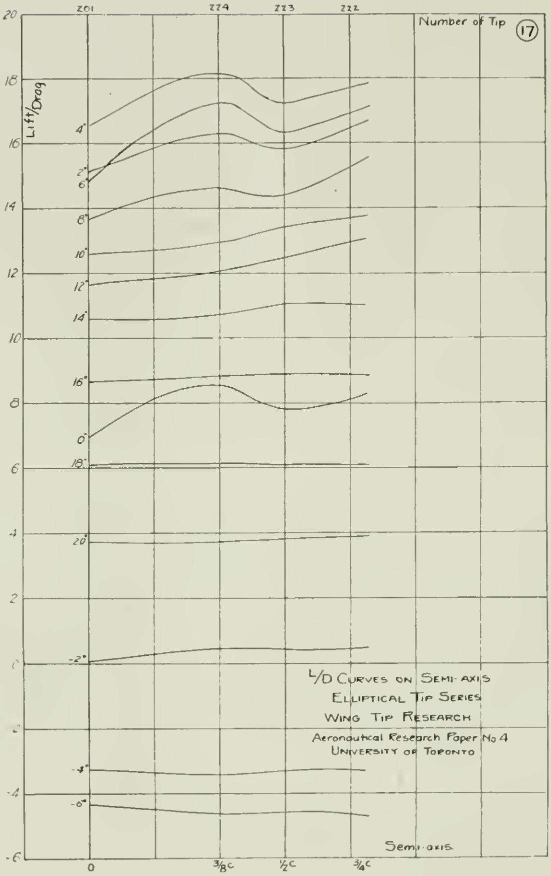
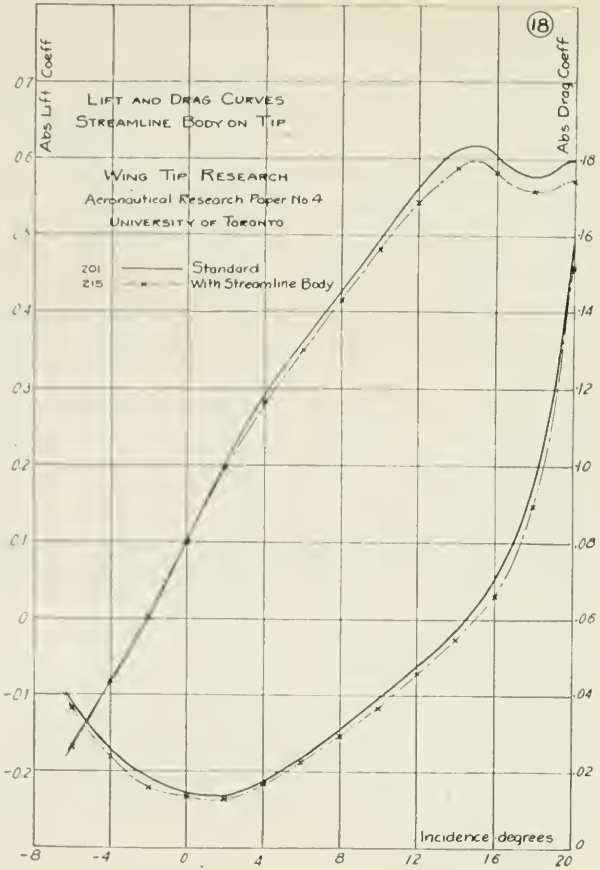
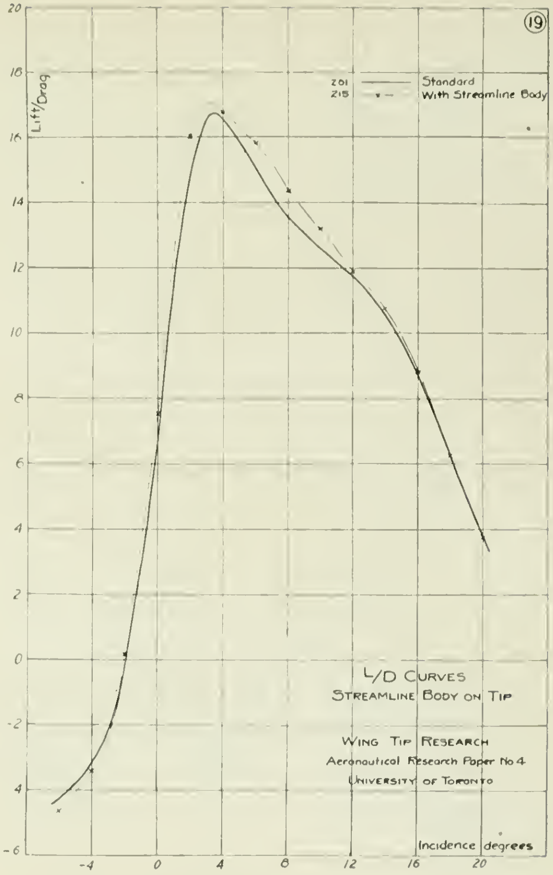


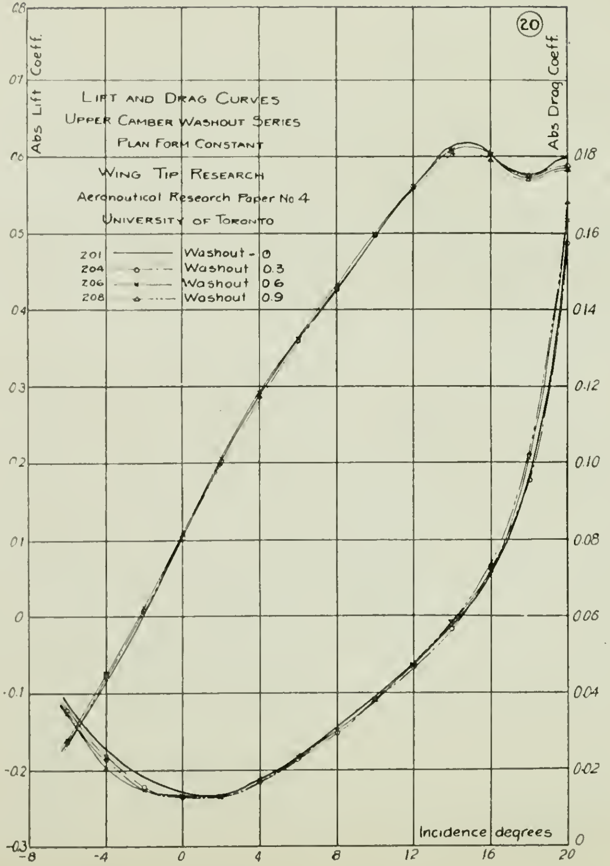
16
201 Semi-axis - 0 Standard
224 Semi axis 3/8 C
223 Semi axis 1/2 C
222 Semi axis 3/4 C
Lift/Drag
L/D Curves
Elliptical Tip Series
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
Incidence degrees
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-4
0
4
8
12
16
20
24

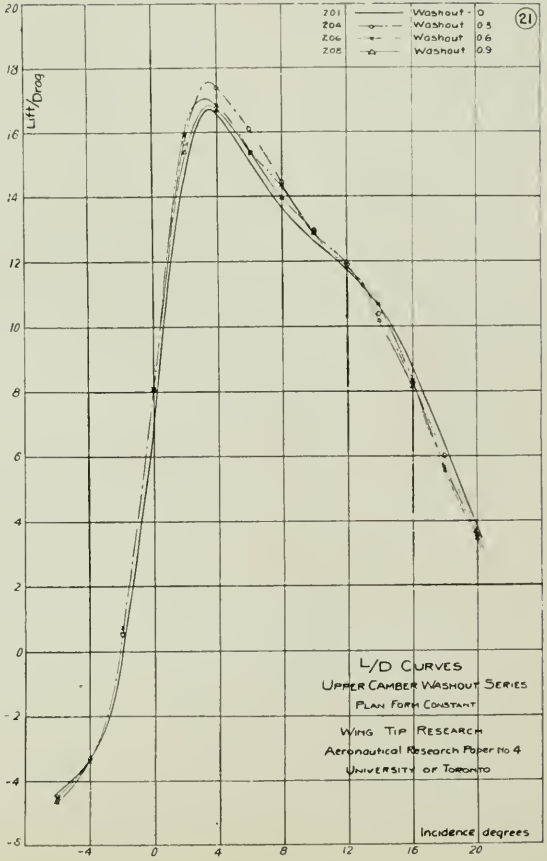


L/D Curves on Semi-axis
Elliptical Tip Series
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto


18
Lift and Drag Curves
Streamline Body on Tip
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
201 Standard
215 With Streamline Body
Abs Lift Coeff
Abs Drag Coeff
·07
·06
·05
·04
·03
·02
·01
0
-·01
-·02
·18
·16
·14
·12
·10
·08
·06
·04
·02
0
-8
-4
0
4
8
12
16
20
Incidence degrees


19
201 Standard
215 With Streamline Body
Lift/Drag
L/D Curves
Streamline Body on Tip
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-4
0
4
8
12
16
20
Incidence degrees


20
Lift and Drag Curves
Upper Camber Washout Series
Plan Form Constant
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
201 Washout - 0
204 Washout 0.3
206 Washout 0.6
208 Washout 0.9
Abs Lift Coeff.
Abs Drag Coeff.
Incidence degrees
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
-8
-4
0
4
8
12
16
20


21
201 Washout - 0
204 Washout 0.3
206 Washout 0.6
208 Washout 0.9
Lift/Drag
L/D Curves
Upper Camber Washout Series
Plan Form Constant
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
Incidence degrees
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-5
-4
0
4
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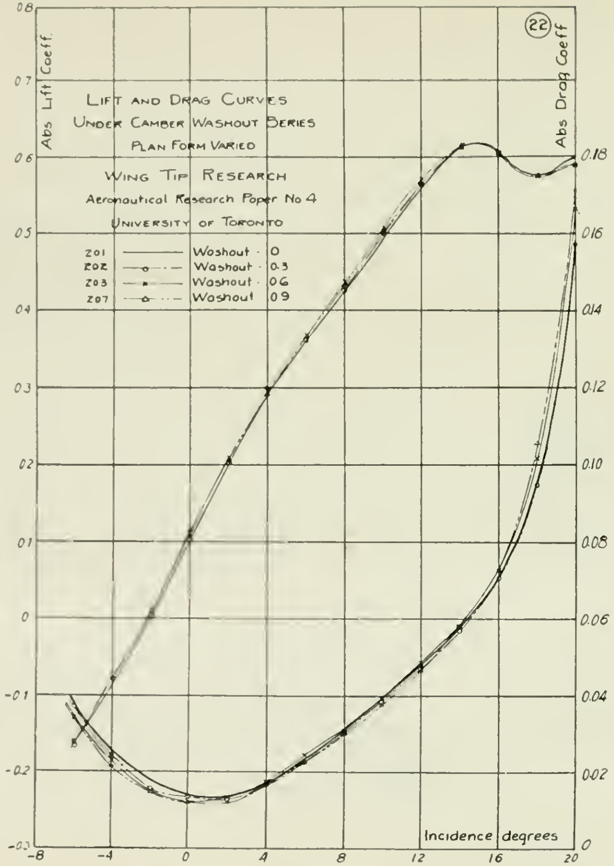
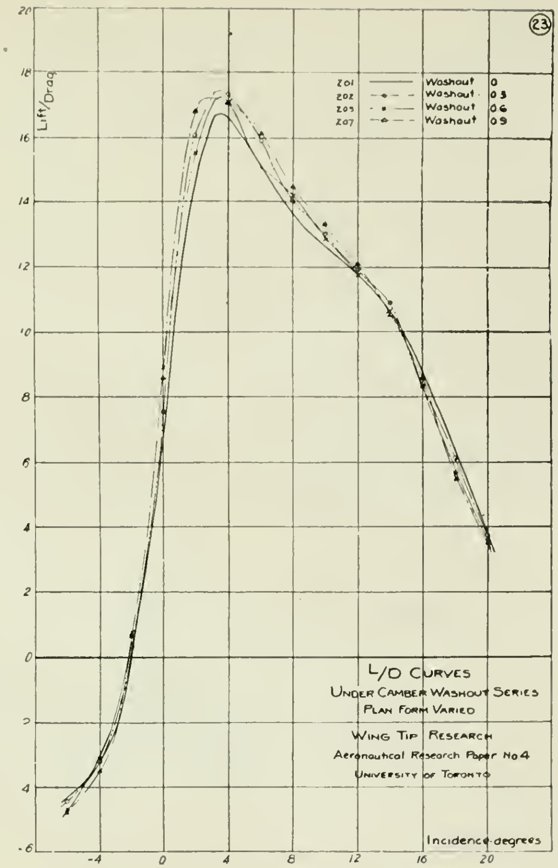
20


22
Lift and Drag Curves
Under Camber Washout Series
Plan Form Varied
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
Z01 Washout · 0
Z02 Washout · 0.3
Z03 Washout · 0.6
Z07 Washout 0.9
Abs Lift Coeff.
Abs Drag Coeff
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
-8
-4
0
4
8
12
16
20
Incidence degrees


23
Lift/Drag
Z01 Washout 0
Z02 Washout 0.3
Z03 Washout 0.6
Z07 Washout 0.9
L/D Curves
Under Camber Washout Series
Plan Form Varied
Wing Tip Research
Aeronautical Research Paper No 4
University of Toronto
20
18
16
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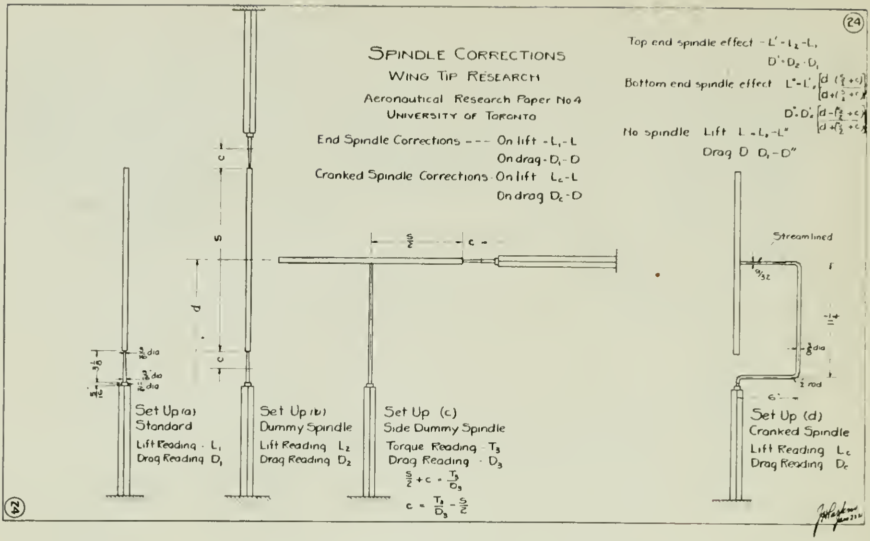
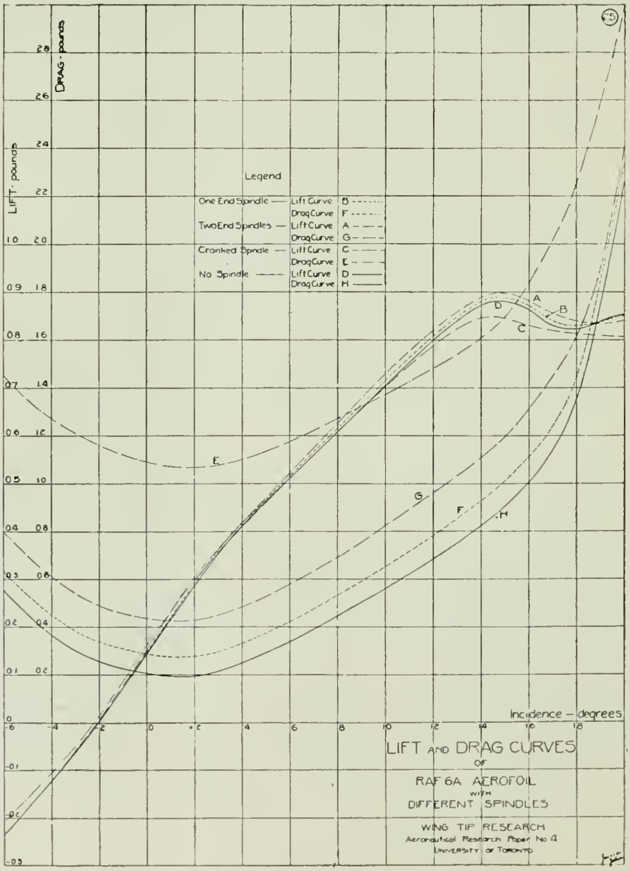
10
8
6
4
2
0
2
4
-6
-4
0
4
8
12
16
20
Incidence degrees


24
SPINDLE CORRECTIONS
WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO
End Spindle Corrections --- On lift - L₁ - L
On drag - D₁ - D
Cranked Spindle Corrections - On lift Lc - L
On drag Dc - D
Top end spindle effect - L' = L₂ - L₁
D' = D₂ - D₁
Bottom end spindle effect L" = L' × [d (s/2 + c)] / [d + (s/2 + c)]
D" = D' × [d - (s/2 + c)] / [d + (s/2 + c)]
No spindle Lift L = L₁ - L"
Drag D = D₁ - D"
c
s
c
d
s/2
c
Streamlined
9/32
1/8 dia
1/2 rod
6"
5/16 dia
3/16 dia
1/16 dia
Set Up (a)
Standard
Lift Reading - L₁
Drag Reading D₁
Set Up (b)
Dummy Spindle
Lift Reading L₂
Drag Reading D₂
Set Up (c)
Side Dummy Spindle
Torque Reading - T₃
Drag Reading - D₃
s/2 + c = T₃/D₃
c = T₃/D₃ - s/2
Set Up (d)
Cranked Spindle
Lift Reading Lc
Drag Reading Dc
24


25
LIFT - pounds
DRAG - pounds
28
26
24
22
1.0 20
0.9 18
0.8 16
0.7 14
0.6 12
0.5 10
0.4 08
0.3 06
0.2 04
0.1 02
0
-0.1
-0.2
-0.3
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
Incidence — degrees
Legend
One End Spindle — Lift Curve B
Drag Curve F
Two End Spindles — Lift Curve A
Drag Curve G
Cranked Spindle — Lift Curve C
Drag Curve E
No Spindle — Lift Curve D
Drag Curve H
A
B
C
D
E
F
G
H
LIFT AND DRAG CURVES
OF
RAF 6A AEROFOIL
WITH
DIFFERENT SPINDLES
WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO

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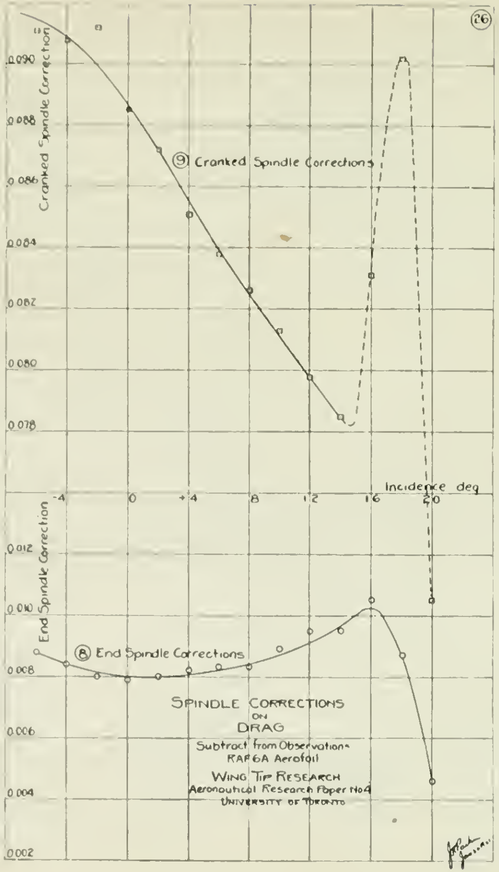

SPINDLE CORRECTIONS
ON
DRAG

Subtract from Observations
RAF 6A Aerofoil

WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO

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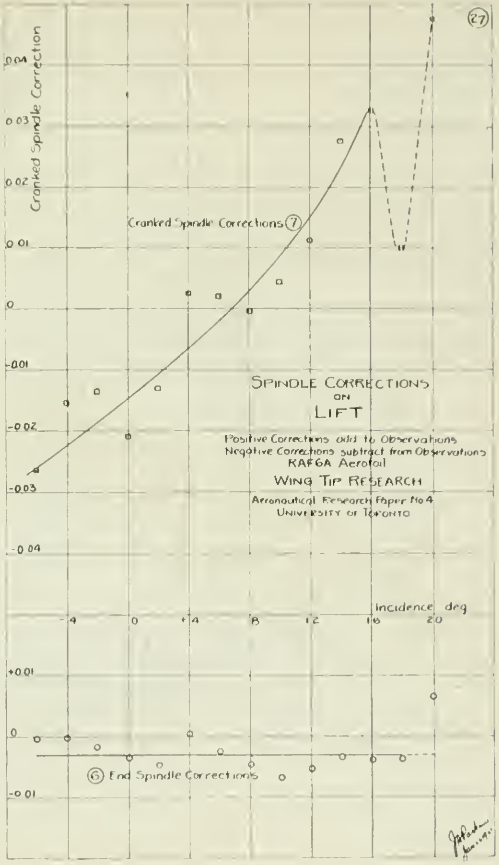

SPINDLE CORRECTIONS
ON
LIFT

Positive Corrections add to Observations
Negative Corrections subtract from Observations
RAF 6A Aerofoil

WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO

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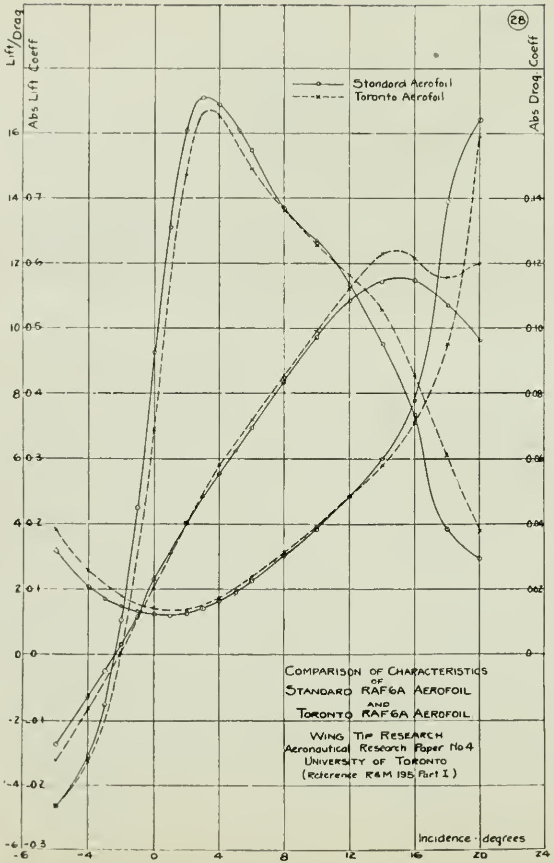

COMPARISON OF CHARACTERISTICS OF STANDARD RAF6A AEROFOIL AND TORONTO RAF6A AEROFOIL

WING TIP RESEARCH
Aeronautical Research Paper No 4
UNIVERSITY OF TORONTO
(Reference R&M 195 Part I)

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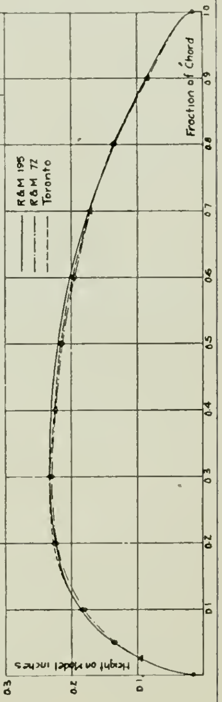
COMPARISON OF RAF6A AEROFOIL DIMENSIONS

All dimensions in fractions of chord

Distance from leading edge	Heights above chord			
	R&M 72 Part III p 89	R&M 110. Part II p 98	R&M 195 Part I p 78	Toronto Average of four sections
0	0.005	0.000	0.000	0.000
0.01		0.020	0.020	
0.02		0.028	0.028	
0.025	0.032		0.032	0.032
0.03		0.035	0.035	
0.04		0.041	0.041	
0.05	0.044		0.0455	0.046
0.06		0.050	0.049	
0.08		0.056	0.057	
0.10	0.060		0.062	0.062
0.12		0.060	0.066	
0.15			0.071	
0.16		0.072	0.072	
0.20	0.074		0.0745	0.076
0.24		0.076	0.0765	
0.25			0.077	
0.30	0.076		0.0775	0.0775
0.32		0.077	0.0776	
0.40	0.075	0.076	0.077	0.075
0.50	0.071	0.073	0.073	0.072
0.60	0.065	0.066	0.068	0.066
0.70	0.057	0.058	0.058	0.057
0.80	0.044	0.045	0.044	0.045
0.90	0.027	0.028	0.028	0.027
0.96		0.017	0.0175	
0.98		0.013	0.013	
0.99		0.010	0.010	
1.00	0.005	0.000	0.000	0.000



Aeronautical Test Report No. 1.
Feb. 1921.

BALANCE PORTION FOR THE RUDDER OF THE F.3 FLYING BOAT.

J. H. PARKIN, H. C. CRANE, AND S. L. GALBRAITH

Published by Permission of the Air Board

The investigation here described was undertaken at the request of the Air Board to determine the effectiveness of a new type of balance portion for the rudder of the F.3 flying boat.

As will be seen from the drawings (see Figs. 1 and 2) the balance portion consists of two fins or blades, 4″ wide, $\frac{3}{4}$″ thick and 36″ long, of streamline cross-section and with semi-circular ends, carried on struts projecting forward and outward from the rudder so that the blades are 15″ apart and their leading edges 7″.5 ahead of the axis of the rudder hinge. The balancer is fitted at the top of the rudder in order to clear the horizontal stabilizer, which is located at the level of the recess in the leading edge of the rudder.

Models. Models of the tail fin and rudder to 1/6th full scale were prepared of mahogany from blueprints furnished by the Air Board. The balance portion was made up in brass, the arms being carefully finished and the blades shaped accurately to template.

The tail fin was mounted in the channel in proper position by means of wires and turnbuckles. The wires were attached to the bottom of the fin and to a point on the inclined entering edge corresponding to the position of the leading edge of the horizontal stabilizer. The interference with the balancer was thus very small and less than would likely occur in practice with the remainder of the tail unit in position.

A 3/16″ threaded spindle was fitted in the rudder at the lower tip in such a way that its axis corresponded with the axis of the rudder hinge. This spindle was screwed into a standard 5/16″ spindle hold in the chuck of the balance (see Fig. 3).

In addition the rudder was fitted with a small hinge at the upper end to steady and maintain it in proper position relative to the fin. The hinge consisted of a small brass block carrying a 1/64″ diameter steel pin turning in a hole of the same size drilled in a 1/32″ thick brass bracket screwed to the fin. The hinge was carefully shaped and fitted to cause a minimum of interference with the air flow and was accurately lined up so that its vertical axis coincided with the vertical axis of the balance. The friction of such a hinge under these conditions is negligible.

Test. Readings of torque about the rudder hinge were taken on the torsion head of the balance for the rudder with and without balancer, every two degrees up to 30° negative angle of yaw of the rudder and 20° positive yaw. It was found impossible to work above 20° positive yaw owing to the torsion spring and screw leaving contact.

The rudder was initially lined up at approximately zero angle by eye, and the true zero afterwards determined by plotting the observations.

The test was carried out at an air speed of 40 f.p.s.

Results. The torques in inch pounds about the axis of the rudder hinge of the model have been plotted on angle of yaw in Fig 4. Tabulated observations have not been given since the readings do not apply to even angles.

Conclusions. The balance portion results in an average reduction in torque about the rudder hinge of about 6% up to the angles of yaw of 25°. For the first 10° of yaw the reduction is about 10%.

The rudder was noticeably less steady with the balance portion fitted than without, due, doubtless, to the disturbing of the air flow by the balancer blades.

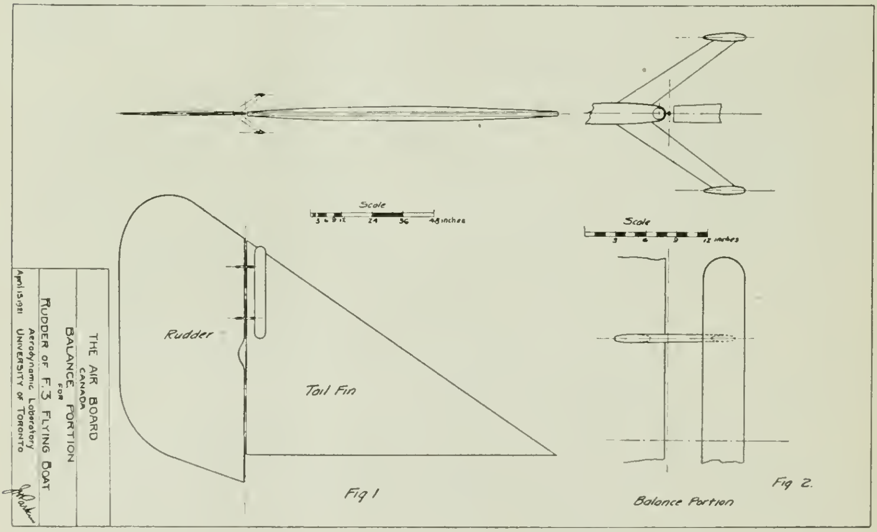


Fig 1

Fig 2.
Balance Portion

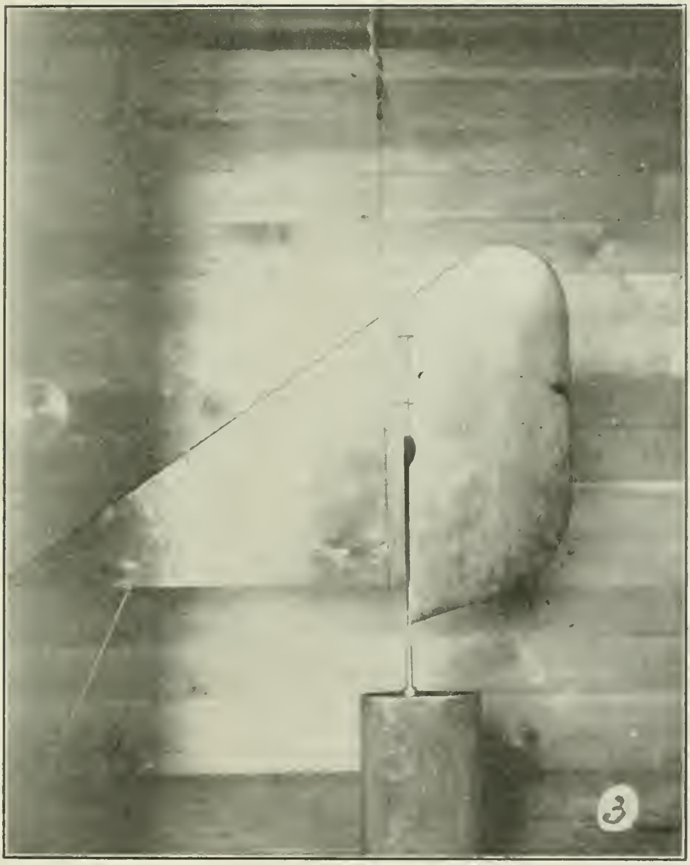
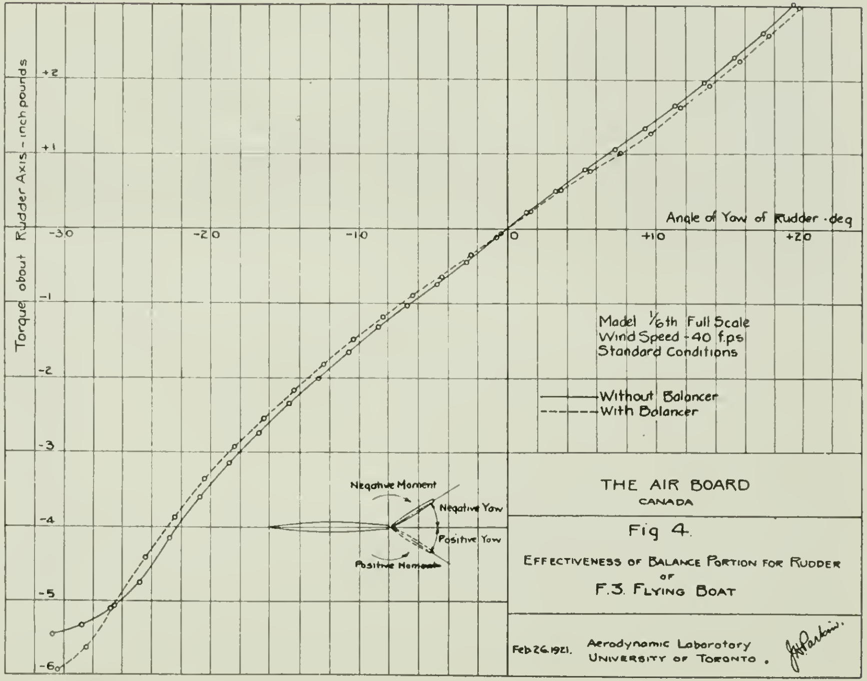


FIG. 3.—MODEL IN WIND CHANNEL.



THE AIR BOARD
CANADA

Fig 4.

EFFECTIVENESS OF BALANCE PORTION FOR RUDDER
OF
F.3. FLYING BOAT

Feb 26.1921. Aerodynamic Laboratory
UNIVERSITY OF TORONTO.
J.H. Parkin.

Aeronautical Test Report No. 2.
April 1921.

MACDONALD AEROFOILS.

J. H. Parkin, H. C. Crane and S. L. Galbraith.

The investigation described below was carried out to determine the effectiveness in increasing the lift of the addition of streamline bodies, termed "intensifiers," to the upper surface of an aerofoil, as proposed by Mr. W. B. Macdonald, M.E.I.S., of Welland, Ont.

Models. The models were supplied by Mr. Macdonald and were as shown in Fig. 1. The models were of plywood, of 3″ chord and approximately 10″ overall span. As will be seen the aerofoil section is coarse and lacking in refinement as compared with ordinary sections in present use.

The distinguishing feature of the Macdonald aerofoils is the use of the "intensifiers" on the upper surface. The intensifiers are of streamline form and of about the same thickness as the main aerofoils. There were six intensifiers, spaced 1″.8 apart and fastened to the aerofoil with wood screws.

Three sets of intensifiers were tested, namely, wide, medium and narrow. The wide intensifiers were about 0″.9 wide with well rounded blunt nose, the medium and narrow, 0″.75 and 0″.5 respectively, with sharp pointed nose and tail. No attempt was made to round the corners at the tops of the intensifiers nor where the intensifiers join the aerofoil.

Tests. Measurements of lift and drag were made on the aerofoil without intensifiers, and with the three different forms of intensifier, over the usual range of incidences, −6° to +20° at 2° intervals. In some tests the readings were taken at angles up to 26° to determine the trend of the curves beyond the critical angle. The air speed was 40 f.p.s. in all the tests.

Results. The results have been expressed in absolute coefficients and plotted, together with the L/D's, on one sheet to facilitate comparison (see Fig. 2).

Conclusions.

(1) The lift is decreased instead of increased by the intensifiers at all angles greater than +3°. Apparently the medium intensifiers were the best of the three forms tried, resulting in least reduction in lift. There

is, however, little to choose between the three. The reduction in maximum lift coefficient is from 10 to 12.5%. For incidences less than +3° the lift is increased by the intensifiers and at +3° the lift is the same both with and without the intensifiers.

An explanation of this effect on the lift may be that below 3° the air flow is jambed down between the intensifiers resulting in the expected increase in velocity, while above this angle the air stream is breaking away from the upper surface of the aerofoil and from between the intensifiers which are thus rendered ineffective. A point to be noted is that while over a small area at the most restricted part of the passage between the intensifiers the lift may be increased, the air flow over the remainder of the aerofoil and the tops of the intensifiers is disturbed with resulting loss of lift.

(2) The intensifiers flatten out the lift curve at the critical angle or burble point, the narrow ones being most effective in this respect.

(3) The drag is increased by the intensifiers. The narrow intensifiers cause least additional drag, while there is little difference in drag for the other two.

(4) The L/D is reduced above −2° by the intensifiers. In the case of the wide and medium intensifiers the reduction in maximum L/D is between 20 and 25% while for the narrow it is about one-half of this.

(5) Two irregularities will be noted in the C_y curves. These have been carefully checked. The explanation of these is not clear.

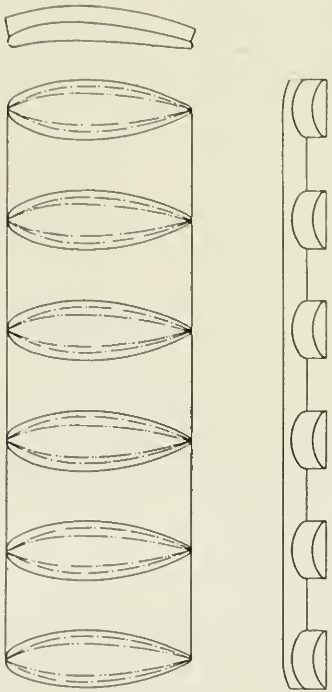


Fig. 1.

MACDONALD AEROFOILS

Scale: Full Size

Aerodynamic Laboratory
University of Toronto
April 8 1921 Canada

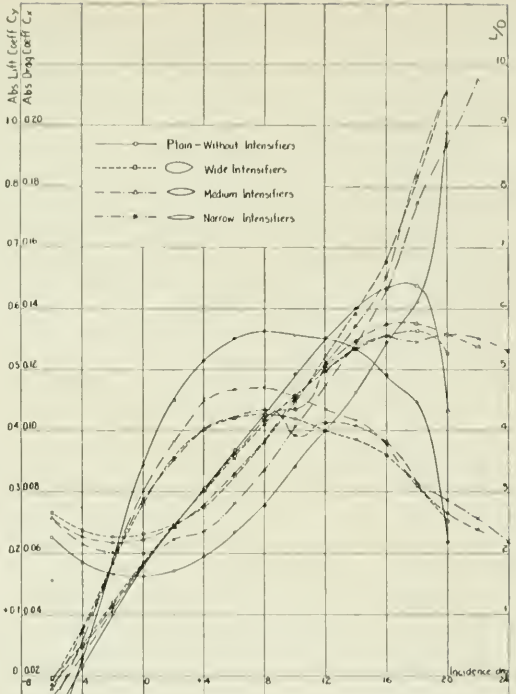


Fig 2

CHARACTERISTIC CURVES
of
MACDONALD AEROFOILS

(Standard Conditions)

Aerodynamic Laboratory
University of Toronto
Canada
April 8 1920

Aeronautical Test Report No. 3.
June 1921.

RUDDER BALANCE FOR F.3. FLYING BOAT.

By J. H. Parkin, E. V. Ahara and J. S. E. MacAllister.

Published by Permission of the Air Board

At the request of the Air Board an investigation was made of a second proposed type of balance for the rudder of the F.3. flying boat; the balancing effect of the first type having proved to be slight. (See Aero Test Report No. 1.)

The new type balance consists (see figs. 1*a* and *b*) of a faired trapezoidal shaped fin attached to the rudder by means of four forward projecting arms in such a way that as the rudder is moved the balance swings through an opening cut in the vertical tail fin just forward of the sternpost at the base of the fin. The balance is considerably thinner than the tail fin at that point and the latter is suitably faired around the opening for the balance. The balance area is approximately 2 sq. ft. or about 6% of the rudder area.

Models. The models of rudder and tail fin were the same ones used in the previous investigation. The balance fin was of mahogany and the supporting arms of brass.

The mounting of fin and rudder was also the same as in the previous tests except for a slightly different attachment of the bottom rear wires (see fig. 2) rendered necessary in order to provide clearance for the balance supporting arms.

Test. The investigation was carried out in the same manner as in the first case, readings of torque about the axis of the rudder hinge being taken every 2° from 30° negative rudder deflection to 20° positive deflection. The air speed was 40 f.p.s.

Results. The torques in inch-pounds about the axis of the rudder hinge of the model have been plotted on angle of yaw for this test, together with those for the first type balance taken from Test Report No. 1, in fig. 3.

Conclusion. The rudder deflections greater than 10° the new type balance is more effective than the first type and for angles greater than 20° is about three times as effective. The area of the new type balance fin is practically equal to the area of the two blades of the first type balance.

From 10° to 20° deflection the new type balance reduces the torque about the rudder hinge axis an average of about 10% and above 20° about 15 to 17%. Below 10° this balance is little if any better than the first type. This is doubtless due to the shielding effect of the thick tail fin. Balance is of course not needed at these small angles.

The steadiness of the rudder appeared to be little affected by the balance except at the large deflections.

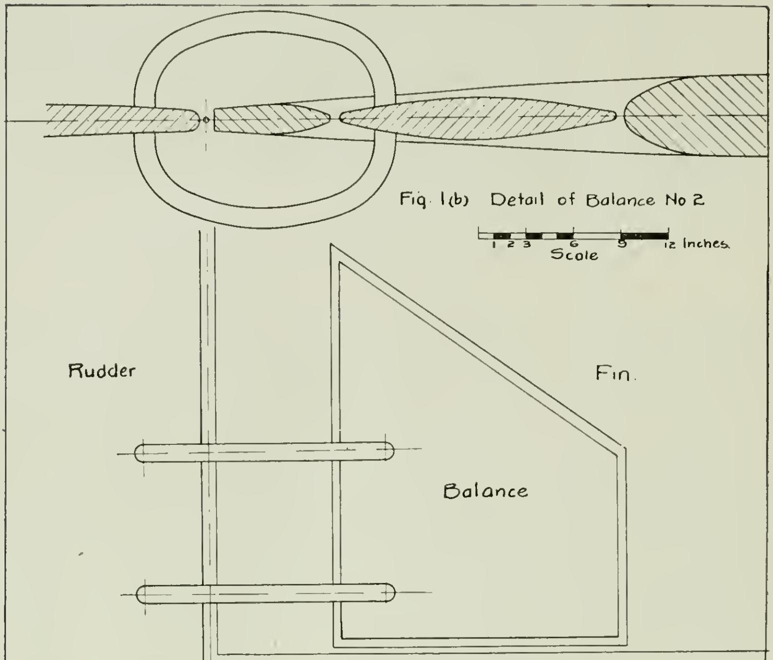


Fig. 1(b) Detail of Balance No 2

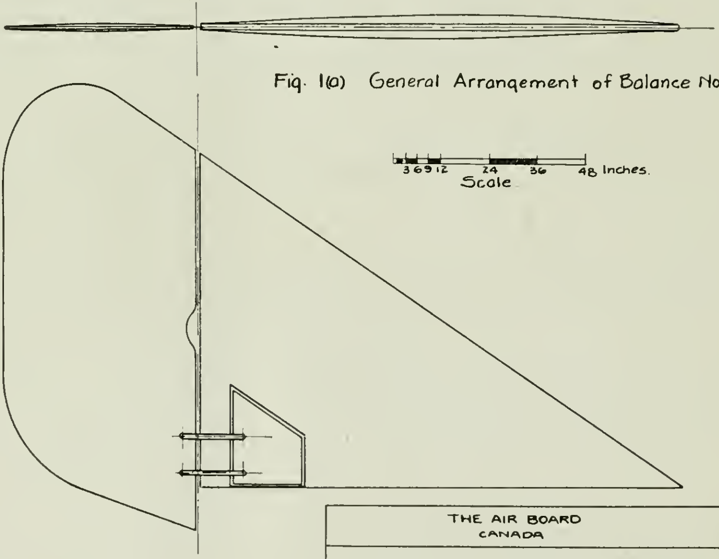


Fig. 1(a) General Arrangement of Balance No 2.

THE AIR BOARD
CANADA

EFFECTIVENESS OF BALANCE FOR RUDDER
OF
F.3 FLYING BOAT.

June 16 1921
Aeronautical Test Report No 3
UNIVERSITY OF TORONTO

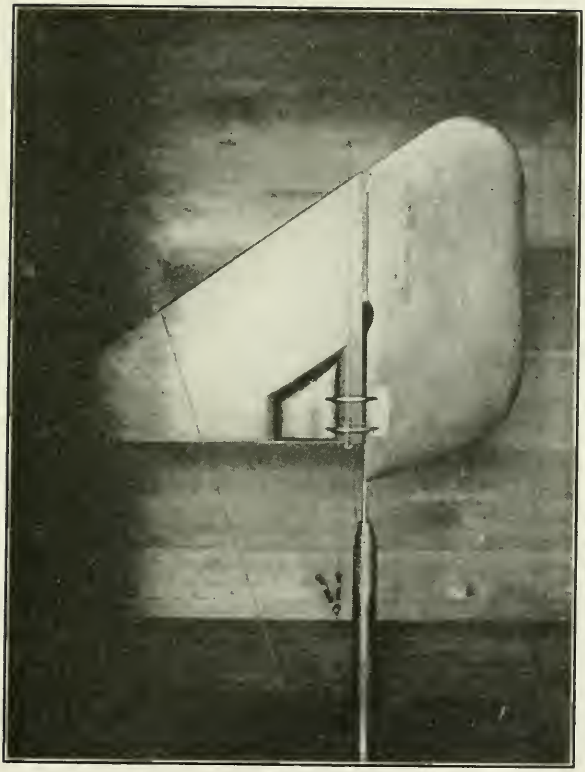
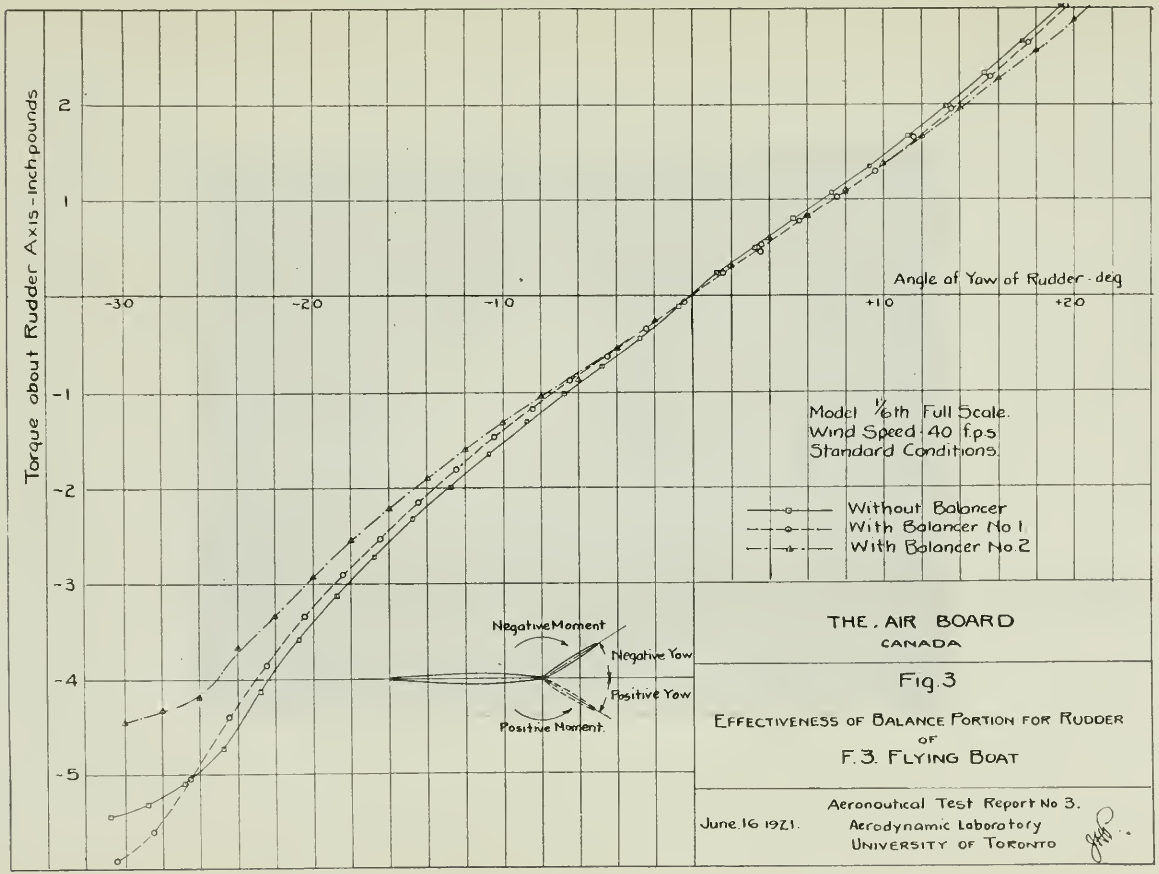


FIG 2.—MODEL IN CHANNEL.



THE. AIR BOARD
CANADA

Fig.3

EFFECTIVENESS OF BALANCE PORTION FOR RUDDER
OF
F.3. FLYING BOAT

June.16 1921.

Aeronautical Test Report No 3.
Aerodynamic Laboratory
UNIVERSITY OF TORONTO

THE BELLISS AND MORCOM SPRING GOVERNOR

By J. H. Parkin,

Assistant Professor of Design

General Description

The governor employed on the self-lubricating quick revolution engines manufactured by Belliss and Morcom, Limited, of Birmingham, England, is of the spring loaded pendulum type and is mounted directly on the end of the engine crank shaft. Single cylinder, compound and triple expansion engines are built. In the single and compound types the governor acts through a bell crank, vertical spindle and balanced or equilibrium throttle valve on the steam entering the high pressure steam chest. In the triple expansion engine the governor in addition controls a variable cut-off gear varying the cut-off in the high pressure cylinder. This is done by rotating the main piston valve, which has inclined ports, thus changing the lap. The throttle valve is first acted on and when its limit is reached the governor commences to act on the cut-off.

For starting purposes there is provided a second hand wheel controlled throttle valve placed beside the governor throttle and on the boiler side of it.

The engines run at high speeds and the ordinary guarantee as to speed variation is "3% permanent and 10% temporary variation" while for special cases 1% permanent and 2% temporary from normal, with whole load suddenly removed, will be guaranteed.

The governor here considered is that of a "two crank compound C type engine" of 125 rated brake horse-power and a normal speed of 525 r.p.m., operating under a boiler pressure of 125 lbs. gauge. The engine is employed to drive through belts two or four turbine pumps in either series or parallel, each pump delivering one cubic foot per second at 125 ft. head. The general arrangement of the governor is seen in fig. 1 from which its construction will be evident. The governor is mounted on one end of the crank shaft and therefore rotates at engine speed. It is enclosed in a case forming part of the engine casing. The governor is exceptionally well lubricated, the pivots *A* on which the load comes being forced feed lubricated by means of oil fed through passages in the crank shaft and governor frame (not shown in the figure). This reduces the disturbing effect of friction in the governor to a minimum.

It will be seen that in this governor the centrifugal force on the weights is resisted by the pull of the two main governor springs and of the compensating spring, the weight of the valve, valve rod, etc., and the pressure of the throttled steam on the valve stem end. These last two are comparatively small and may, as will be seen later, be neglected without serious error. The compensating spring is provided with a hand adjusting wheel by means of which the compensating spring may be shortened or lengthened, thus increasing or decreasing the mean speed of the engine whilst running (see results of this later) or to restore the speed to normal after a change in load has caused an alteration. By this means the average speed of the engine may be varied 4 or 5% on either side of normal or a similar variation due to load changes may be corrected. This is a desirable feature for load adjustment purposes or in electric driving for paralleling.

ANALYSIS OF FORCES ACTING

At any speed the forces acting in the governor just balance each other, neglecting friction. For a change in the load on the engine there is a corresponding change in the speed which causes an increase or decrease in the centrifugal force, under which the weights move outward or inward, altering the spring extensions and the throttle valve position until a new balance is effected at a slightly different speed.

The centrifugal force on the weights is balanced by four other forces, namely, those due to (*a*) compensating spring Sc, (*b*) valve and valve rod weight Wv, (*c*) steam pressure on the end of the valve stem Pe, which is the only unbalanced force in the so-called balanced throttle valve employed, and (*d*) the main springs S. Each of these four produces a turning moment or torque about the governor pin A in a contrary sense to that produced by the centrifugal force and thus the moment due to the latter balances those due to the former. These moments are as follows:

COMPENSATING SPRING MOMENT.

(*a*) The turning effect about the weight pivot caused by the compensating spring pull Sc is given by the formula,

$$\frac{Sc}{2} \times \frac{g \times c}{d}$$

VALVE AND VALVE ROD WEIGHT.

(*b*) If the total weight of the valve, spindle and fittings be Wv lbs. the moment due to this about each pivot A is,

$$\frac{Wv}{2} \times \frac{f \times c}{d}$$

STEAM PRESSURE ON END OF ROD.

(*c*) The steam pressure on the end of the valve rod acts in the same way as the valve weight and its moment therefore is,

$$\frac{Pe}{2} \times \frac{f \times c}{d}$$

Ordinarily, (*b*) and (*c*) are so small compared with the others as to permit of their being neglected without serious error.

MAIN SPRINGS.

(*d*) The moment of the pull of the main springs about the pins *A* is evidently, since there are two springs,

$$2 \times S \times b.$$

CENTRIFUGAL FORCE.

(*f*) Opposing these spring moments is the moment due to the centrifugal force of the weights. The centrifugal force *C* is equal to

$$\frac{w}{g} r \omega^2 = m r \omega^2$$

where w is the weight of the revolving weight in pounds or m the corresponding mass, r the radius of rotation of the centre of gravity in feet and ω the angular velocity of rotation in radians per second.

$$\omega = \frac{2\pi \ R.P.M.}{60}$$

The moment of C about the pin A is contrary in sense to that of the four listed above and equal to

$$C \times a.$$

SUMMARY.

Summing up, the four moments tending to keep the weights next the shaft are resisted by that due to the centrifugal force tending to throw the weights away from the shaft, and therefore,

$$M.Sc + M.Wv + M.Pe. + M.S = M.C$$

$$\text{or } \frac{Sc}{2} \times \frac{g \times c}{d} + \frac{Wv}{2} \times \frac{f \times c}{d} + \frac{Pe}{2} \times \frac{f \times c}{d} + 2 \times S \times b = C \times a.$$

Friction has been neglected in the above, which may be safely done in this case due to the efficient lubrication employed.

APPLICATION TO PRACTICAL CASE.

The above system of analysis will now be applied in investigating the forces acting in, and the operation of, the particular governor already mentioned.

In the first place the governor dimensions were accurately determined together with the magnitude of the weights and the position of the centre of gravity of the revolving weights. This data is tabulated in the top line of Table 1 and also below this table. The springs were also carefully calibrated, that is the weights causing different extensions determined and calibration curves plotted as in Curve Sheet No. 1.

With reference to the unbalanced steam pressure on the end of the valve stem, this is the throttled steam pressure and the maximum that can possibly act at even heaviest loads is 125 lbs. per sq. in. or boiler pressure. The area of the valve stem end is 0.15 sq. in. so that the maximum pressure on the end is only 18.75 lbs. while for ordinary loads the pressure is so small in comparison with the other forces acting that it can be neglected.

A carefully conducted test was then run on the governor in actual operation to determine the speeds corresponding to different radii of rotation of the weights. The engine was started up slowly with the smallest load (12% of rated) applied, control being by hand throttle, and the speed at which the governor just started to operate was taken on a calibrated sensitive tachometer. It was obviously impossible to measure the radii while the weights were rotating, but instead the position of some definite mark, say the centre of the pin at the bottom of the valve spindle was noted, which, in conjunction with an accurate scale drawing of the governor enabled not only the radii but all the corresponding moment arms necessary in calculating the turning moment as outlined above to be determined. The different dimensions so obtained are tabulated in Table 1.

The speed was increased slightly, after getting the starting speed, and another set of measurements taken and the process repeated at different speeds until full speed was reached with the hand throttle wide open, the speed being controlled by the governor. In order to eliminate possible errors due to slight lag in getting measurements, the process was repeated with decreasing speeds and the averages tabulated. The differences were very slight.

At each of the test speeds the moments about the governor pins *A* were than calculated. These have been tabulated separately in Tables 2, 3, 4 and 5, while the sum of the spring moments is tabulated in Table 6 together with the centrifugal force moment for purposes of comparison. According to the above theory the total spring moment should equal the centrifugal force moment and in this practical case it is seen that the agreement is quite good. The difference is within the limits of experimental error. It will be noted, however, that the centrifugal force moment in every case *exceeds* the total spring moment by an average of 7-8 in. lbs. This would seem to indicate that it is not due to experimental error but to another cause, possibly friction, although it is considered more probable that the difference is due to the centrifugal effect on the main governor springs. This is referred to later.

Characteristic Curves

It is more usual in governors of this type to plot curves of moments about a principal pivot rather than forces as in the true characteristic curve. The moment curves give a great deal of useful information about the governor while, to find the sensitiveness and power, the true characteristic curve must be plotted. In this case Moment Characteristic Curves have been ploted as shown in Curve Sheet No. 2.

The curves plotted cover the full range of action of the governor, from the time the weights start to move out until full speed, at this small load, is reached. The normal operating range of the governor from no load to full load or 25% overload will, of course, be much less than this total range. As predicted from governing curves plotted later the active operating range, no load to 25% overload for this setting, would be from 533 to 503 r.p.m., the full load speed being 517½. This range has been indicated on the curve sheet. In the case of this test which was run with about 12% rated load, the speeds were all controlled by the hand throttle in order to get measurements which otherwise would have been impossible due to the rapidity of the changes if under the control of the governor.

The moment curves of the different forces acting in the governor indicate the relative importance of those forces. Evidently the valve weight moment is practically negligible, while the moment due to the steam pressure on the valve stem end, which is less than the latter, may, as has been done here, be quite safely neglected.

The R.P.M. curve has been plotted on this sheet to render easy references to speeds.

The valve movement curve is of interest. It will be noted that the total movement of the valve is 0.535 or 17/32 inches, while the movement during the active governing range is only 0.215 or 7/32 inch. This corresponds to a change of radius of rotation of the weights of only 0.485 inches.

To determine (from the curves) the power and sensitiveness of the governor, the true characteristic curve must be plotted. This has been done on Curve Sheet No. 2 (*a*) from which the power (area under the curve) available in the governor from no load to 25% overload is 13.17 ft. pds. The sensitiveness from the curve is found to be (412.5-387.5)/(412.5+387.5) = 3.12% for the no load to full load range, or (412.5-367.0)/(412.5+367.0) = 5.88% for the no load − 25% overload range. The corresponding sensitivenesses as determined from the speeds are 2.95% and 5.79% respectively. The agreement is good considering the fact that the full load and 25% overload speeds were predicted by extending a curve.

ACTION UNDER VARYING LOAD

In order to study the action of the governor under varying loads the engine was operated under three different loads and the corresponding speeds noted. It was impossible to fully load the engine, the greatest load being about 90% of full rated power which consisted of the four pumps operating in parallel at maximum discharge. The other two loads consisted of one pair of pumps in parallel at maximum discharge or about 60% rated load, and one pair of pumps running empty or about 10% rated power. The power developed at each speed was determined by indicator and the speeds as before by tachometer. These results are plotted on Curve Sheet No. 3.

As would be expected, as the load increases the output from the engine increases causing a slight drop in speed which reduces the centrifugal force on the governor weights and the excess spring pull then available pulls the weights inward opening the throttle slightly thus increasing the entering steam pressure to take care of the greater load. But slight variation in speed is required to effect this as is seen from the curve. For instance, for an increase in load from 12 to 61% the speed drops only from 532 to 527 r.p.m., while an increase from 61 to 87% causes a further drop to 522 r.p.m. Both curves of r.p.m. fluctuation and percentage speed fluctuation are shown.

By producing these curves it is possible to predict, with some degree of accuracy, the speeds at no load, full load and 25% overload, and also to find the speed range or sensitiveness of the governor. Doing this the no load speed is 533 r.p.m., and the full load or normal speed for this particular setting of the governor is seen to be 517½ r.p.m., while for 25% overload the speed is 503 r.p.m. The percentage variation in speed then for full load suddenly removed is 533-517½/533+517½ = 2.95% which is within the guarantee as stated. The corresponding "governing curve" from the catalogue is also shown on the curve sheet for purposes of comparison and the agreement is fairly good. Whilst these results above 90% rated load are liable to error the governor is evidently extremely sensitive to small speed changes. One reason for this is, doubtless, the fact that the governor is mounted directly on the crank shaft without intermediate reduction gearing.

During this test the throttled steam pressure was also noted at the different loads and speeds. These have been plotted on this curve sheet resulting in an interesting curve as shown. The pressures as given by the gauge were checked from the indicator cards.

A complete set of indicator cards under the various loads are given on Sheet No. 4 to illustrate the action of the throttle valve, in reducing the steam pressures entering the high pressure cylinder, and the resulting effects on the pressures in the low pressure cylinder and on the load distribution between the cylinders.

HAND ADJUSTMENT DEVICE

The second point worthy of note is the action of the hand adjustment device. As already seen this consists of a hand wheel by means of which the length of the external compensating spring may be altered thus altering the pull inward on the governor weights. This will result in a change in speed and change in radius of rotation until the new centrifugal force just balances the new spring pull. Evidently shortening the spring increases the spring pull and, therefore, the spring moment and raises the speed of rotation.

The speeds corresponding to any setting of this adjusting device can be deduced with considerable accuracy from the speeds at normal adjustment. When the compensating spring is, say, tightened the spring pull is increased, the total spring moment is increased and the throttle valve is opened and the speed increases until a balance is again effected at a higher speed. The position of the valve at the new speed may, however, be slightly different, for, although the load is the same, the speed being higher results in less steam pressure being necessary to develop the required power, while, on the other hand, friction losses will be greater with the higher speed requiring slightly more steam pressure. These two likely very nearly neutralize each other, and it is assumed in deducing the speeds in the following that the valve occupies the same position after adjustment as before, in other words the radius of rotation is the same. That this is a safe assumption will be apparent from the results.

The adjusting wheel and spindle have 16 threads per inch so that one turn results in a change in length of the spring of .0625 ins. or in spring pull of 14 lbs. For any adjustment that it is desired to invesitgate then, say 2 turns above normal, 28 lbs. is added to each of the different spring pulls in Table 2, the corresponding moments calculated, the total moments found and finally the centrifugal force moments determined. From the centrifugal force moment the centrifugal force and finally the speed may be calculated and a speed curve plotted. This has been done for various adjustments and the curves plotted to different bases on Curve Sheet No. 5.

Tests were run on the governor to determine experimentally similar curves to those just deduced. At the three loads previously dealt with the speeds were taken for a series of adjustments of the hand wheel, and the results are plotted at *B* on Curve Sheet No. 5. It is seen that, allowing for experimental errors, the experimental results agree very well with the deduced values.

It is evident that a fairly wide range of speeds is made available by this device although 4 or 5% either way from normal is given as the limit in practice. Any attempt at a wide variation by hand adjustment results in a heavy pull on the bell crank by the spring which is transmitted through the remainder of the governor causing excessive friction and wear on the working parts, affecting the governor performance. Or if the spring be loosened the tension of the compensating spring may be removed altogether, resulting in very unsatisfactory governor operation, irregular speeds and serious hunting; as was seen in the tests, the speeds corresponding to 3 turns below normal being very erratic and unreliable.

A similar set of governing curves to those plotted on Curve Sheet No. 3 may be plotted from these results for any other hand wheel adjustment such as 3 turns below normal or 2 turns above, further showing the effect of this hand adjustment.

CENTRIFUGAL FORCE ON THE GOVERNOR SPRINGS.

A disturbing element in this type of governor and one liable to cause errors in the present analysis is the effect of the centrifugal force on the two main governor springs. Referring to Sheet No. 6 each coil of the spring will be subject to a certain centrifugal force *C* acting radially outward and depending in magnitude on its radius from the shaft axis, its weight and the speed. The eyes at each end of the spring are considered as part of the governor weights. The weight and speed factors are the same for all the coils so that the centrifugal force on the different coils will vary as their radii (see figure).

The centrifugal force acting on each coil may be resolved into two components, one *A* in the direction of the axis of the spring tending to bring the coils together at the ends of the spring and to separate them or stretch the spring at its centre, as indicated by the arrows. The other component *P* is perpendicular to the spring axis and in the plane of rotation and tends to distort or bow the spring outward from the shaft axis as indicated. This latter component on each coil is of the same magnitude as is seen from the figure and hence the spring under the action of these latter components corresponds to a beam under uniform loading and the deflection may be determined in the same way as for the beam. The result of this deflection is to stretch the spring bringing to bear a further pull inward on the spring pins of the weights. The net result on the governor of these two components of the centrifugal force on the spring coils would be difficult to determine accurately. Apparently the two components to some extent neutralize one another. However, the actual determination of these effects carries the analysis of the governor to a degree of refinement not contemplated nor justifiable with the data available on which to work.

TORONTO, CANADA,
Dec. 29, 1919.

TABLE 1.—DIMENSIONS AND MOMENT ARMS ABOUT PIVOTS

No.	(*a*)	(*b*)	(*c*)	(*d*)	(*f*)	(*g*)	Elev. of V	Extension		Rad.	R.P.M.	Valve Move't
								S	Sc			
0	3.56	3.19	3.47	10.31	4.68	5.31	......	......	......	......	....	....
1	3.55	3.19	3.42	10.31	4.63	5.25	0.88	8.38	6.14	3.70	415	0.00
2	3.56	3.17	3.45	10.31	4.64	5.26	0.99	8.82	6.27	3.93	454	0.11
3	3.55	3.13	3.48	10.31	4.65	5.28	1.12	9.30	6.42	4.20	487	0.24
4	3.53	3.07	3.48	10.31	4.66	5.30	1.24	9.76	6.56	4.47	508	0.36
5	3.48	2.99	3.46	10.31	4.67	5.31	1.36	10.20	6.70	4.73	526½	0.48
6	3.47	2.97	3.45	10.31	4.67	5.31	1.38	10.29	6.72	4.78	529	0.50
7	3.46	2.96	3.44	10.31	4.67	5.31	1.40	10.38	6.74	4.83	532	0.52

Miscellaneous

Weight of each weight	10.57 lbs.	e	4.00 inches
Mass of each weight	0.327	α	18° 30′
Weight of valve and spindle	20.00 lbs.	β	4° 30′
Scale of main spring	56.00 lbs./inch	θ	4° 00′
Scale of compensating spring	220.0 lbs./inch		

In the above table the first line gives the lengths of the arms, the remainder are the perpendicular distances from pivot to line of action of force.

TABLE 2.—COMPENSATING SPRING

No.	Extension	Pull	$\frac{g \times c}{2 \times d}$	Moment about *A*
1	6.14	39	0.831	32.4
2	6.27	67	0.880	58.9
3	6.42	100	0.891	89.1
4	6.56	129	0.894	115.4
5	6.70	160	0.891	142.6
6	6.72	163	0.889	144.9
7	6.74	167	0.888	148.3

TABLE 3.—WEIGHT OF VALVE AND SPINDLE

No.	Weight	$\frac{f \times c}{2 \times d}$	Moment about *A*
1	20	0.769	15.4
2	20	0.776	15.5
3	20	0.785	15.7
4	20	0.786	15.7
5	20	0.783	15.7
6	20	0.780	15.6
7	20	0.779	15.6

TABLE 4.—GOVERNOR SPRINGS.

No.	Extension	Pull	Pull of 2 Springs	(*b*)	Moment about *A*
1	8.38	97.5	195	3.19	622
2	8.82	122.5	245	3.17	776
3	9.30	150.0	300	3.13	939
4	9.76	176.5	353	3.07	1083
5	10.20	200.0	400	2.99	1196
6	10.29	205.5	411	2.97	1223
7	10.38	211.0	422	2.96	1250

TABLE 5.—CENTRIFUGAL FORCE

No.	Radius		Speed			Mass	Cent'l Force	(*a*)	Moment about *A*
	Inches	Feet	R.P.M.	ω	ω^2				
1	3.70	0.308	415	43.5	1890	0.327	190.5	3.555	677.5
2	3.93	0.327	454	47.5	2255	0.327	241.0	3.56	859.0
3	4.20	0.350	487	51.0	2601	0.327	297.5	3.555	1058.0
4	4.47	0.373	508	53.2	2830	0.327	345.5	3.53	1220.0
5	4.73	0.394	526½	55.1	3040	0.327	392.0	3.48	1364.0
6	4.78	0.398	529	55.4	3070	0.327	400.0	3.47	1388.0
7	4.83	0.403	532	55.7	3105	0.327	410.0	3.46	1418.0

TABLE 6.—SUMMARY OF MOMENTS

No.	Comp'g Spring	Valve Weight	Main Springs	Total	Cent' Force
1	32	15	622	669	676
2	59	16	776	851	859
3	89	16	939	1044	1058
4	115	16	1083	1214	1220
5	143	16	1196	1355	1364
6	145	16	1223	1384	1388
7	148	16	1250	1414	1418

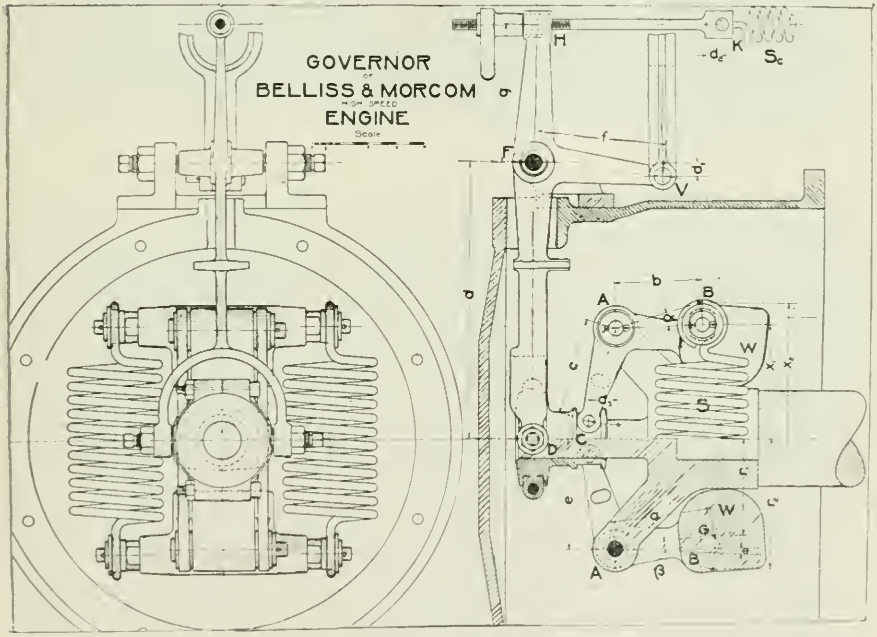
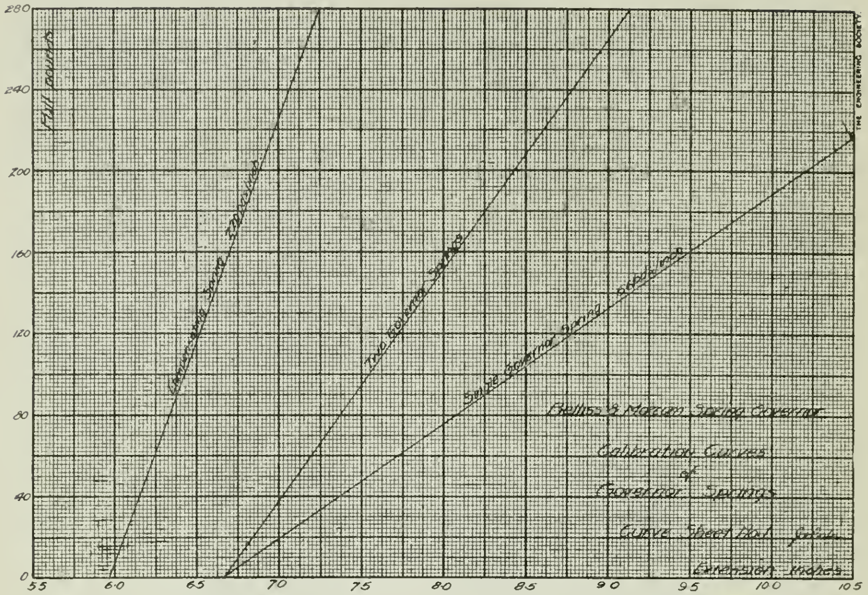
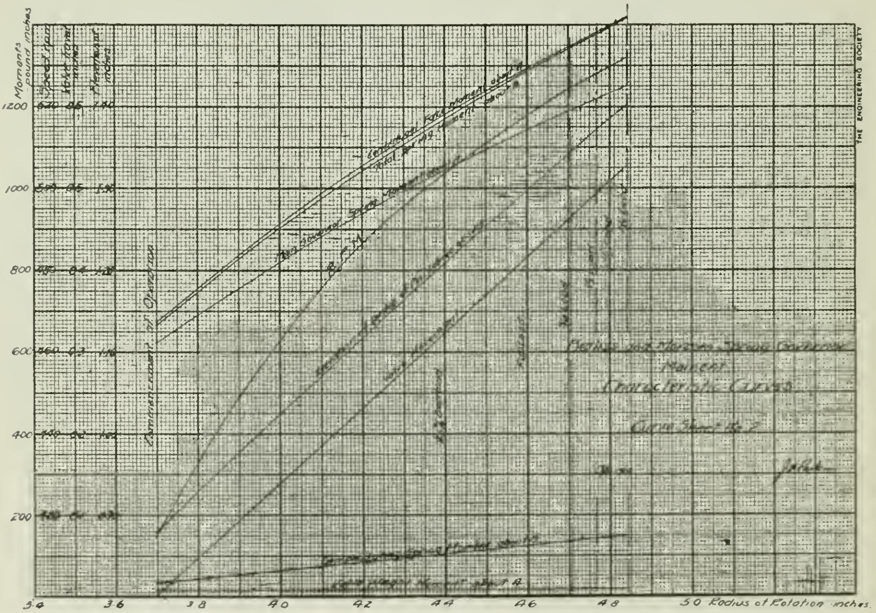
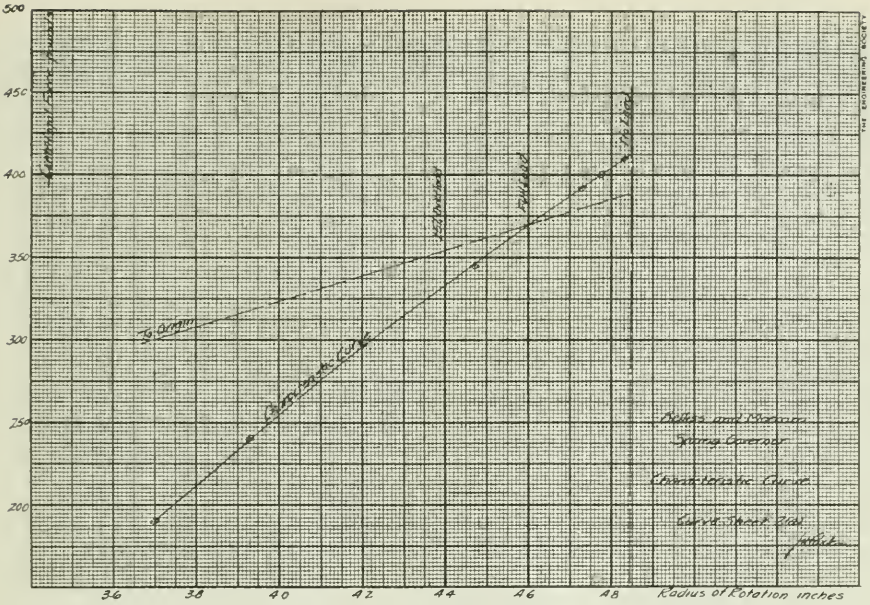
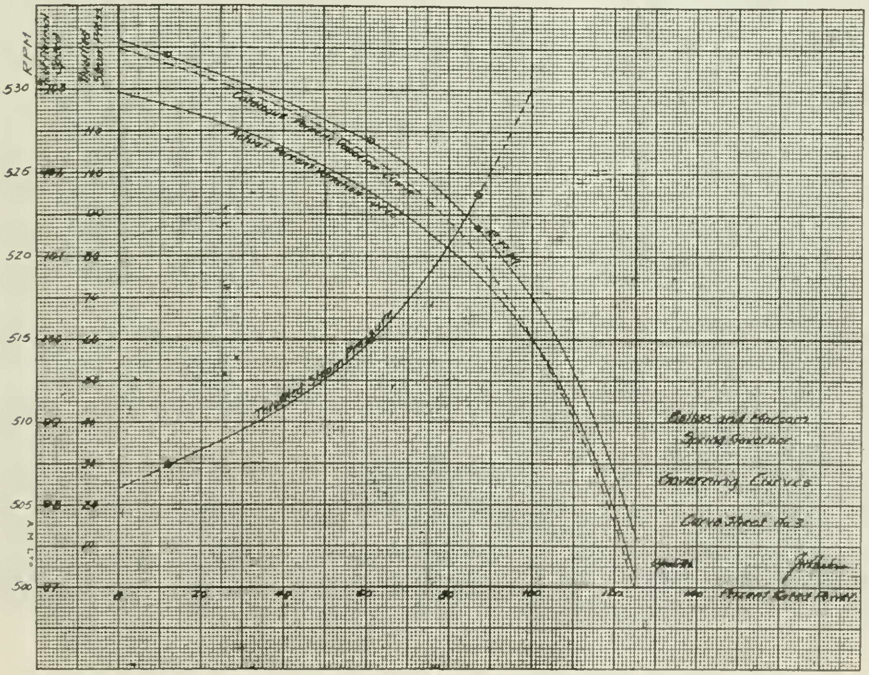


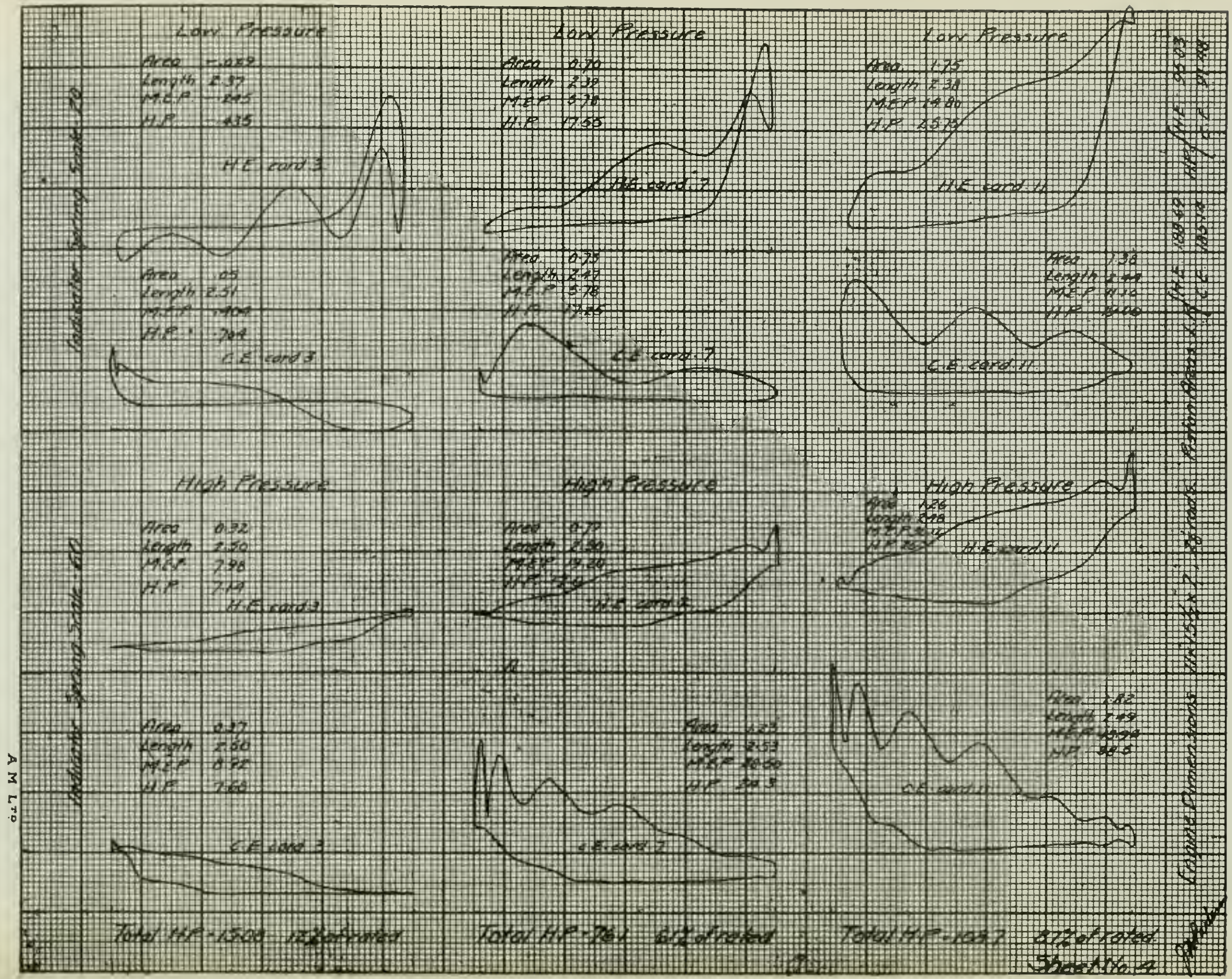
FIG. 1.

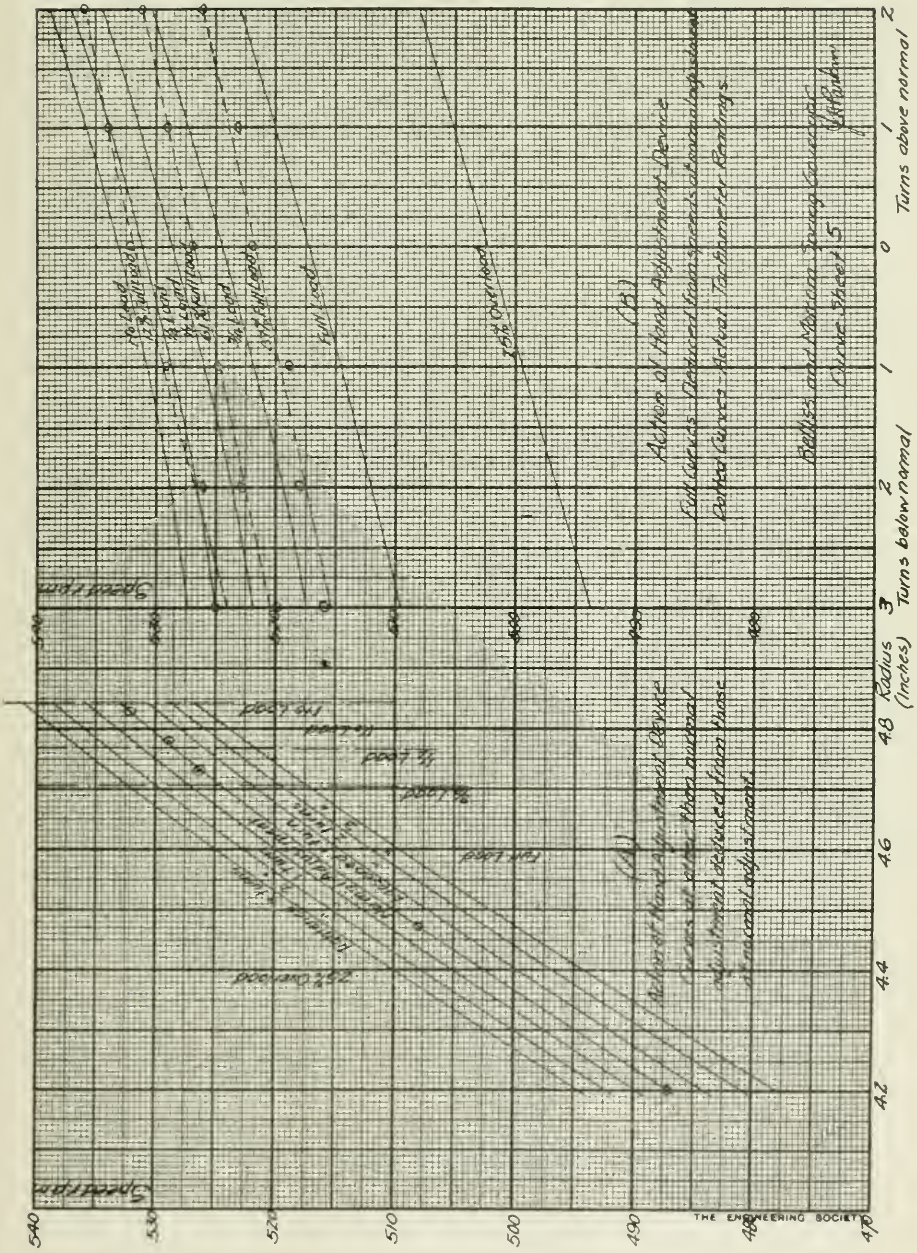

Pull pounds
280
240
200
160
120
80
40
0
55
6·0
65
70
75
80
85
9·0
95
10·0
10·5
Compensating Spring Extension
The Governor Spring
Single Governor Spring
Belliss & Morcom Spring Governor
Calibration Curves
of
Governor Springs
Curve Sheet No. 1
Extension inches
THE ENGINEERING SOCIETY


Moments pound inches
Speed r.p.m.
Valve travel inches
Movement inches
1200
1000
800
600
400
200
3·4
3·6
3·8
4·0
4·2
4·4
4·6
4·8
5·0 Radius of Rotation inches.
Commencement of Operation
Centrifugal Force Moment
Total Spring Moment
Belliss and Morcom Spring Governor
Moment
Characteristic Curves
Curve Sheet No. 2
THE ENGINEERING SOCIETY


500
450
400
350
300
250
200
Centrifugal Force Pounds
To Origin
Characteristic Curve
Full Load
No Load
36
38
40
42
44
46
48
Radius of Rotation inches
Bellis and Morcom
Spring Governor
Characteristic Curve
Curve Sheet No. 2
THE ENGINEERING SOCIETY


RPM
Per Cent Normal Speed
Throttled Steam Pressure
530
525
520
515
510
505
500
Calculated Percent Regulation Curve
Actual Percent Regulation Curve
R.P.M.
Throttled Steam Pressure
0
20
40
60
80
100
120
140
Percent Rated Power
Bellis and Morcom
Spring Governor
Governing Curves
Curve Sheet No. 3


Low Pressure
Area -.039
Length 2.37
M.E.P. .245
H.P. .435
H.E. card 3
Area .05
Length 2.51
M.E.P. .404
H.P. .704
C.E. card 3
Low Pressure
Area 0.70
Length 2.39
M.E.P. 5.76
H.P. 17.55
H.E. card 7
Area 0.75
Length 2.47
M.E.P. 5.78
H.P. 17.25
C.E. card 7
Low Pressure
Area 1.75
Length 2.38
M.E.P. 14.80
H.P. 45.75
H.E. card 11
Area 1.38
C.E. card 11
High Pressure
Area 0.32
Length 2.50
M.E.P. 7.98
H.P. 7.14
H.E. card 3
Area 0.37
Length 2.60
M.E.P. 8.72
H.P. 7.68
C.E. card 3
High Pressure
Area 0.77
Length 2.90
M.E.P. 14.20
H.E. card 7
Area 1.23
Length 2.53
M.E.P. 26.50
H.P. 24.5
C.E. card 7
High Pressure
Area 1.26
H.E. card 11
Area 1.82
Length 2.49
H.P. 38.5
C.E. card 11
Indicator Spring Scale 20
Indicator Spring Scale 60
Total H.P. 15.08 12% of rated
Total H.P. 76.1 61% of rated
Total H.P. 103.7 87% of rated
Sheet No. 4
A M Ltd


Speed rpm
540
530
520
510
500
490
480
470
Radius (Inches)
4.2
4.4
4.6
4.8
Turns below normal
3
2
1
0
Turns above normal
No Load
Full Load
25% Overload
(A)
Action of Hand Adjustment Device
(B)
Action of Hand Adjustment Device
Full curves Derived from speeds at normal adjustment
Dotted curves Actual Tachometer Readings
Bellis and Morcom Spring Governor
Curve Sheet 5
THE ENGINEERING SOCIETY

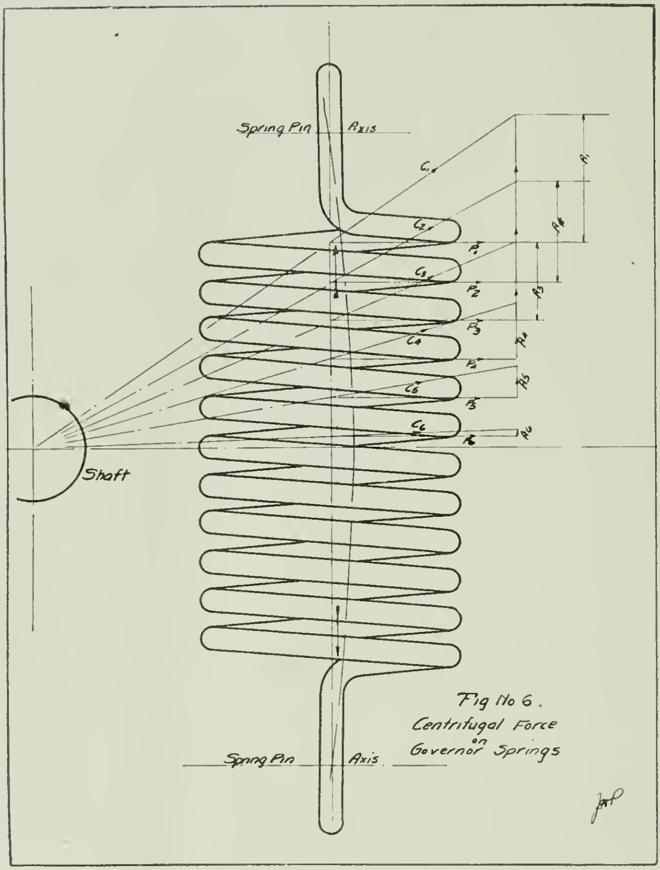


Fig No 6.
Centrifugal Force on Governor Springs

THE IMHOFF SEWAGE TANK IN AMERICA: A DECADE OF OPERATION REVIEWED.

Peter Gillespie,
Professor of Civil Engineering.

Introduction.

Sewage sludge which has been adequately rotted is inoffensive in appearance and odour, usually black in colour and possesses the property of parting with its water readily and of drying quickly on properly prepared and underdrained sand beds. There are two preferred methods of producing sludge of this character. The one, represented by the plants at Birmingham, England, and Baltimore, Md., employs what are known as tanks for separate digestion. By this is meant that sedimentation of raw sewage proceeds in one series of tanks, while the sludge rotting processes are carried on in another series, the sludge having been drained or pumped periodically from the former to the latter. The other method of bringing about the digestion of sludge is by the use of the so-called two-story Emscher or Imhoff tank. In this tank the upper chamber called the sedimentation chamber, communicates with the lower or sludge digestion chamber by narrow slots only, through which settling solids may readily pass downwards. The flow of sewage passing through the upper chamber does not enter the lower chamber at all and conversely, no fluid in the lower chamber has a chance to enter the upper. These statements while correct for all ordinary purposes, are of course, not absolutely so. Because of the conditions indicated it will be apparent that a sewage reaching the tank in a fresh condition leaves it practically in the same condition since the decaying masses in the lower chamber are not permitted to contaminate the contents of the room above.

The first attempt to give to sludge separate tank treatment was made in 1890 by Mr. Dibdin at that time chemist to the London County Council. A quantity of sludge was placed in an old up-ended steam boiler 30 feet high. This was permitted to remain there until vigorous fermentation had taken place. This fermentation caused the solids to float and was accompanied by vigorous foaming and the evolution of most disagreeable odours.

Mr. H. W. Clark, Chief Chemist to the Massachusetts State Board of Health at Lawrence, Mass., in 1899 constructed what is probably the first two-story sewage sedimentation tank. The upper or sedimentation chamber was operated on the fill-and-draw principle, the lower being the receptacle for the sedimented solids. Sludge rotting proceeded in regular fashion in this lower room and it was known that much of the organic matter therein was converted into liquid and gas. This chamber having only a limited capacity, became filled in the course of a short time and the flow of sewage was discontinued. The contents, however, were retained for several months during which the process of rotting and reduction in volume proceeded in the regular manner.

In 1903 Dr. W. Owen Travis of Hampton, England, constructed his first hydrolytic sewage tank. This was a two-story tank, through the upper chamber of which sewage was made to flow slowly and in which sedimentation took place. Through the lower chamber, primarily designed to be the sludge digestion chamber, a small portion of the sewage was made to flow so that the effluent from the tank to some extent was contaminated by contact with the decaying matter in the lower room. A number of these hydrolytic tanks were subsequently constructed by municipalities in England.

In 1905 a tank somewhat after the pattern of the Travis Tank was constructed by the Engineers of the Emscher Drainage Board in the province of Westphalia, Germany, after an examination of the Hampton model. The Emscher or Imhoff tank however differed in one important respect from its prototype in that while there were two stories in both tanks, in the former none of the flowing through sewage was purposely passed through the lower chamber as was done in the Hampton tank. The Emscher tank was found to be a very satisfactory appliance for producing a well-rotted inoffensive quick-drying sludge and thus became a favourite with the Engineers of the Emscher Drainage Board. By 1910-11, it had been introduced into America and since then, but especially during the five or six years following that date, many municipalities in the United States and Canada have constructed this type of tank for the treatment of their sewage. The reception of the tank on this continent was to a great extent due to the reports of American engineers who had observed its operation in Europe and in some cases who had been employed by the Emschergenossenschaft, the Board which had been created by the Prussian Government to administer in all its phases, the drainage of the Emscher Valley. Experience has shown that the European designs were not altogether suitable on this side of the Atlantic, that in some cases, trouble was met with and that care and vigilant attention are necessary if the best results are to be attained. And while it was recognized that in itself it did not constitute a complete solution for the perplexing sewage disposal problem it was nevertheless believed that if reasonable care were taken in its design, and pains and intelligence in its operation, it did offer a solution for the *bête noire* of this ancient problem, the disposal of sludge. In America it has now had a decade of trial and it is the purpose of this paper to review briefly that period of operation and to estimate in part the success and failure that have accompanied it. To this end there is given a brief but it is believed, reliable history of a number of Imhoff installations in the United States and Canada. Some of the information contained therein has been obtained from the Reports of municipal officials, some of it from articles in the Engineering journals, some of it from correspondence with engineers in charge of plants and much of it from personal observation.

AMERICAN EXPERIENCE.

AKRON, OHIO.

The plant at Akron, Ohio, was completed in 1916 and serves a city of over 90,000 people. During the summer of 1919, considerable trouble from foaming was experienced. The remedy adopted was spraying with a hose under high pressure. This was not entirely effective but assisted materially. During 1920 foaming did not occasion any trouble. It should be remembered in this connection that the plant is being operated with a 100% overload, a circumstance which affects any conclusion that might otherwise be drawn.

The sludge has been free from odour and dries readily.

ALBANY, N.Y.

This plant was designed to serve a population of 110,000 and was put in service in August 1919. Two weeks later marked ebullition of gas at the vents developed. In October of the same year the foam broke over the walls of the gas vents and spread over the surface of the sedimentation chambers. This was especially marked in the downstream end of the tanks. Following this, the flow was reversed and

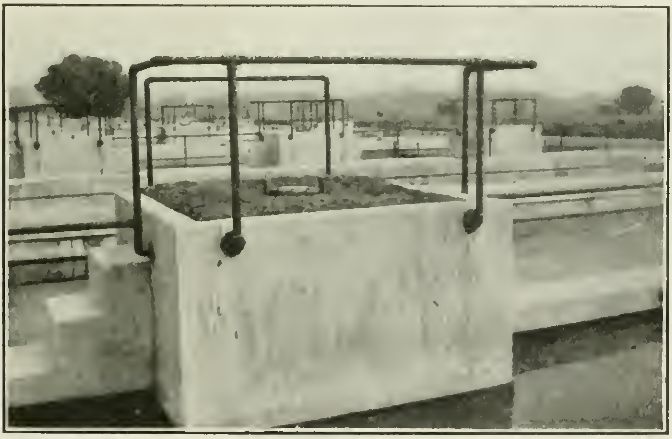


Fig. 1.—Scum Formation in Gas Vents. Albany, N.Y., 1920.

sludge drawn after which the trouble subsided. Three weeks subsequently foam again appeared, once more at the down-stream end of the tanks. The same remedy was again applied and with the same results. Since then little trouble has been encountered. The sludge is normal in colour and odour and contains about 83% of water. Only on muggy summer nights is the odour here objectionable.

ALBIA, IOWA.

The plant at Albia, Iowa, was built nine years ago, the contributing population being 1,500.

About two years after the plant was completed it went through a period of foaming but after the top scum had been removed the trouble ceased and has not occurred since. This plant probably suffered from insufficient care.

ANAHEIM, CAL.

Anaheim, Cal. completed in 1912 a plant designed for a population of 4,500.

Foaming is reported as being troublesome on occasions, perhaps on the average, a week each year. The remedy consists in spraying with a pressure hose which gives temporary relief only. The trouble always disappears however in the course of a few days. On one occasion it was attributed to the presence in the sewage of large quantities of transmission oil from a garage.

The sludge is almost entirely free from odour and dries quickly. In the dried condition, it is sold at 2 cents per cubic foot for fertilizing purposes.

ATLANTA, GA.

Atlanta, Ga. was the first large city in America to adopt the Imhoff tank for the preliminary treatment of its sewage, its three plants at Proctor Creek, Peachtree Creek and Intrenchment Creek having been completed in 1912, 1913 and 1914 respectively. Each installation comprises bar screens, grit chambers, Imhoff tanks, and trickling filters. The Imhoff tanks are 26 feet deep and have longitudinal flow. Each sedimentation chamber overlies three circular sludge wells and in order to equalize the deposits in these wells, crossover channels for reversing the flow have been provided. The mean flowing through time is three hours; the capacity of the sludge wells is 5 to 6 months' accumulation; the mean velocity in the sedimentation chamber is .55 feet per minute. After the Peachtree Creek plant had been in operation for about two years, some of the ventilators were hooded over and the gas collected and stored in a floating steel receiver. Since then the gas has been used for heating a dwelling on the premises, for illuminating the plant instead of electric light and for general laboratory use. It is a mixture of methane or marsh gas, carbon dioxide, nitrogen and hydrogen with the first mentioned predominating. In the early days of the Imhoff tank in America, no installation attracted as much attention as that at Atlanta and many visits of inspection were made thereto by municipal officials from all parts of the continent.

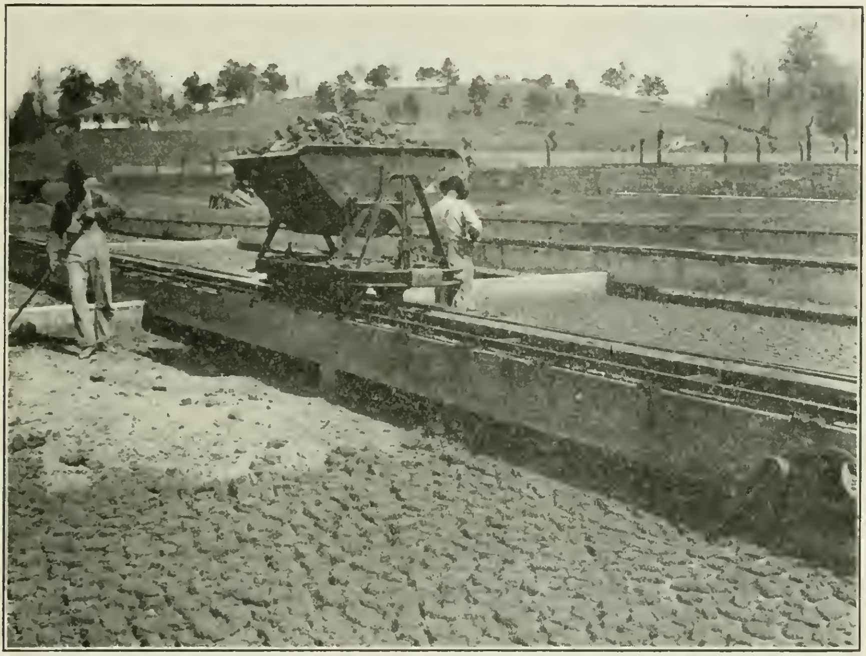


Fig. 2.—Sludge Drying Beds. Atlanta, Ga., 1921.

It has been observed at Atlanta that the accumulation of sludge is very much greater in the first sludge well than in the second. In the third in turn it is much less than in the second. By reversing the flow at regular intervals, the sludge level can be kept fairly uniform in the two end wells but not in the centre one. As a result the end wells are overtaxed, or what is the same thing, the storage period is reduced below what it would have been had there been uniform distribution of sludge in the three chambers. Generally the sludge has been of excellent quality both as to odour and texture and under favourable atmospheric conditions, dries in from 7 to 9 days. Gas house tar at Proctor Creek has been known to inhibit bacterial action on several occasions.

Foaming troubles have been encountered at the Atlanta plants. These first manifested themselves during the ripening period when a tenacious foam appeared at the gas vents in large quantities and in time overflowed into the settling chambers. Spraying with a pressure hose gave temporary relief but the foam re-appeared after the spraying had been stopped. It happened that the Intrenchment Creek plant was undergoing construction at the time that the Proctor Creek plant was in its most turbulent state. With a view to preventing in the new plant the troubles that had developed in the earlier installation, the gas vents in the former were materially increased in area. This effect was not what had been looked for and during the ripening period the Intrenchment Creek plant was very troublesome. Mr. Hommon, Chemist and Bacteriologist, believes that the small gas vents are preferable to large ones since the concentration of ebullition in the former and its resultant mechanical agitation have a tendency to break up the froth.

The remedies resorted to are to rest the troublesome units for a period of time and to draw off sludge. The operation of these plants was sorely neglected during the war and they have not entirely recovered yet from that neglect. Mr. Hommon believes that no troubles arise at Atlanta or elsewhere which cannot be made to yield to careful and intelligent treatment. That the civic authorities are evidently well satisfied is shown by the following statement issued from the office of the Chief of Construction in 1919, seven years after the first plant was put into operation:

"All the plants have given excellent results and perfect satisfaction and it is intended that when funds can be obtained, three additional plants be constructed on the same design, one at Utoy Creek, one below Lakewood Park at South River and one to take care of the Druid Hill sewage at Peachtree Creek".

AVON, N.Y.

This plant, built in 1913 for a population of 2,500, has given no trouble through foaming at gas vents. Sludge is reported as not being free from offensive odour, though drying quite readily in summer. The formation of a surface scum, a common trouble in many of the smaller plants, was reported.

BALTIMORE, MD.

Prior to 1915, the sewage treatment plant at Baltimore consisted of plain sedimentation tanks, separate sludge digestion tanks, sprinkling filters and final settling tanks. In the year mentioned, this was supplemented by the completion of a series of 28 radial flow Inhoff tanks each intended to serve 4,000 people and of a nominal capacity of 500,000 gallons daily. These tanks were started at a very low rate and were slow in reaching the ripened stage. When this condition had been attained the quantity of sewage was increased which resulted in excessive foaming and scum in the gas vents. It seemed at times as though the entire sludge content of the digestion chamber had floated to the surface and attempts to draw sludge from the hoppers were quite futile because no sludge remained at the bottom. The only effective remedy seemed to be to take the offending tanks out of service for a period varying from 3 days to 3 weeks. It was finally discovered that if the flow were halved (250,000 gals. instead of 500,000 gals. per day) the trouble mainly disappeared. It may be stated that the sludge room capacity was less than 1 cubic foot per person based on the originally assumed contributing population. The gas vents were also considered entirely too small. These circumstances and the fact that the sewage on arrival at the works was quite septic have been considered as causing the trouble experienced at Baltimore.

Of the 28 tanks constructed, some 9 have since been slightly modified by raising the walls and otherwise improving the hydraulic conditions. This together with their operation at half nominal capacity has resulted in the production of a good sludge and normal behaviour generally. The management is of the opinion that the cost of operation of Imhoff tanks at Baltimore is greater per volume of sewage treated than it is for a combination of plain sedimentation tank and separate digestion tank giving a final sludge of practically the same quality.

BEXLEY, OHIO.

This plant was completed seven years ago to serve a community of 4,000. No trouble from foaming was reported until the summer of 1920. The sludge had been entirely removed some three months previously and it was thought that the foaming which occurred after this was due to the fact that the biological processes had to be started from the beginning again. No remedy was applied and the trouble soon ceased.

The sludge is free from odour but the drying has not been as rapid as desired. This is attributed to the condition of the filtering material in the bed. Generous sludge drying area, a sludge room that is easily accessible and a good hydrostatic head on the sludge outlet are recommended.

BOCA GRANDE, FLORIDA.

This is a plant which for eight years has served the small town of Boca Grande. No trouble of any kind has been reported.

CHAMBERSBURG, PA.

This plant was constructed in 1911 and was designed to serve a population of 20,000. Very little foaming has occurred. Sludge dries very nicely in from 8 to 10 days and normally has no appreciable odour. Very complete satisfaction with the plant generally is expressed by the engineer.

CHICO, CAL.

The Imhoff tank at Chico, Cal. has served a community of 6,000 people since 1914.

It is reported that scum forms quite heavily at the gas vents but apparently no effort is made to keep the gas vents clear. The odour from the tank during the summer time is said to be perceptible at a considerable distance. Apparently careful supervision and the keeping of accurate records are not attempted.

CLEBURNE, TEXAS.

No trouble has been experienced in this plant either with foam or sludge that has not been remedied by care on the part of the operator.

COLUMBUS, OHIO.

During the years, 1915-16 the city of Columbus, Ohio, changed the septic tanks, until then in use by that city, into Imhoff tanks. The new system was put in operation during 1917 and foaming began one month subsequently. It had previously been observed that during warm weather, it was possible to operate septic tanks advantageously from four to six weeks only after which no real improvement in clarification took place. The tank effluent from then on was black and the active ebullition of gas resulted in large volumes of sludge being lifted to the surface.

In subsequent operation of the new two-story tanks, foaming appeared in from four to six weeks after starting. This led to the belief that the causes in the two cases were similar if not identical. Nevertheless, the effluent from the Imhoff tanks was generally superior to that from the septic tanks that preceded them.

The foaming trouble was remedied by drawing off sludge whenever this was possible, since there is a dearth of sludge drying area at the

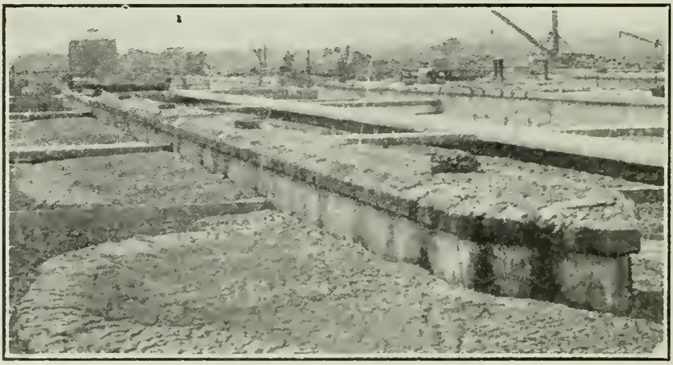


Fig. 3.—Foaming at Gas Vents.

Columbus plant. When this is done foaming ceases. The sludge has a normal appearance and dries well.

The trouble here has been attributed to overloading the tanks and to the scarcity of sludge drying facilities. The capacity of the sludge drying space is about adequate for a long dry season and if, due to rains or other cause, the removal of wet or dry sludge is interfered with, the successful operation of the tanks is very difficult. With this corrected, it is believed the trouble can be controlled.

FAIRFIELD, IOWA.

Fairfield, Iowa, is a town of 2,000 inhabitants. The existing sweage treatment plant was completed nine years ago. No trouble from foaming has occurred. The sludge is free from offensive odours and no trouble has been experienced in drying it.

FALLON, NEVADA.

This plant was completed seven years ago to serve a population of 2,000.

Crust is reported as forming in both the sedimentation chamber and the gas vents. This has been attributed to the flowing of liquor from one sedimentation compartment through the sludge room to the other sedimentation compartment, the construction being such as to render this possible. At no time has foaming been serious enough to require any special attention.

FITCHBURG, MASS.

Fitchburg, Mass. is a city of about 65,000 people. In 1914 a five-tank Imhoff plant was constructed. During the first two years' operation there was no serious foaming in any of the tanks. One unit, however, gave a little trouble but this was temporarily stopped by wetting the surface of the gas vents with a hose attached to a hydrant and by drawing some of the sludge. The foaming in the instance referred to was attributed to the fact that the sludge well had become entirely filled. When sludge was withdrawn the trouble disappeared. During the summer of 1919, foaming caused some trouble in two of the tanks. Investigation showed that there was in the sludge room of these two tanks an accumulation of solid matter that the air lift refused to move, and which practically filled the hopper shaped bottoms of the sludge wells. It also developed that such sludge as could be drawn was not well digested which circumstance was attributed to the restricted capacity resulting from the accumulation just referred to. After cleaning out this mass and seeding the sludge rooms with ripe sludge from another unit, normal action was restored.

The surface of the water in the sedimentation tanks is skimmed weekly or oftener to remove grease, etc., that refuses to settle. The sloping bottoms of the upper chamber and the slots are also cleaned weekly.

Secondary settling tank sludge is also treated in the Imhoff tanks. When it becomes necessary to pump this back to the primary tanks, the water level in the latter is lowered, the sewage is by-passed and the secondary tank sludge is pumped directly into the Imhoff tanks.

The sludge is described as possessing a tarry odour and showing an alkaline reaction to lacmoid. Its water content varies from 92 to 80%. It is reported as being normally inoffensive and at no time has it caused a nuisance. The minimum period for drying is 15 days. Sludge levels in the lower story of the Imhoff tank are determined by the use of a pitcher pump with a graduated suction hose which is gradually lowered into the sludge room while the operation of pumping proceeds.

LAKEWOOD, OHIO.

This plant was constructed in 1916 to serve a population of about 15,000.

No difficulty has been experienced because of foaming. The plant has a much greater capacity than the present demands upon it, the sludge is agitated weekly by water jets through a system of piping in the bottom of the sludge hopper and the scum on the gas vents is broken up and diluted every second day. To these details of operation the absence of trouble is attributed.

The sludge produced is almost absolutely free from odour and dries quite readily on open beds. Sludge is drawn twice a year, in April and October.

LEBANON, PA.

Lebanon, Pa. completed the first unit in this plant nine years ago. It was then designed for a population of 6,000. Foaming in one of the units was reported early last spring and late last fall but at no time did the foam rise higher than two feet above the surface of the water. The trouble was remedied by skimming off all material in the gas vents and by drawing sludge frequently in small quantities.

The sludge produced has been absolutely free from offensive odours at all times. The plant as a whole has given very good satisfaction.

MADISON-CHATHAM, N.J.

This plant, the first Imhoff installation in America, was built in 1911 to serve a community of 7,000 people. The tanks were at first provided with reinforced concrete covers but these prevented the removal of scum and the cleaning of sloping bottoms and slots, and in 1913 were removed. Since this change very little trouble with foaming has occurred although a great deal of difficulty because of it had been experienced previously. The present practice is to apply milk of lime at the gas vents and to draw small quantities of sludge frequently. This apparently prevents the recurrence of the trouble. No offensive odours are noticeable at about 50 feet from the plant.

The sludge produced during the past seven years has been of excellent quality possessing all the characteristics of good Imhoff sludge as to colour, texture and odour. It dries to a condition permitting handling in from three to seven days. Farmers use it to some extent as a fertilizer.

The engineer believes that intelligent and faithful operation are required to obtain satisfactory results in any Imhoff installation.

MARYSVILLE, OHIO.

This plant was designed for a population of 6,000 people and was completed seven years ago.

No trouble from foaming has been experienced. The sludge is free from offensive odour and dries easily.

PENNYPACK CREEK, PHILADELPHIA, PA.

This plant, said to be the second commercial installation of the Imhoff tank in America, was completed in 1912. Although it was designed for a flow of 2,000,000 gallons per day and a population of 10,000 people, it serves a community to-day of only 3,500 comprising the village of Holmesburg and a number of penal and eleemosynary institutions located in the district. One quarter of the sewage is domestic; three quarters are institutional. The sewage is very fresh on arrival at the treatment plant and contains an unusual quantity of bread, rags and fat. To the presence of the latter is attributed the tendency of the sludge to become acid.

There are two circular radial flow Imhoff tanks operated in parallel. They are each 30 feet in diameter and 32 feet 6 inches deep. The flowing through time is about two hours.

Prior to 1917, scum appearing in the settling compartment was removed and buried. Since then it is settled with a spray from a pressure hose, this being used for a 5-minute period daily in hot weather. Perforated lead pipes with outside connections were laid in the sludge rooms at the time of construction. To dislodge sludge that adheres to the bottom slopes of the sludge room and to liberate entrained gas, water under pressure is forced into these pipes. These have proved themselves very useful for this purpose. Since 1914, lime water is added to the digestion chamber to prevent the development of acidity in the sludge. This has had the desired effect, producing a good alkaline sludge where before an acid sludge was obtained. The drying period varies from 15 to 40 days; the water content is from 75 to 87 per cent.

PLAINFIELD, N.J.

The plant at Plainfield, N.J., serves a population of about 40,000 people in the three municipalities of Plainfield, North Plainfield and Dunellin.

Each tank is constructed with five sludge pockets in series and provision is made for reversing the direction of flow periodically. It was found, however, that the bulk of the sludge was deposited in pockets 1 and 5 and comparatively little in the others. To remedy this in part, only one-third of the tanks were kept in service at a time. This increased the flowing through velocity three-fold and produced something like an equality in the distribution of sludge in the five sludge pockets. Another change was the installation of Riensch-Wurl fine screens preliminary to the Imhoff tanks. The result has been that foaming at the gas vents which had been very troublesome before the changes just referred to were made, has given very little trouble since.

A feature of the performance of the secondary sedimentation tanks has been their persistent foaming. Little sludge appears to settle in them. For the most part, it rises to the surface as scum.

The sludge from the Imhoff tanks is run on to sand beds to a depth of nine inches, dries readily in a week or ten days, and is said to be of the very best quality.

ROCHESTER, N.Y.

The city of Rochester, N.Y. possesses in the Irondequoit plant the largest Imhoff tank installation in the United States. A smaller plant known as the Brighten plant has Imhoff tanks also.

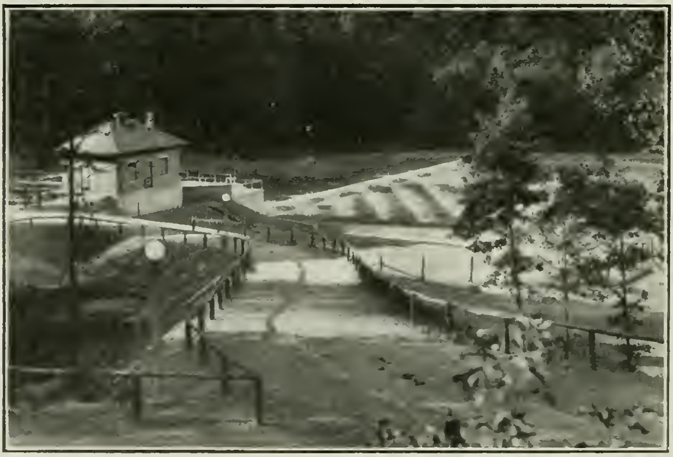


Fig. 4.—Brighton Plant, Rochester, N.Y.

The main or Irondequoit plant consists of five double units, one half of each double unit being designed for the sewage of 20,000 people. The Imhoff settling chamber for this 20,000 unit consists of two parallel channels each 10 ft. wide by 110 ft. long in which the detention period is about one hour.

The sludge space below a plane lying three feet lower than the slot and not including the hopper shaped bottoms, is equivalent to two cubic feet per head of tributary population. This has been found to be adequate. The sludge drying area is one-third of a square foot per person.

The tanks are preceded by Riensch-Wurl screens and grit chambers.

At the Brighton plant the flowing-through period is three hours and the sludge storage capacity is 1.76 cubic feet per person. The tanks at both plants are 35 feet deep. This was easily secured during construction as ground water was not encountered at that depth. At both plants, the effluent is used to operate water turbines for the generation of electric current. Both plants are working excellently and no troubles have been encountered.

The excellent quality of the sludge obtained at both plants is attributed partially to the unusual depth of the tanks as stated above. This depth of water is equivalent to one atmosphere.

The plants are most carefully maintained and many evidences of intelligent operation are apparent.

SCHENECTEDY, N.Y.

The plant at Schenectady, N.Y., has been in operation since the month of January, 1915. Because of certain difficulties encountered during construction, it was decided to lessen the depth of tank from 21 ft. 4 in. as originally planned, to 13 ft. 5 in.

The first sludge was drawn in June of 1915 and was found to be brown in colour, and very offensive. As a corrective, lime was added, and almost immediately large volumes of gas were given off. In July large quantities of foam arose in the gas vents and overflowed the walls into the sedimentation chambers. Sprinkling with water from a pressure hose gave only temporary relief and with the advent of cold water, the trouble ceased.

During the spring of 1916 foaming continued as in the previous year and lime was again used. This apparently stopped the foaming but the sludge was reported as exceedingly offensive. Later in the summer on the advice of Mr. Harrison Eddy, Consulting Engineer, the following changes were made:

Raising the walls of all gas vents one foot by the addition of wooden boxes.

Increasing the size of equalizing openings in transverse partitions between sludge wells.

Increasing the width of the slot in the bottom of the sedimentation chambers.

The troubles were mitigated somewhat by these alterations and on the arrival of cold weather, the foaming again ceased. In 1917 foaming troubles again occurred but the regular removal of sludge from the sludge wells and of scum from the gas vents caused a very material reduction of the nuisance. There was also some foaming during 1918, but this was much less than during the three previous years. Experience seemed to indicate that the use of water from a pressure hose to break down the froth produced ultimately the opposite effect from what was intended, and this practice was discontinued. The regular withdrawal of the scum and sludge finally resulted in a condition of the plant which was quite satisfactory both foaming and obnoxious odours having practically disappeared. It was now generally accepted that the troubles at Schenectady were due to the overloading of the shallow sludge rooms.

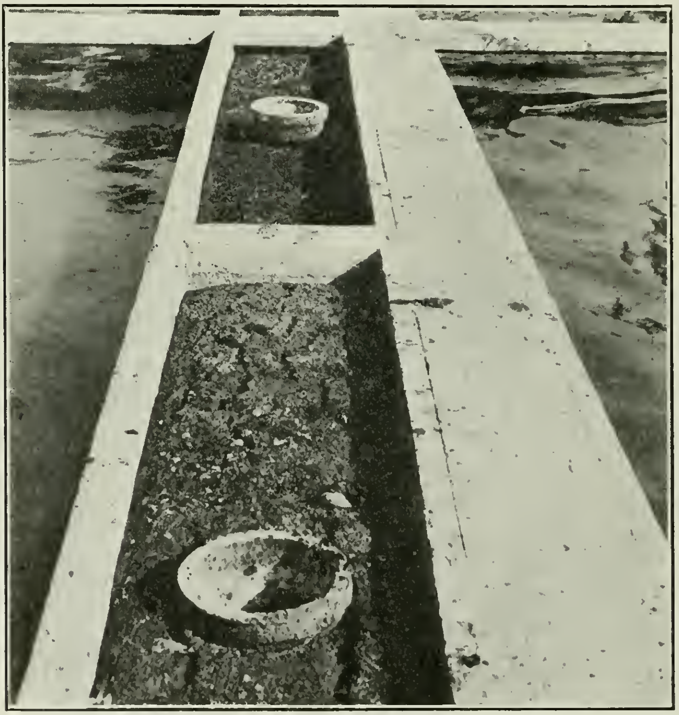


Fig. 5.—Foam and Scum on Gas Vents. Schenectady, N.Y. 1915.

The operation of the tanks during 1919 was more satisfactory than in any previous year. Believing that the quantity of sewage coming to the works was twice as much as the plant could properly assimilate, the management adopted the practice of diverting half the flow into the Mohawk River without treatment. Partly no doubt, as a result of this, foaming was negligible and well-digested inoffensive sludge was drawn at regular intervals of about two weeks in warm weather. The point that seems worthy of emphasis is that there has been a progressive improvement in the behaviour of the plant since 1915 largely the result of experience and of intelligent supervision and operation.

STRATFORD, ONT.

This plant was designed for a population of 17,000 and was completed seven years ago. No trouble of any kind has been experienced. The sludge is drawn at regular intervals.

URBANA, OHIO.

This plant was completed in 1915 and serves a community of 8,000 people. Some trouble with foaming has been experienced especially at times prior to the drawing of sludge when the hoppers are filled to capacity. To remedy this, sewage from the sedimentation chamber is poured over the surface of the foam using a long handled dipper for the purpose. This operation consumes an hour each morning and gives the tanks a clean and tidy appearance for the rest of the day.

It is reported that the sludge has a slight odour for a few days after being discharged on the drying beds but this is said not to be offensive. It is run on to these beds to a depth of 14 ins. and in about four weeks is dry enough to be hauled away.

WESTFIELD, NEW JERSEY.

The Westfield, N.J. plant was put in service in 1913, the contributing population being 7,500.

It is reported that during the last two years, little or no trouble due to foaming has been met with. It is the custom to break the scum in the vents three times a day and at the same time, to saturate it with liquid taken from the sedimentation chamber. This seems to have put an end to the foaming troubles.

The sludge is practically odourless and dries rapidly when the weather is favourable.

MR. KIRCHOFFER'S EXPERIENCE.

The experience of Mr. Kirchoffer, Sanitary and Hydraulic Engineer of Madison, Wis., is interesting. After several unfortunate experiences with the operation of Imhoff tanks which he had designed, he conceived the idea of having the raw sewage first enter the lower chamber. After depositing part of its solids there it was then to pass upward through the slots to the regular outlet. Imhoff tanks for the Southern Wisconsin Home for Feeble-Minded were modified by him in accordance with this idea. He placed a transverse cross-wall in the sludge room dividing it thereby into halves. The raw sewage was admitted to the first of these two compartments, flowed upward through the slots to the upper chamber and thence to the outlet. Any settleable solids brought up by the liquid had thus an opportunity to settle out and slide down into the second compartment of the sludge chamber.

After three months' operation no scum had appeared on the surface of the liquor in the upper story or in the gas vents and the plant is reported as giving good results since the change was made in 1918.

At Ripon and at Geneva, Wis., tanks of similar design were constructed and sludge possessing good drying qualities was produced. It is further stated that the effluents from these tanks as modified can be more readily treated on percolating filters than could the effluents from the same plants operating as they did before the changes were made.

DEFECTS AND REMEDIES THEREFOR.

The objections to the Imhoff tank to which expression has been most frequently given are the formation of scum on the surface and the collection of solids on the sloping bottoms of the sedimentation chamber both of which in time become septic; the choking of the sludge pipe due to partial solidification of sludge; the failure of sludge to move readily toward the sludge pipe when sludge is being withdrawn; the production of sour and offensive sludge and the foaming at gas vents resulting in the spreading of sludge over walls and surface of sedimentation chambers. Some of these troubles can be traced to faulty design; others to lack of careful operation.

Grease and oil must be regularly skimmed from the surface of the sedimentation tank and scum which cannot be settled by sprinkling or other means, from the surface of the gas vents. Solids must be prevented from adhering to the sides and bottom of the sedimentation chamber and slots must be kept open. If slopes be made sufficiently smooth and sufficiently steep these troubles need not occur. Mr. Hammond at Brooklyn found that no solids lodged on a smooth wooden floor inclined at an angle of 42° above the horizon although 50° is generally preferred. He attaches much importance to having an unobstructed passage without change of direction for the settling solids as they move downward. In settling tanks of the V-shape, this necessitates two openings as shown in Fig. 6. In any case the design should be such that the slots and sloping bottoms are readily accessible so that an attendant may be able to clear away obstructions and remove the inevitable floating matters when necessity arises. Obviously the low cover is a mistake. To control possible odours, however, especially in muggy weather, a housing should be provided if the plant be in the vicinity of a built-up district.

If the sludge pipe be back-filled with water after each drawing of sludge and especially if it be arranged so that water under pressure may be forced into it from the upper end, clogging is unlikely to occur and if it does occur it can readily be remedied. Moreover if perforated water pipes be laid in position at the bottom of the sludge well, the movement of the digested sludge to the outlet is much facilitated by the water discharged therefrom. It is a good policy to agitate the sludge in this way once a week. This aids in the prevention of scum by releasing entrapped gases from the sludge. It is now considered inadvisable to have three or four sludge wells underneath a single sedimentation chamber because of the difficulty of securing an even distribution of sludge in the various hoppers. Present tendencies favour a sludge room capacity of two cubic feet per person for separate systems and of three cubic feet for combined. It is always better to draw small quantities of sludge at short intervals than to draw a large quantity at one operation since the sludge level will then adjust itself between operations.

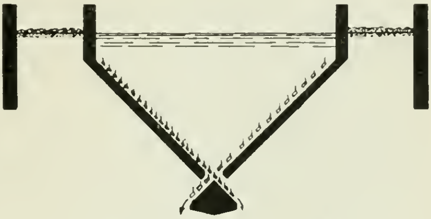


Fig. 6.—Mr. Geo. T. Hammond's Method of Facilitating Removal of Settling Solids.

The best experience favours leaving some ripened sludge in place always to give continuity to the bacterial action. Dr. Imhoff at one time suggested artificial stirring of sludge in order to hasten the ripening. Another method proposed by him to the same end was to pump out the entire body of sludge and afterward return it to its hopper in order to give it a thorough mixing. It is a notorious fact that many Imhoff tanks, which for long periods after being put into operation, produced a sour sludge, finally produced a normal sludge, the only remedy being patient waiting. Milk of lime as a corrective seems to be uncertain in its action.

FOAMING.

Of the thirty Imhoff tanks whose history is here reviewed, twenty have had foaming troubles of a more or less serious character. Of these twenty, eight have obtained relief by sparging the foam with water or by drenching with sewage; seven by drawing sludge; two by adding milk of lime; two by taking the tanks out of service and one, Plainfield, N.J., by the introduction of fine screens. It is interesting in this connection to observe that at the large plant at Rochester, N.Y., which has had no trouble at all, the sewage is prepared for the Imhoff tanks by passing through Riensch-Wurl screens. The causes of foaming as assigned by the superintending engineers, are also various. Four attribute it to overloading the plant; one to gas vents being too small; one to an excessive accumulation of sludge; one to oil from a garage and one to the septic condition of the sewage on arrival at the works. Inoffensive quick-drying sludge is reported as being obtained at twenty-one plants, a circumstance indicating that in spite of defects the Imhoff tank performs this function pretty satisfactorily.

Foaming results when anything prevents the free escape of gas from the liquid in which it is generated. It is usually attributed to the presence of dissolved matters which decrease the surface tension of the liquid and augment its viscosity and thereby reduce the readiness with which the bubbles break. In steam boilers, foaming is known to be due to the presence of sodium and potassium salts in the feed water when these reach a certain concentration and the trouble is intensified when a precipitated sludge is present also. In the operation of brewing, the wort contains certain constituents coming from the malt such as dextrin which with the carbon dioxide evolved in the process of fermentation, produce the head of foam in the vats. Other frothing agents are tannin, urine, egg albumen, saponin, amyl alcohol, soap, etc.

It seems very trite to liken again the behaviour of a foaming Imhoff tank to the modern and very useful process of concentrating valuable ores in the mill by the so-called flotation process. And yet the similarity is very striking. The mineralized body consisting of worthless rock and the valuable ingredient (say a sulphide) is ground by suitable apparatus in the mill and then converted into a pulp by the addition of water and a foam-forming oil followed by thorough mixing. It appears that owing to some cause not entirely understood, the metallic sulphides are readily wetted by oil while the rock or gangue is as easily wetted by water. Both of these phenomena are intensified by the addition of a little acid to the pulp. The pulp is then fed into a cell or tank in which bubbles of air are produced by agitation, pneumatic pressure or other means. The foam or froth which appears on the surface and which constantly overflows the outlet lip, for the operation is continuous, is found to contain the metallic sulphides, while the worthless rock settles to the bottom of the vessel. The oiled sulphides exhibit a marked tendency to adhere to gas films and though usually heavier than the gangue and always heavier than water, they float up with the foam to the surface and are segregated thereby in preparation for further treatment.

If a quantity of cooked blood albumin in a divided granular condition be placed in a glass beaker half filled with clear water, it will normally settle to the bottom of the glass. If it be agitated by air bubbles blown through a glass tube, no foaming will occur because of the absence of a frothing agent in the liquid. If a little soap bark or saponin solution be added to the beaker and the experiment repeated, it will be found that the particles of albumin will be lifted by the foam to which they readily attach themselves, and in a few minutes, the sediment will have been completely carried over the sides of the glass. The froth will gradually disappear after the blowing has ceased. A few drops of alcohol or ether will "cut" the froth instantly.

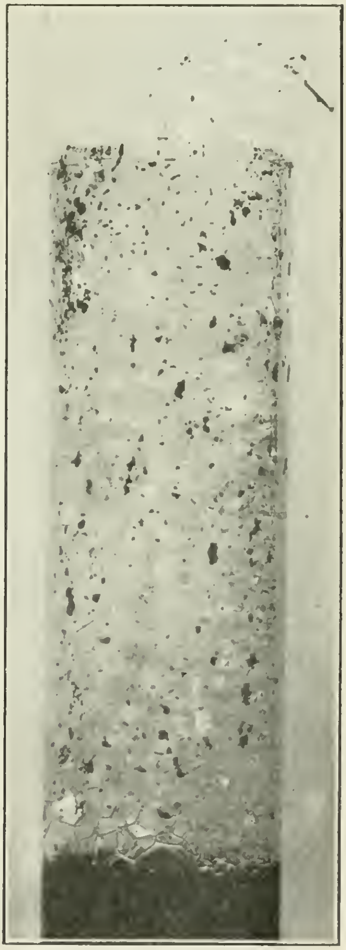


Fig. 7.—Particles of Blood Albumin Lifted by Saponin Froth.

Imhoff tanks have been observed to foam when the sludge is acid and when it is alkaline. They have foamed when sludge is present in excessive quantity and when very little sludge has accumulated. Generally speaking, the trouble has been most pronounced in the summer season when the temperature of both atmosphere and sewage is higher than in winter and when in consequence, the evolution of gas is most active. While the trouble frequently occurs in the early history of tanks during the so-called "ripening period" and with fresh sewage it has also developed after the ripening period has passed and in cases where the sewage on arrival at the works is very septic.

The foaming phenomenon is probably due more to the condition of the liquor above the sewage in the digestion chamber than to any other single circumstance. It should be remembered that this liquor normally remains unchanged except for the insignificant displacement that occurs continuously, due to the incoming deposits from above and for the relatively greater displacement that accompanies the withdrawing of sludge. In the former case, septic liquor equal to the volume of the deposits, passes from the lower to the upper chamber; in the latter, a quantity of fresh sewage liquor equal to the volume of the withdrawn sludge passes through the slots from the upper to the lower story. Presumably this may have an effect on the liquor above the sludge in the lower well somewhat analogous to dilution. This septic liquor having been acted upon by micro-organisms, the substances held in solution and in suspension, have been greatly altered and it would appear that as a result there is an increase of those substances which particularly favour the formation of froth. The evolution of gas through the normal processes of decay and the concentration of this at the more or less restricted gas vents would seem to be about the only other essentials for a troublesome frothing condition.

Ammonia is produced by the partial decomposition under anaerobic conditions, such as obtain in sewage tanks, of such substances as urea, albumen, fibrin, etc. Alkalinity in sludge liquor is due to ammonia, lime, soda, or soap. In a bacterial tank, the greases are emulsified or saponified by the ammonia or other alkalis present and it is indeed conceivable that the fatty content of fecal matter may be the better prepared for these changes by the action of digestive juices during its passage through the alimentary canal. It is a matter of common knowledge that when air is blown into a vessel containing an emulsion of a fat such as corn oil, oleic acid, lard oil, or whale oil with either ammonia or ammonium carbonate, a very persistent soapy froth is produced. In some instances the union of the fatty acid and the ammonia is aided by a solvent such as methyl alcohol, following which froth appears on aeration. Stearic acid is one of these. On the other hand, carbolic acid or rosin oil are said to produce good froths on aeration without the aid of other reagents. In the processes of mineral flotation, the use of acids is rather common and it has been observed that the froth from pine oil or creosote oil is substantially improved by the addition of a little hydrochloric acid. And just as the metallurgist has found that the tenacity of his froth is improved by the presence of a finely divided sulphide in the flotation cell, so it has been observed that brittle froths become persistent if sludge be present in the liquor. A selective tendency similar to what obtains in the flotation of minerals has also been recognized. It appears as though not all of the constituents of the sludge are capable of attaching themselves to the froth and of being lifted thereby. The residuum is therefore analogous to the gangue which remains at the bottom of the flotation cell.

In the majority of the cases previously considered, where foaming troubles have occurred in Imhoff tanks and where a remedy has been sought, either the drawing of sludge or the pouring of sewage or water into the gas vents has afforded some measure of relief. If sludge be drawn, it follows that an equivalent volume of fresh sewage is admitted to the sludge digesting chamber. In addition to this it is probably true that the activity of the escaping gas is thereafter diminishing owing to the lessened quantity of organic matter undergoing decomposition. If either fresh sewage or water be poured into the gas vents, an equivalent volume of septic liquor will pass upwards from the sludge room into the sedimentation chamber by way of the slots. The effect of either of the two favoured remedies is therefore to dilute the septic liquor lying above the sewage in the sludge room and doubtless also to diminish its froth-forming properties. This has suggested a modification in the design of present day Imhoff tanks whereby the foaming difficulties may either be prevented or corrected when they do occur. If the tank be constructed so that at such times as the necessity arises, the septic liquor above the sludge can be replaced more or less completely by fresh sewage or by water, there is, it seems to the writer, very good reason for supposing that the foaming difficulty could be controlled. This is to some extent the scheme that was proposed and actually carried out by Mr. Krichoffer at Ripon and Geneva and elsewhere in the State of Wisconsin. Here the entire inflow is discharged into the lower chamber, the liquor, after partial sedimentation, reaching the upper chamber by way of the slots. The consequence is that the effluent, because of its contact with the decomposing sewage, is always septic, and therefore not well adapted, according to present knowledge, for subsequent treatment on filters or otherwise. A method that would pass the fresh sewage into the sludge room only when foaming troubles had to be corrected would seem to possess some advantage over that of Mr. Kirchoffer since it would insure a non-septic effluent except at those times when trouble had developed. For an Imhoff tank, with two sludge wells and having facilities for reversing the flow, it would seem to be a good plan to separate the two sludge hoppers by a transverse wall rising as high as the underside of the sloping bottom of the sedimentation chamber. An arrangement of piping could be worked out so that at any time a portion of the incoming sewage could be diverted only into that particular sludge room which for the time being is toward the entrance end of the tank, it being assumed that a common sedimentation tank lies above the two sludge hoppers. If this be done the liquor after partial sedimentation in the first sludge room would pass up through the slots into the common sedimentation chamber and any suspended matters carried up with it would have an opportunity to settle into the second sewage well as the liquor flowed on to the outlet weir. Indeed the second half of the Imhoff tank would be always operating in the regular Imhoff manner. Such an arrangement would suffice to renew the liquid contents of the sludge well at such a rate and at such times as the operator desired.

The so-called Hampton tank—the prototype of the Emscher tank—is constructed so that the sludge well and the sedimentation chamber both communicate with the same outlet weir. This has the effect of drawing a portion of the sewage liquor (usually about 20 per cent.) through the sludge room, the rest passing directly to the outlet weir from the upper story. This results in the liquid contents of the sludge room being completely changed every twelve hours. It should be stated that the quantity of sewage liquor drawn through the sludge room was always under the control of the operator and that while one-fifth was a common value it was by no means a fixed one. The outlet on occasions was closed entirely in which case the flow through the sludge well ceased altogether. After many years of experience with these tanks at Hampton, Worcester and Luton in England, Dr. Travis, the inventor, assures the writer that "there has never been in any of the hydrolytic tank installations, anything of the nature of foaming nor any marked scum accumulation. In every installation, provision had been made for the periodical removal of the scum but it has rarely been used".

Since foaming occurs only in conjunction with vigorous evolution of gas, vents should be more generous in area than is common practice in order that the intensity of the ebullition may be lessened. The use of elongated vents to serve the ridge-like division between two parallel V-shaped sedimentation chambers is much to be preferred to the once common square or nearly square chimney. The former in addition to providing a larger area for the escape of gas, eliminates the chance of gas- and sludge-pockets occurring underneath the ridge. Another useful feature is to provide an emergency drain so that when desired, the liquor in the tank may be drawn down to at least the level of the slots. This will obviate the need of pumping when any emergency requiring examination on repair of the interior of the tank occurs.

RECAPITULATION

1. Most of the Imhoff tanks whose history is reviewed, produce an inoffensive quick-drying sludge.

2. The troubles complained of are:
    - (*a*) collection of solids on sloping bottoms of sedimentation chambers,
    - (*b*) formentation of scum on surface of sedimentation chambers,
    - (*c*) choking of slots,
    - (*d*) choking of sludge pipes,
    - (*e*) failure of sludge mass to move to apex of conical bottom of sludge room,
    - (*f*) production of sour sludge, and
    - (*g*) foaming at gas vents.

3. The remedy for (*a*), (*b*), (*c*), (*d*) and (*e*) lies firstly in the hands of the designer and after that in the hands of the operator. Intelligent and careful operation are essential.

4. For acid sludge, milk of lime is an uncertain remedy. In many plants, time has eventually accomplished the change from an acid to an alkaline condition.

5. Foaming is the most troublesome of the evils indicated above and occurs in two-thirds of the installations whose history is reviewed. It is thought to be due to the froth-forming constituents of the liquor overlying the sludge and for it the partial or complete change of this liquor by substituting for it fresh sewage or water, is the remedy proposed. This substitution is to be made only when the foaming manifests itself and by means easily provided by the designer of the plant.

EFFECTS OF MAGNETIC LEAKAGE IN CURRENT TRANSFORMERS

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Introduction

In power transformers and instrument voltage transformers magnetic leakage and resulting reactance are of importance because of effects on regulation and other properties. In many types of current transformer leakage reactance is vastly greater in relative importance than in any constant-voltage transformer, being comparable in magnitude with the secondary burden. It is remarkable that magnetic leakage has usually been passed over briefly when mentioned at all in articles on current transformers.

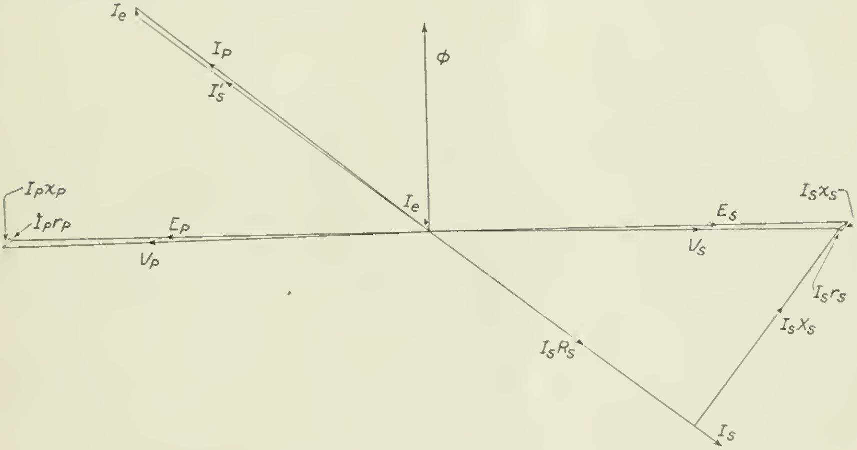

Fig. 1.—Vector diagram of an average power transformer with 1 per cent. resistance, and 5 per cent. reactance drop.

The general transformer theory is usually explained by the aid of the vector diagram shown in Fig. 1 in which;

I_s is the secondary current;

V_s is the secondary terminal voltage equal to $I_s\ R_s + I_s\ X_s\ j$, where R_s and X_s are the resistance and reactance respectively of the load;

E_s is the secondary induced voltage equal to $V_s\ + (I_s\ r_s + I_s\ x_s j)$, where r_s and x_s are the resistance and reactance respectively of the secondary winding of the transformer;

E_p is the component of primary terminal voltage consumed by the primary induced voltage which is E_s multiplied by the turns-ratio;

ϕ is the mutual flux or working flux in the core, which induces voltages in the primary and secondary windings;

I_e is the exciting current necessary to produce the flux ϕ;

I_p is the primary current, being the vector sum of I_e and I_s', the reversed secondary current;

V_p is the primary terminal voltage equal to $E_p + (I_p\, r_p + I_p\, x_p j)$.

In Fig. 1 all voltage and current vectors are drawn to scale to represent rated-load values for an average power transformer with 1 per cent. resistance and 5 per cent. reactance drop.

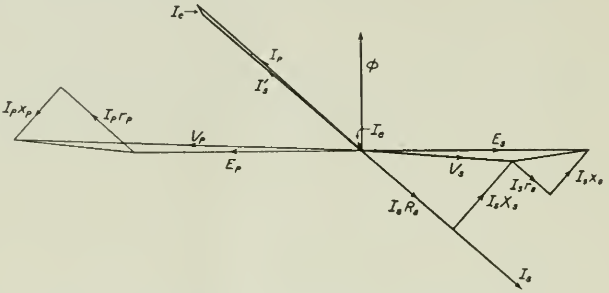

Fig. 2*a*.—Vector diagram of a typical current transformer with burden of 25 volt-amperes at 80 per cent. power factor, 25 cycles.

For a typical current transformer (No. 4 in Table I), with a burden of 25 volt-amperes at 80 per cent. power factor, the diagram assumes the form shown in Fig. 2*a* at 25 cycles, and Fig. 2*b* at 60 cycles, both to scale. In this case the leakage reactance is of much greater relative importance than in the power transformer, especially at the higher frequency. The quantities represented by light lines are seldom considered in the current transformer as they do not affect the current ratio and phase angle.

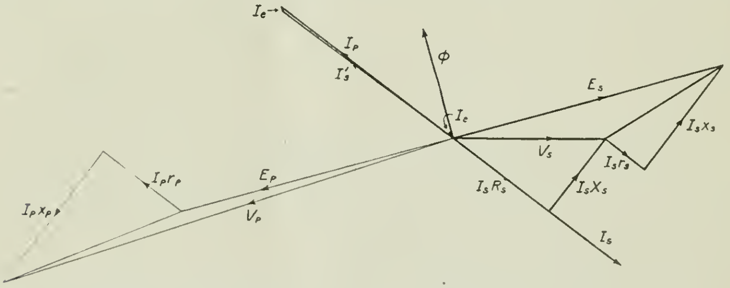

Fig. 2*b*.—Vector diagram of a typical current transformer with burden of 25 volt-amperes at 80 per cent. power factor, 60 cycles.

Further, in the power transformer, the impedance drops indicated by the triangles in Fig. 1 decrease with decreasing load, not only in absolute value but also in relative importance to the primary and secondary voltages which remain practically unchanged. In the current transformer, however, all voltages vary together, the burden being constant, and hence the leakage reactance is just as important at light loads as at full load.

The quantities in Fig. 2 which are affected by magnetic leakage are the primary leakage reactance, x_p; the secondary leakage reactance, x_s; and the core flux, ϕ. The secondary leakage reactance influences the secondary induced voltage, hence the flux, hence the exciting current, and hence the primary current. The core flux, which is assumed to be of constant value and uniform distribution in the core, is also affected by the superposition of leakage fluxes as will be explained later, and indirectly affects the exciting current and the primary current. The primary leakage reactance does not directly affect the primary or secondary currents, but the primary and secondary leakage reactances are more or less interdependent, and a change in one generally involves a change in the other.

LEAKAGE REACTANCE

In order to give a definite conception of the magnitude of leakage reactance in a number of typical current transformers Table I has been prepared. The transformers chosen are all commercial switchboard or portable types, or especially constructed transformers which are copies of commercial designs. Excepting the transformer with the ring core and uniformly distributed secondary, these figures show that in most commercial designs secondary leakage reactance is quite appreciable, being frequently comparable in magnitude with the rated secondary burden; while in a few cases the rated secondary burden is small compared to it. The latter result will evidently occur in transformers of large primary current rating, such as furnace busbar transformers, if the secondary is not distributed as on a circular core or on the four sides of a rectangular core. Other things being equal, the leakage reactance will vary as the square of the number of turns in the winding and directly as the frequency.

The primary leakage reactance is also given in Table I except in the case of through-type transformers, in which the portion of primary conductor to be considered as the primary winding is indefinite. These figures show that in a general way primary and secondary leakage reactances increase or decrease together, but it will be noticed that the total reactance is not always equally divided between the two windings as is often assumed, and in some cases the division is far from equal.

TABLE I

C.T. No.	Plate No.	Nom. Ratio	Max. Line Voltage	Type	Core	Windings	Sec. Turns	Rated Sec. Burden (Ohms)	Sec. Internal Resistance	Sec. Leakage Reactance at 60 ~	Prim. Leakage Reactance at 60 ~ in Terms of Secondary
1	1	5/5	3500	Similar to commercial switchboard types	Shell-type	Superposed link-shaped coils around middle leg of core	223	1.0	0.35	0.54	0.75
2	2	5/5	3500	"	"	Link-shaped secondary, long rectangular primary	223	1.0	0.35	1.14	5.2
3	3	75/5	15000	Switchboard	Rectangular	"	296	1.6	0.46	2	9
4	4	5/5	8000	"	"	Superposed link-shaped coils around one leg of core	222	1 to 2	0.33	0.87	1.1

TABLE I.—*Continued.*

5	5 and 6	1600/5	3500	"	"	Two link-shaped secondary coils on opposite legs of core. Single turn primary	317	1 to 2	0.32	0.86 to 1.07 †	*
6	7	1200/5		Similar to portable or switchboard types	Circular	Secondary uniformly distributed. Single-turn primary	238	1.0	0.25	Small ‡	*
7	8	1600/5	500	Portable	Rectangular	One link-shaped secondary coil on one side of core. Single-turn primary	320	0.4	0.39	3 to 7 †	*

*Values of primary leakage reactance in through-type transformers are not given because the portion of primary conductor to be considered as the primary winding is indefinite.

†Depending on the position of the return primary conductor.

‡The secondary leakage reactance may be positive or negative depending upon the arrangement of the primary conductor, but in all cases is quite small.

LEAKAGE FLUX

Leakage reactance is the result of leakage flux; that is, flux which fails to pass through all the turns of both windings due to the strong opposing magnetomotive forces of the two windings. Since leakage reactance is in many cases of very considerable magnitude it may be expected that leakage flux is also worthy of notice. It will be seen that if the primary and secondary windings of a current transformer can be made to produce exactly equal and opposite magnetomotive forces, there will be no mutual flux (or working flux) but leakage fluxes will be present practically the same as in normal operation. The distribution of the leakage fluxes can then be explored without interference from the working flux.

Convenient conditions for this experiment are found in a current transformer of one-to-one ratio, with equal turns in the two windings. In other cases the primary winding must be replaced by another winding of identical form and dimensions but having the same number of turns as the secondary. The new primary winding, when producing the same magnetomotive force as the original, will cause the same flux conditions in the transformer. The primary and secondary windings, when connected in series with opposite polarity, will carry the same current and will produce equal and opposite magnetomotive forces as required. Exploring coils of one or more turns may then be used in conjunction with a suitable voltage indicating instrument to determine the magnitude and phase of the flux at any point. The instrument used in the experiments here described was a separately excited suspension dynamometer with mirror, deflections being read by telescope and scale. The moving coil had a resistance of about 1,000 ohms, and the sensibility of the instrument with full field excitation was about 500 microvolts per millimetre at 1 metre distance.

The distribution of leakage flux was in this way investigated in a number of different types of current transformer under various conditions. Single-turn exploring coils were located at intervals around the core, and in some cases the core was divided up into three sections in the plane of the laminations for the placing of exploring coils. The results of these measurements are given in more detail further on, but in general it may be stated:—

(1) that both primary and secondary leakage fluxes utilize parts of the core as part of the leakage path;

(2) that the maximum primary leakage flux in the core is found at the core section which is most influenced by the primary winding, and similarly that the maximum secondary leakage flux in the core is found at the section of the core which is most influenced by the secondary winding;

(3) that the leakage flux in the core varies in magnitude at points between the two maxima, and at one point which is influenced equally by the two windings the leakage flux is zero;

(4) that there is more leakage flux in the outer laminations of the core than in the inner;

(5) that the leakage fluxes are nearly in phase with the currents in the respective windings, but that they lag by a small angle (of the order of magnitude of one or two degrees in the cases observed);

(6) that the leakage fluxes are practically proportional to the magnetomotive forces of the respective windings, or in other words, for a given current transformer the leakage fluxes vary only with the currents in the respective windings;

(7) that the leakage fluxes in their path through the core pass through the laminations not only in the normal direction along the length of the core, but also transversely, and even at right angles to the plane of the laminations.

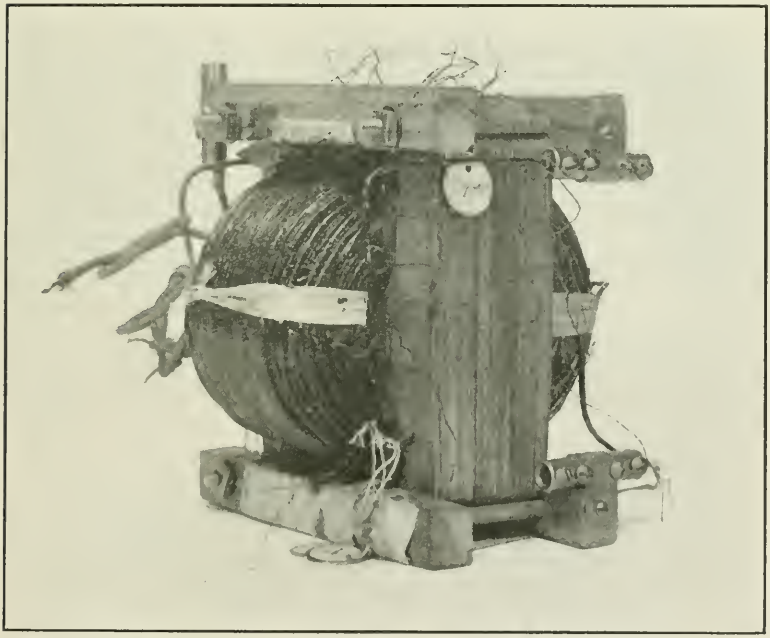

Plate No. 1.

These points may be illustrated by current transformer No. 1 of Table I, which is shown in Plate No. 1. In this case the exploring coils were located not only around the whole core but also around the individual thirds of the core, thus dividing it into three parts in the plane of the laminations as indicated in Fig. 3*a*. Fig. 3*b* shows a section through the windings and indicates the paths of a few typical lines of leakage flux. Fig. 3*c* shows vectorially the flux values observed. Vectors *D* to *J* are double the actual values which they represent because the core area at these sections is only half that at sections *A* to *C*. It will be noticed that the leakage flux in either of the outer sections of the core is approximately twice as much as that in the inner section. The maximum value of primary and secondary leakage fluxes in the core in this case was about 8,000 lines, the current being 6 amperes. That the leakage fluxes are nearly proportional to and in phase with the current in the windings is due to the fact that the reluctance of the leakage path is mostly in air, although the effect of the iron loss is noticeable in the small angle by which the current leads the leakage flux. Thus the true leakage reactance includes a small fictitious resistance due to the iron loss caused by the leakage flux.*

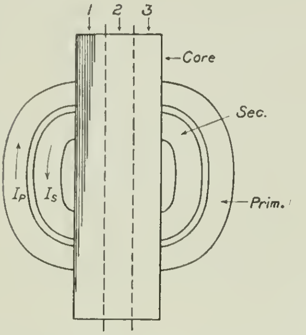

Fig. 3*a*.—Elevation of current transformer No. 1, showing division of core into three sections.

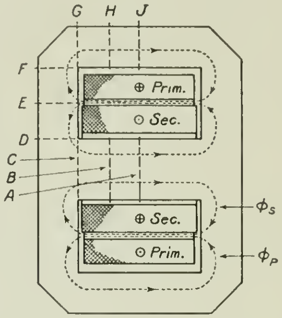

Fig. 3*b*.—Section of C.T. No. 1, indicating exploring coil positions and typical lines of leakage flux.

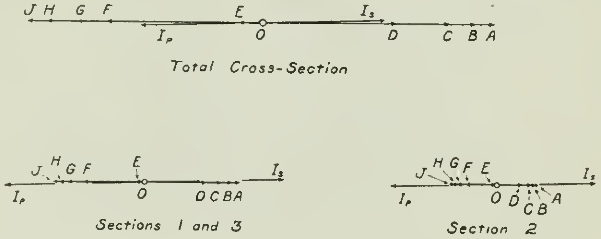

Fig. 3*c*.—Leakage flux observed in the core of C.T. No. 1, at sections shown in Fig. 3*b*.

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*Steinmetz, *Theory and Calculation of Electric Circuits*, sec. 114, page 224.

TOTAL RESULTANT FLUX

The actual flux existing in the core of many current transformers during normal operation is found to be widely different in different parts of the core. There are variations not only in magnitude and phase, but also in distribution over the core cross-section.

This is illustrated in the case of transformer No. 1 of Table I, before referred to, by the vector diagrams in Fig. 4*a* which represent the flux in the core with the primary carrying 6 amperes at 60 cycles with a burden of 0.99 ohms non-inductive resistance. Comparing these diagrams with those of Fig. 3*c*, it will be noticed that the total flux at each point is the vector sum of the leakage flux originally observed at that point plus a constant component ϕ_m. The latter is obviously the working flux or mutual flux which is said to cause in both primary and secondary windings a "nominal induced voltage" proportional to the turns in each.

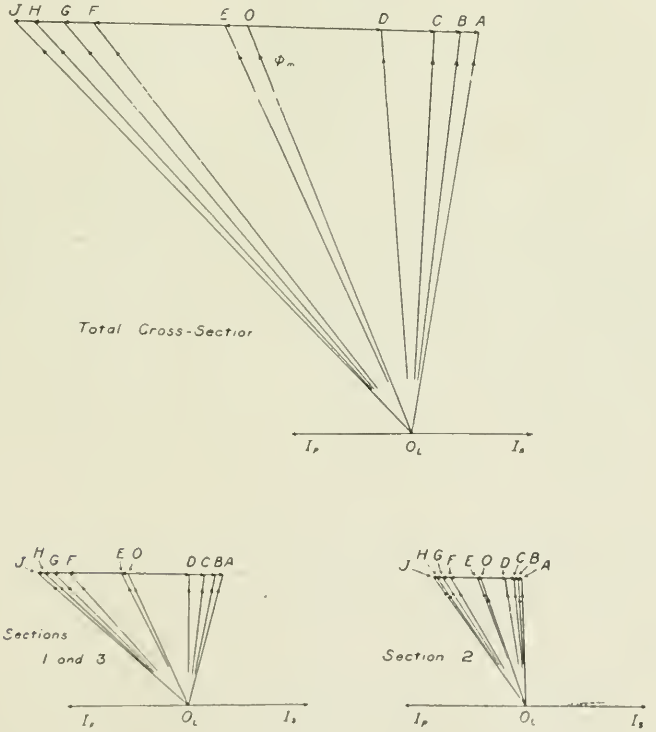

Fig. 4*a*.—Flux vector diagrams of C.T. No. 1, with 0.99 ohm, non-inductive burden at 60 cycles.

The secondary induced voltage, E_s, Fig. 4*b*, lags 90 degrees with regard to the working flux, and forces the secondary current through the total impedance of the secondary circuit. The component of E_s in phase with I_s overcomes the resistance of the circuit, while the quadrature component provides for the reactance drop. These two components, E_r and E_x, may be considered as due to ϕ_r and ϕ_x respectively, the two components of the working flux with regard to I_s. Thus ϕ_r is proportional to the total resistance in the secondary circuit and ϕ_x to the total reactance.

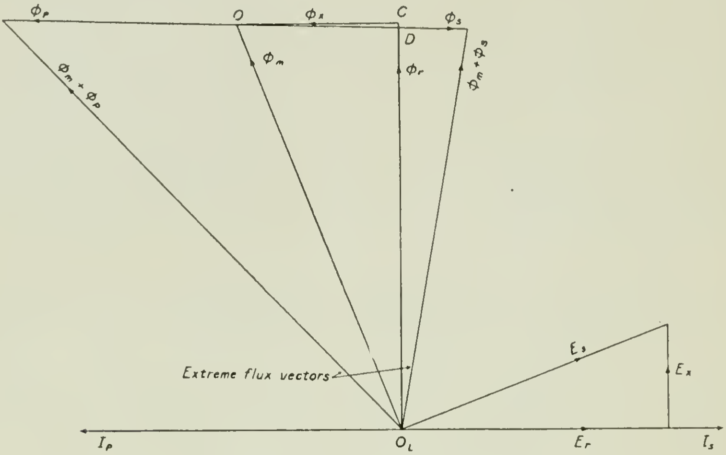

Fig. 4*b*.—Transformer diagram of C.T. No. 1, showing components of ϕ and Es.

In this particular case the resistance of the burden was 0.99 ohm and of the secondary winding 0.35 ohm, and their sum, 1.34 ohms, is proportional to $O_L\,D$ in Fig. 4*b*, while DC represents the small fictitious resistance due to the iron loss caused by the leakage flux. OC represents the effective leakage reactance since no other reactance existed in the secondary circuit. The secondary leakage reactance is therefore equal to $1.34 \times \frac{OC}{DO_L}$ which is found to be 0.54 ohm in this case.

Thus the leakage reactance is determined as a by-product of an investigation of flux distribution. It is unnecessary to know the calibration constant of the dynamometer if the resistance of the secondary winding and the resistance and reactance of the secondary burden are obtainable.

It is interesting to notice how the flux diagram of Fig. 4*b* will be influenced by changing various conditions. Increase in the resistance of the burden moves the point O_L vertically downward. Increase in the reactance of the burden moves the points O_L and C horizontally to the right. Increase in current with constant secondary burden* enlarges

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*Constant burden meaning in this case constant resistance and *inductance*.

the whole diagram in proportion. Increase in frequency with constant burden* and current moves the point O_L vertically upward.

As already stated, the leakage flux is greater in the outer laminations of the core, and in transformer No. 1 was found to be twice as great in either of the two outside sections as in the inner section. The working flux is more uniformly distributed over the cross-section, but is nevertheless noticeably greater in the outer laminations. This is probably due to the fact that the leakage flux density in the outer laminations is greater, thus increasing the permeability of this part of the core, it being kept in mind that the density was below the point of maximum permeability of the iron, as is usual in well-designed current transformers.

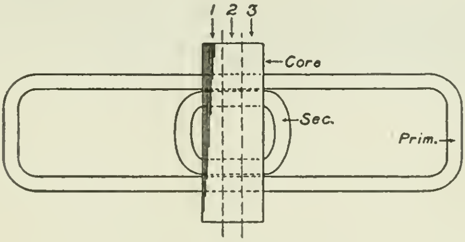
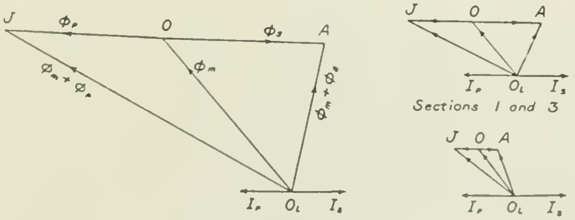


Fig. 5*a*.—Sketch of C.T. No. 2.

The flux distribution in other transformers of Table I will be given with briefer comment.

Fig. 5*a* is a sketch of transformer No. 2, which is shown in Plate No. 2. Fig. 5*b* represents the flux in different parts of the core. Transformer No. 2 is No. 1 with the previous primary coil replaced by another of long rectangular form. Plate No. 3 of transformer No. 3 shows a commercial transformer with this style of primary winding. This increases both primary and secondary leakage flux and leakage reactance, and further changes the ratio of secondary to primary leakage reactance from the proportions 1: 1.39 to 2.1: 9.6. Also the ratio of the leakage flux in the outer sections of the core to that in the inner section



Total Cross-Section

Sections 1 and 3

Section 2

Fig. 5*b*.—Flux vector diagrams of C.T. No. 2 with 0.99 ohm non-inductive burden at 60 cycles.

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* Constant burden meaning in this case constant resistance and *inductance*.

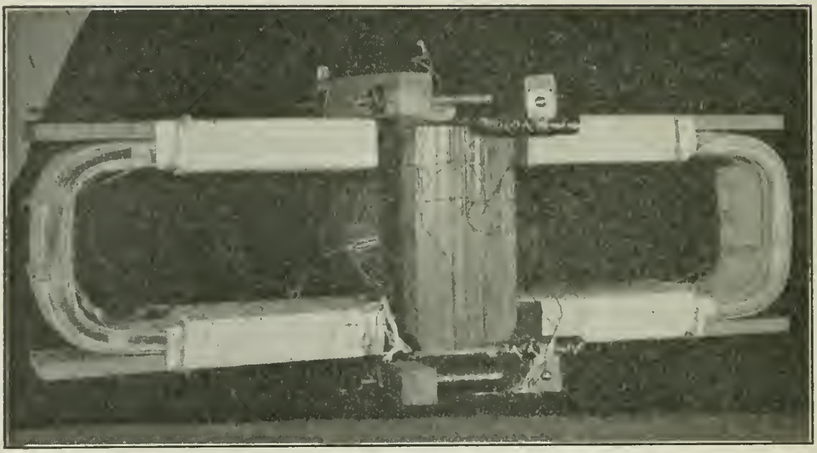

Plate No. 2.

becomes approximately three to one instead of two to one as in transformer No. 1. Conditions for transformer No. 2 were:—

I_p = primary current = 6 amperes.

F = frequency = 60 cycles.

r_s = secondary internal resistance = 0.35 ohm.

R_s = secondary external resistance = 0.99 ohm.

x_s = secondary internal reactance = 1.14 ohm.

X_s = secondary external reactance = 0.

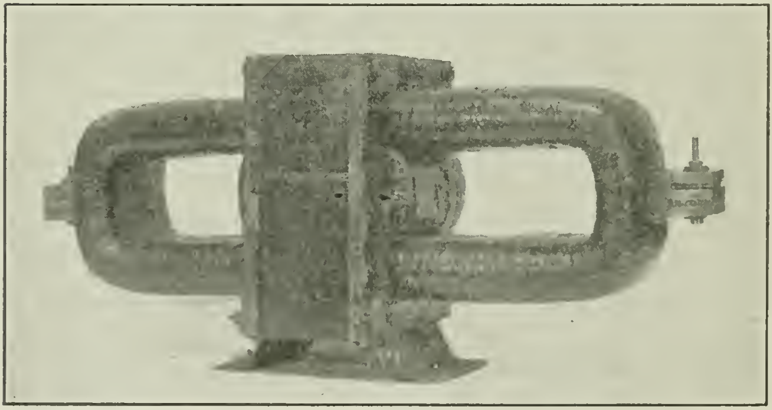

Plate No. 3.

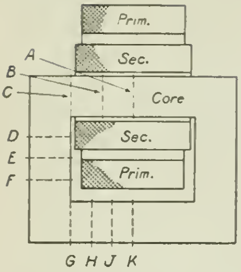

Fig. 6*a*.—Section of C.T. No. 4, showing exploring coil positions.

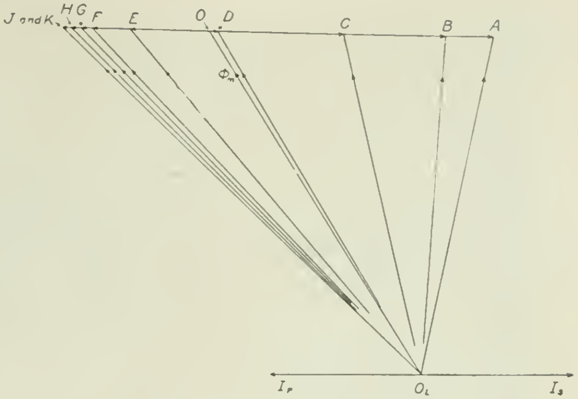

Fig. 6*b*.—Flux vector diagram of C.T. No. 4.

Fig. 6*a* shows the arrangement of the windings and the exploring coil positions on transformer No. 4, Plate No. 4, with the flux distribution as shown in Fig. 6*b* for the following conditions:—

$I_p=6$ $r_s=0.33$ $x_s=0.87$
$F=60$ $R_s=0.99$ $X_s=0.$

Transformer No. 5 is shown in Plate No. 5 with a distributed primary winding which will give an approximation to the effect of a straight through conductor. On the right of the picture is seen the metal case and insulating tube with which the transformer is provided

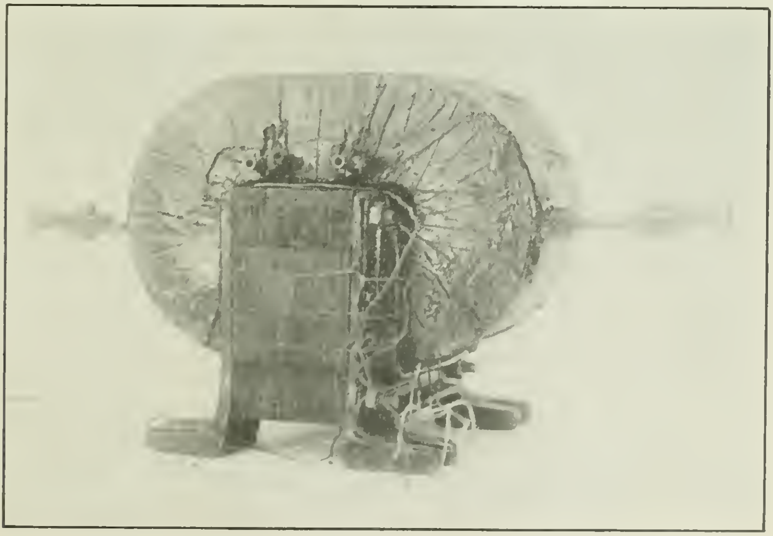

Plate No. 4.

for installation on a busbar. The exploring coil positions are indicated in Fig. 7*a* and the flux distribution in Fig. 7*b*. Conditions are as follows:

$I_p=6$ $r_s=0.32$ $x_s=0.96$.
$F=60$ $R_s=0.99$ $X_s=0$.

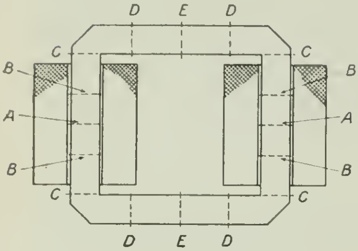

Fig. 7*a*.—Section of C.T. No. 5, showing exploring coil positions.

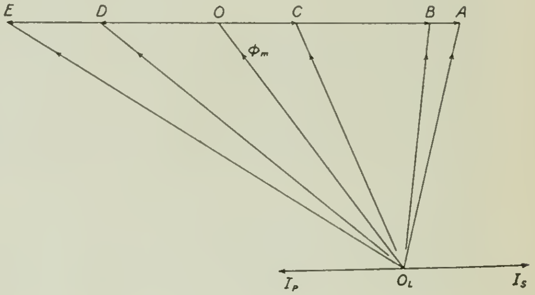

Fig. 7*b*.—Flux distribution in core of C.T. No. 5, with 0.99 ohm non-inductive burden, 60 cycles.

This transformer, with other positions of the primary, has secondary leakage reactances at 60 cycles varying from 0.86 to 1.07 ohm, the former value being obtained with half of the primary passing around each secondary coil as in Plate No. 6 and the latter with half of the primary passing around each end of the core. These two conditions would correspond to the use of two or more turns of cable in a portable through-type transformer of this construction. With a metal case the

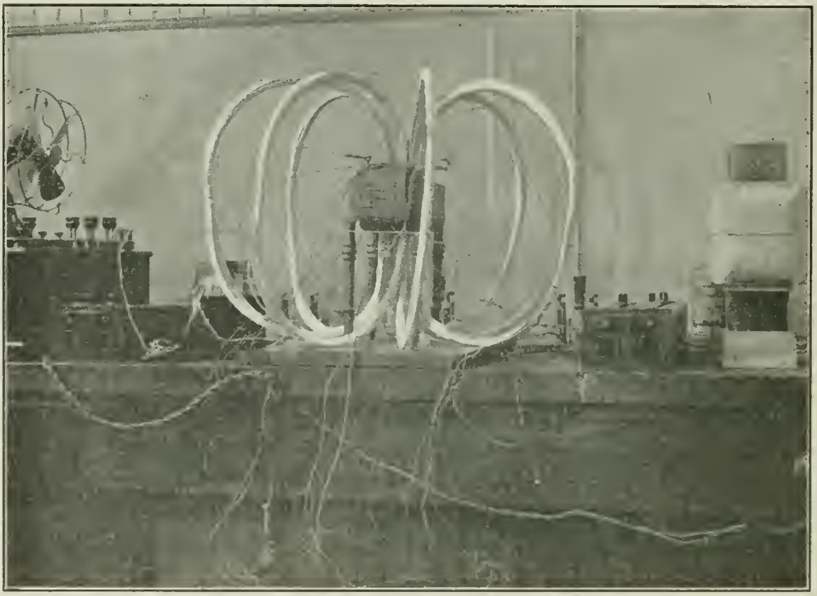

Plate No. 5.

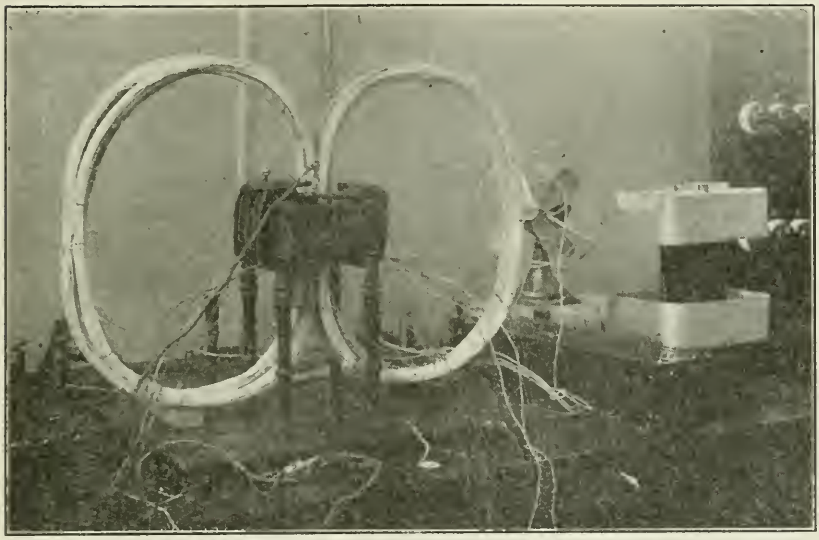

Plate No. 6.

flux distribution and leakage reactances in this transformer would probably be appreciably different.

Transformer No. 6 is a style sometimes used in through-type switchboard and portable current transformers. With a symmetrical well-distributed primary, or its equivalent a straight through conductor with return conductor at a considerable distance, no leakage flux enters the

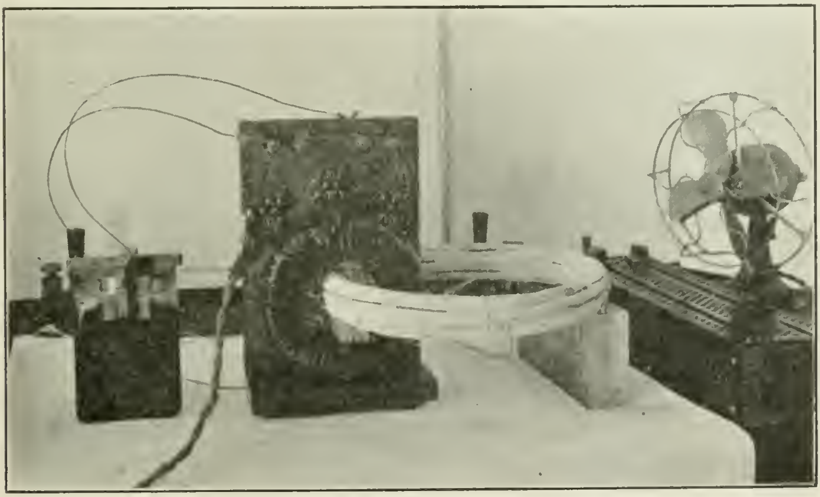

Plate No. 7.

core, and the core flux is the same at all cross-sections. In actual switchboard installations, and in use as a portable transformer, the primary is seldom completely distributed; and in such cases the core flux is not uniform. In Plate No. 7 of this transformer the primary winding is shown as a circular loop about a foot in diameter passing around one side of the core. With this primary and normal operating conditions the leakage flux in the core and the total flux were found to be as ahown in Fig. 8*b*. The working flux and leakage fluxes must, therefore, travel somewhat as suggested in Fig. 8*a*. It will be seen that the working flux, when resolved into its components with regard to the secondary current, appears to indicate a *negative reactance* in the secondary circuit. The resistance of the secondary was 0.25 ohm, and the burden in this case was 0.99 non-inductive resistance. Since the portion of the secondary circuit external to the transformer is non-inductive the reactance shown by Fig. 8*b* must be that of the secondary winding, and further must be negative.

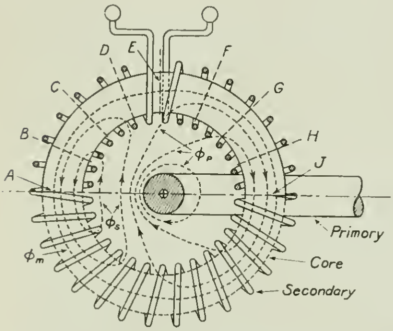

Fig. 8*a*.—Section of C.T. No. 6, showing. exploring coil positions and leakage paths.

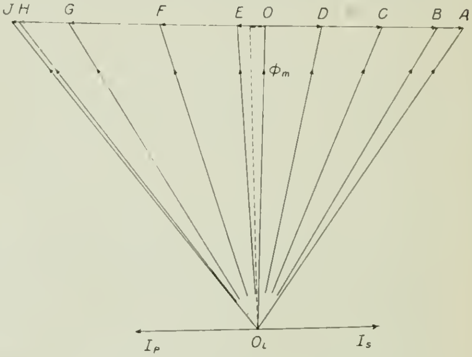

Fig. 8*b*.—Flux distribution in core of C.T. No. 6, with 0.99 ohm non-inductive burden, 60 cycles.

An explanation of this negative leakage reactance is to be found in the fact that the primary leakage flux in passing through the core also passes through some turns of the secondary winding which is uniformly distributed on the core. These primary leakage lines, linking with secondary turns, tend to neutralize the reactive effect of the secondary leakage lines which are opposite in sense; and since a greater part of the core carries primary leakage flux, and also because the primary leakage flux density is greater the integrated linkages of primary leakage lines with secondary turns is greater than that of secondary leakage lines with secondary turns, with the result that the secondary reactance appears to be negative.

The question may be asked whether flux which passes through part of both primary and secondary turns is not really working flux or mutual flux. It is true that such flux is strictly mutual flux with regard to the parts of each winding to which it is common, but the leakage fluxes mentioned above differ in several important respects from the ordinary working flux which is common to all turns of both windings. The following reasons are given for considering flux which is common to the primary and only part of the secondary as leakage flux and not as working flux:

(1) Unlike working flux, it is present when equal and opposite magnetomotive forces are produced by the primary and secondary windings.

(2) Unlike working flux, its path is not completely within the core and it does not completely interlink both windings.

(3) Unlike working flux, its magnitude and phase are practically independent of the secondary burden.

(4) Like leakage flux, its magnitude and phase are determined by the magnitude and phase of the current in the windings.

This apparently negative secondary leakage reactance was confirmed by a ratio and phase angle test at 60 cycles with secondary short-circuited. It was found that even with small values of secondary current such as 0.75 and 1.0 ampere, and with equal turns in primary and secondary the observed ratio was 1.000; which shows that the exciting current was in quadrature with the secondary current. This could not have been the case unless the core losses were zero or the secondary current was leading the secondary induced voltage. Although the core loss was small at such a low density, it was not zero; which leads to the conclusion that the effective secondary leakage reactance was negative for that arrangement of primary and secondary coils.

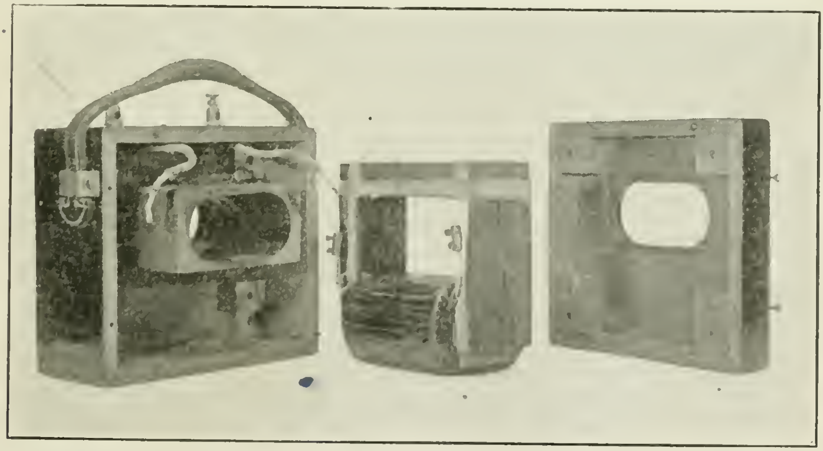

Plate No. 8.

The construction of transformer No. 7 of Table I is clearly shown in Plate No. 8. This transformer possesses the greatest leakage reactance of any thus far considered. The reactance depends very much upon the arrangement of the primary. If the primary is wound around the secondary coil in a circular loop about a foot in diameter the secondary leakage reactance at 60 cycles is about 3 ohms, while if the primary is looped around the opposite side of the core as in Fig. 9*a*, the value is 7 ohms. Fig. 9*c* indicates the flux distribution and the appearance of the transformer diagram for the latter case with the following conditions:—

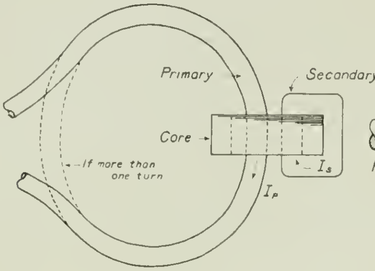

Fig. 9*a*.—Sketch of C.T. No. 7, showing position of primary.

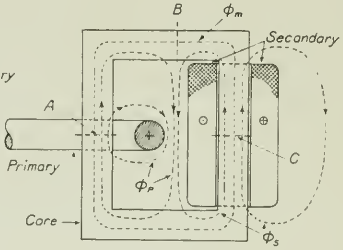

Fig. 9*b*.—Section of C.T. No. 7.

$I_p = 2.6$	$r_s = 0.4$	$x_s = 7$
$F = 60$	$R_s = 0.84$	$X_s = 0.$

Notice that the actual flux $\phi_m + \phi_p$ present at section *A* is approximately eight times that at section *C*, and about 75 degrees out of phase with it. In this case, due to the low power factor of the secondary circuit, the secondary current lags with regard to the primary current, and there is a corresponding phase difference in the leakage fluxes ϕ_p and ϕ_s.

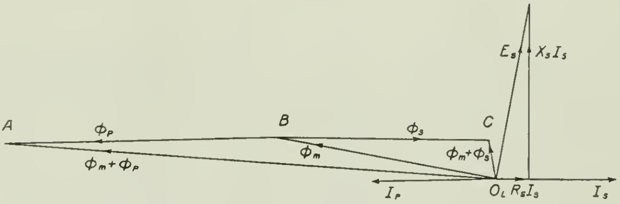

Fig. 9*c*.—Transformer diagram and flux distribution of C.T. No. 7.

EFFECT OF LEAKAGE FLUXES ON THE EXCITING CURRENT

Although the primary leakage flux in the core is usually of the same order of magnitude as the secondary leakage flux, and the mutual flux is, roughly speaking, midway between the two extremes of flux observed in the core, still the exciting current required by a core carrying mutual flux with superposed leakage fluxes is in general not the same as that which would be required if the same mutual flux existed without the leakage fluxes. This is shown in a very marked manner in the case of transformer No. 7 by comparing observed ratio and phase angle curves with those calculated in the usual way on the assumption of uniform flux in the core. See Fig. 10. The calculated curves were based on exciting current measurements made on the actual core of transformer No. 7, using the secondary winding as an exciting winding, the primary circuit being open. Under these conditions the flux will be practically uniform throughout the core, and not variable from point to point as observed under actual operating conditions.

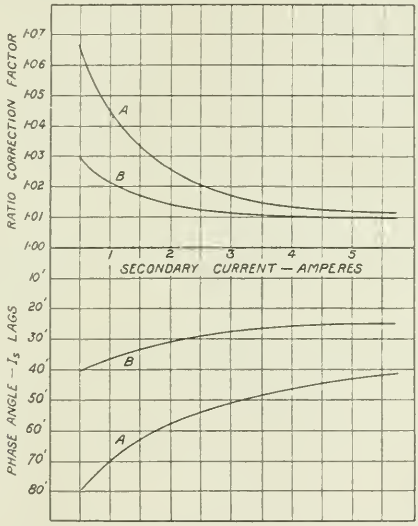

Fig. 10.—Ratio and phase-angle curves of C.T. No. 7, with primary distributed and secondary short-circuited. *A*, calculated; *B*, observed.

For the purpose of analysis the actual exciting current of a current transformer may be considered as the vector sum of the exciting currents required by a large number of small sections into which the core may be divided along its length. Any expectation of agreement between the observed ratio and phase angle curves and those calculated as outlined above is based on the general assumption that the exciting current under the flux conditions existing in actual operation will be the same as that which would be required if the same mutual flux existed without the leakage fluxes. This general assumption involves the following particular assumptions, many of which are shown to be untrue.

(1) It is assumed that the mutual flux is the vector average of the maximum and minimum fluxes observed in the core. In no observed

case has this been found exactly true although the mutual flux is always somewhere between the maximum and minimum fluxes.

(2) It is assumed that half of the core is affected by primary leakage flux, and the other half by secondary leakage flux, and to an equal extent. This assumption has also been found to be inaccurate in all observed cases, although it holds in a very rough manner.

(3) It is assumed that the flux in the core is uniformly distributed over the core cross-section, and travels in a path parallel to the core axis. As has been stated and shown in the diagrams, the flux is not uniformly distributed over the cross-section, nor does it travel in the normal direction, *i.e.*, parallel to the core axis.

(4) It must be assumed that the core reluctance is constant in order that the lower magnetizing current required by the sections of the core in which the flux is less than the average may be exactly counterbalanced by the higher magnetizing current required by the other parts of the core. Actually the core reluctance of the iron varies considerably with the flux density, well designed current transformers operating normally at maximum crest densities below the point of maximum permeability of the iron and below the straight part of the saturation curve.

(5) It is assumed also that the core-loss current varies directly with the density in all parts of the core. Otherwise the total exciting current for all parts of the core could not be that corresponding to an assumed uniform average flux density. If the core-loss current is to vary directly with the flux density, then the core losses must vary directly as the square of the density. The eddy loss does vary as the square of the density because eddy currents vary as eddy voltages, which in turn vary as the density at given frequency. Usually, however, 50 to 80 per cent. of the core-loss is due to hysteresis. This part of the loss usually does not vary as the square of the density and since it is the major part of the core-loss the assumption under discussion can be justified only as an approximate one.

With these facts in view it is not surprising that the calculated and observed characteristics do not agree. The complexity of the problem renders it exceedingly difficult to formulate a correction factor which will make the calculated results agree with the observations.

The discrepancies are not usually so great as in transformer No. 7, which has a very large leakage reactance. Fig. 11 gives a comparison of the observed and calculated curves of transformer No. 5 with distributed primary. Fig. 12 shows the results on transformer No. 4. The discrepancies in these two cases might be considered negligible for many purposes. A greater discrepancy might easily be caused by a less accurate knowledge of the value of the secondary leakage reactance than was obtained in these experiments. Ordinarily the secondary leakage reactance would be computed from the physical constants of the windings by the aid of some empirical formula, or estimated by making an arbitrary division of the total reactance of the transformer into

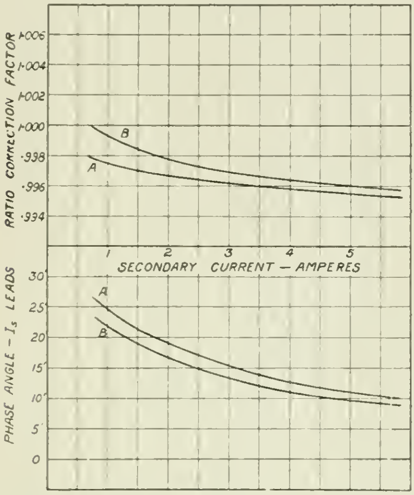

Fig. 11.—Ratio and phase-angle curves of C.T. No. 5, with distributed primary and 0.99 ohm non-inductive burden, 60 cycles. *A*, calculated; *B*, observed.

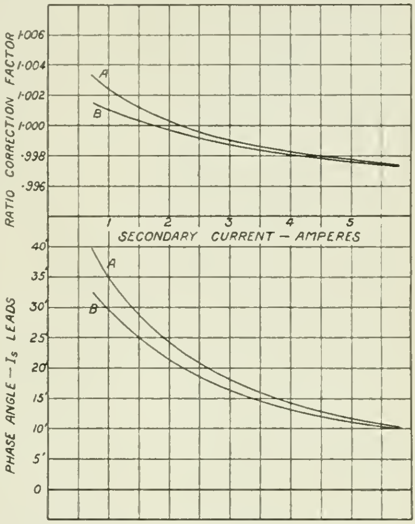

Fig. 12.—Ratio and phase-angle curves of C.T. No. 4, with 0.99 ohm non-inductive burden, 60 cycles. *A*, calculated; *B*, observed.

primary and secondary reactances, by which means only an approximate idea of the secondary reactance can be obtained. However, it seems improbable that the discrepancies observed in these experiments are due to errors in the determination of the secondary leakage reactance. They must, therefore, be due to the effects of leakage flux in the core as before explained.

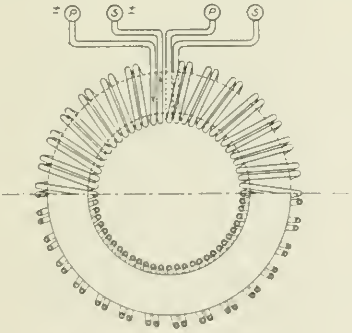

Fig. 13.—C.T. No. 6, with uniformly distributed primary winding.

This statement is supported, though not absolutely proved, by an experiment made on transformer No. 6. This transformer has two identical core windings of 119 turns each, both uniformly distributed around the core, and arranged as shown in Fig. 13 so that each turn of one winding has by its side a turn of the other winding. One of these windings may be used as the primary and the other as the secondary of a one-to-one current transformer. This construction was chosen to make the magnetic leakage small, and to prevent as far as possible those leakage lines which did exist from entering the iron core. From the arrangement of the windings, all leakage flux must pass through the narrow space occupied by the insulation between turns. The leakage reactance is very small, about 0.002 ohm per winding at 60 cycles, and no perceptible leakage flux enters the core as would be expected from the construction. The observed and calculated ratio and phase angle curves of this transformer are practically coincident, the maximum discrepancy between individual points being 1/20 of one per cent. in ratio and 2 minutes in phase angle from one-fifth to full load current.

In order to determine in more detail the effects of leakage flux on the exciting current the components of the latter were obtained by inverse calculation from observed ratio and phase angle curves of various transformers. These results were compared with the exciting current measured with uniform flux as before described. From a large number of calculations of this kind, on transformers of different types, few conclusions can be drawn to which there are no exceptions. However, the following statements may be made:—

(1) In a current transformer in which no leakage flux enters the core the ratio and phase angle may be accurately calculated in the usual way, *i.e.*, from measurements of core-loss and magnetizing currents made with uniform flux distribution, a knowledge of the turn-ratio, and the constants of the secondary circuit. The accuracy of the calculated results depends only on the accuracy of the data from which they are computed.

(2) In a current transformer in which leakage flux exists in the core, superposed on the working flux, the ratio and phase angle can not be calculated with consistent accuracy by the usual method referred to above.

(3) The magnitude of the discrepancy increases with the magnitude of the leakage flux in the core, or in other words with the leakage reactance, other things being equal.

(4) The calculated ratio is usually too high and the calculated angle of lead of I_s with regard to I_p is usually greater than the actual.

(5) The actual core-loss current may be higher or lower than core-loss current measured with uniform flux.

(6) The actual magnetizing current is lower than that measured with uniform flux over a considerable range of densities below the density corresponding to maximum permeability of the iron.

This last mentioned point appears to be the explanation of the bulk of the discrepancy. The magnetizing component is usually larger than the core-loss component of the exciting current, and seems to be more affected by leakage fluxes. The reluctance of the iron is reduced by the addition of leakage flux to the mutual flux, since the densities are usually lower than that at which the permeability of the core iron is maximum.

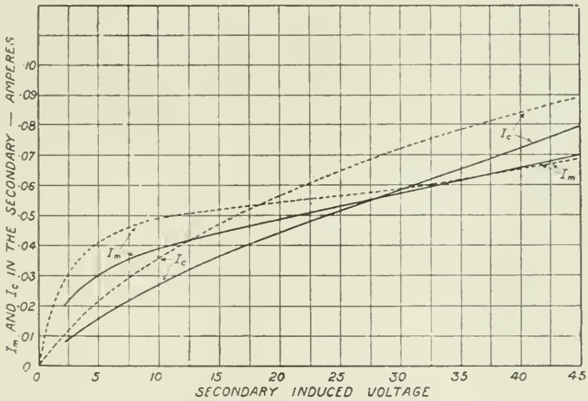

Fig. 14.—Magnetizing and core loss components of exciting current of C.T. No. 7, with primary as in Fig. 9*a*. *I*m, magnetizing component; *I*c core loss component. *Dotted lines*, by direct measurement with uniform flux in core; *full lines*, actual values when leakage fluxes are present.

As a sample of the discrepancies which are observed, Fig. 14 shows the magnetizing and core-loss components of the exciting current of transformer No. 7. The broken lines are plotted from the results of direct measurement of the exciting current with uniform flux in the core while the full lines are obtained by inverse calculation from the observed ratio and phase angle curves of Fig. 10. The discrepancies in this transformer are rather extreme since the leakage reactance is so large.

EFFECT OF LEAKAGE REACTANCE ON RATIO AND PHASE ANGLE

Although leakage reactance, accompanied by leakage fluxes in the core makes calculations of ratio and phase angle unreliable if high accuracy is desired, it does not follow that the effects on the actual ratio and phase angle curves obtained are always undesirable. These curves are affected in different ways by leakage reactance and will be considered separately.

Increase of leakage reactance in a current transformer, other conditions being unchanged, will reduce the angle of lead of I_s with regard to I_p. This is because the power factor of the total secondary circuit is decreased, causing the secondary current to lag more with regard to the secondary voltage. In most cases the power factor of the total secondary circuit will be higher than that of the core, so that increase of leakage reactance brings the secondary current more into phase with the exciting current, thus reducing the phase angle between primary and secondary currents.

Increase of leakage reactance raises the ratio, and also usually increases the slope of the ratio curve. There are two reasons for this. One is that increase of leakage reactance increases the total impedance of the secondary circuit, and an increase of the exciting current is necessary, which raises the ratio. The power factor of the secondary circuit is at the same time reduced, which causes the ratio to be more dependent on the magnetizing component of the exciting current than on the core-loss component. Since the magnetizing current is usually larger than the core-loss current, and since it is not generally so near to a linear function of the density as the core loss current at the densities used in well designed current transformers the ratio is raised due to this cause also, and the slope of the ratio curve is increased. These effects are somewhat undesirable but may be partially corrected by turns compensation.

In consideration of the reduction of the phase angle errors these effects on the ratio may be considered permissible, but if phase angle errors are considered unimportant for the purpose for which the current transformer is to be used, leakage reactance should be reduced to a minimum in order to obtain minimum ratio errors.

We are indebted to the Hydro Electric Power Commission of Ontario for the loan of certain apparatus and for other assistance.

APPENDIX TO

Effects of Magnetic Leakage in Current Transformers

BULLETIN NO. 2, 1921, SECTION NO. 4, SCHOOL OF ENGINEERING RESEARCH, UNIVERSITY OF TORONTO

The relative values and phase positions of the fluxes in various parts of the cores of the current transformers investigated have been plotted to scale in vector diagrams, for non-inductive burdens only, Figs. 4*a* to 9*c*. The only flux ordinarily assumed to exist in the core is flux ϕ_m. The actual extreme values of fluxes $\phi_m+\phi_p$ and $\phi_m+\phi_s$ differ so much in value and phase with regard to ϕ_m that it seems worth while also to tabulate their relative values and phase angles corresponding to the vector diagrams:

TABLE II

Vector Diagram	C.T. No.	Relative Values			Phase Angle $\phi_m+\phi_p$ to $\phi_m+\phi_s$
		ϕ_m	$\phi_m+\phi_p$	$\phi_m+\phi_s$	
Fig. 4*a*, total section..........	1	1.00	1.30	0.93	53°
Fig. 4*a*, sections 1, 3..........	1	1.00	1.38	0.93	63°
Fig. 4*a*, section 2.............	1	1.00	1.15	0.94	34°
Fig. 5*b*, total section..........	2	1.00	1.65	0.76	73°
Fig. 5*b*, sections 1, 3..........	2	1.00	1.75	0.81	88°
Fig. 5*b*, section 2.............	2	1.00	1.31	0.84	31°
Fig. 6*b*......................	4	1.00	1.23	0.86	58°
Fig. 7*b*......................	5	1.00	1.52	0.83	71°
Fig. 8*b*......................	6	1.00	1.29	1.20	74°
Fig. 9*c*......................	7	1.00	2.21	0.18	75°
Fig. 13......................	6	1.00	1.00	1.00	0°

It will be noticed that all the values tabulated above, and the corresponding vector diagrams, are for non-inductive secondary burdens only. This was done to bring out the comparative effects of leakage flux in the various styles of transformer and arrangements of windings, without the further effects of varying the power factor of the burden.

If burdens of highly reactive or condensive character be connected to the secondaries, the relative phase positions of ϕ_m, ϕ_p, ϕ_s change greatly, so that very wide variations in $\phi_m+\phi_p$ and $\phi_m+\phi_s$ are possible with the same secondary current and terminal voltage. Those familiar with vector diagrams for current transformers can easily construct, for any type of transformer and desired burden, approximate comparative flux diagrams adapted from one or other of those in the text.

The authors wish to acknowledge the assistance of Mr. G. F. Tracy, research assistant, in preparing all line drawings in these papers, and the vector diagrams and curves exactly to scale from our records of observations and calculations.

METHOD OF MEASURING RATIO AND PHASE ANGLE

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The methods of measuring the ratio and phase angle of current transformers in the experimental work connected with the research described in the preceding paper fall under the general classification of comparison methods.

In some cases the current transformer to be investigated is compared with a standard transformer of the same ratio by the deflection method described by F. B. Silsbee.* The primaries of the two transformers are connected in series and the secondaries are both connected to the current coil of a Weston single-phase low-power-factor wattmeter in such manner that the coil carries the difference between the two secondary currents. This difference is a vector difference. By exciting the voltage circuit of the wattmeter from two different phases, two readings are obtained which give the components of the difference-current. The ratio and phase angle of the transformer in question, with regard to the standard transformer, may then be calculated; and by correcting for the known errors of the standard transformer, the ratio and phase angle of the other are obtained.

In most of the transformers dealt with in this research the ratio was one-to-one, or was made so by using the required number of primary turns. This was necessary in order to determine the leakage flux distribution and the leakage reactance under different conditions, and incidentally it made possible a simplification of the comparison method for measuring ratio and phase angle. The secondary current may be compared directly with the primary current which may be thought of as the secondary current of a standard transformer with no errors. Corrections for the errors of a separate standard transformer are thereby avoided.†

The connections used are shown in Fig. 1. The vector difference between primary and secondary currents is i, which flows through the current coil of a wattmeter. The voltage coil of the wattmeter is supplied first from V_P and then from V_Q, two readings being obtained from which the components of i in phase and in quadrature with the primary current are calculated. V_P is in phase with the primary current

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Electrical World, 70, p. 667, Oct., 1917.

†Testing of Instrument Transformers, III, H. M. Crothers, *Electrical World*, 75, No. 6, Feb., 1920.

since it is the voltage across the primary lamp bank which is non-inductive. V_Q is adjusted in quadrature with the primary current by rotating the phase shifter until the wattmeter in the main line reads zero.

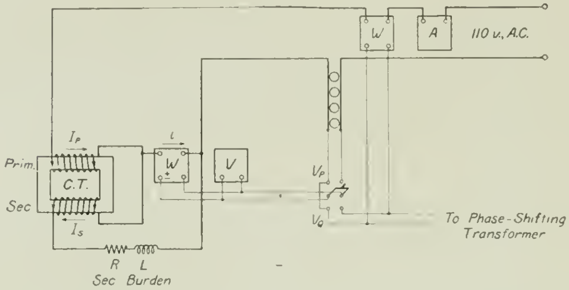


Fig. 1.—Connections for measurement of ratio and phase angle of current transformer.

The vector diagram, Fig. 2, indicates the various phase relations. From the two wattmeter readings are derived i_p and i_Q. The phase angle θ is $\sin^{-1}\frac{i_Q}{I_s}$. Unless an ammeter is included in the secondary circuit, I_s will be unknown. However, if the ratio error is small, the phase angle is approximately $\sin^{-1}\frac{i_Q}{I_p}$, or more correctly $\sin^{-1}\frac{i_Q}{I_p - i_p}$. The ratio is $\frac{I_p}{I_s}=\frac{I_p}{\frac{I_p - i_p}{\cos\theta}}=\frac{I_p\cos\theta}{I_p - i_p}$. If the phase angle is small, $\cos\theta$ is very nearly one, and the formula becomes $\frac{I_p}{I_s}=\frac{I_p}{I_p - i_p}$.

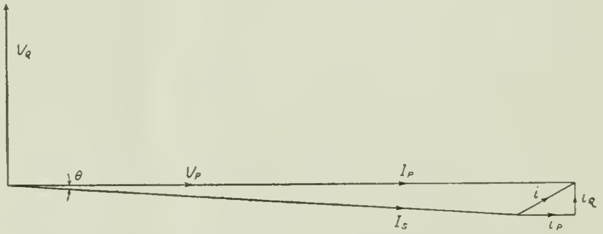


Fig. 2.—Vector diagram of currents in ratio and phase-angle measurement.

The wattmeter used for measuring the difference between primary and secondary currents was a Weston, Model 310, single-phase, low-power-factor wattmeter, 5 or 10 amperes, 75 or 150 volts, 75, 150 or 300 watts, with 150 line scale. The 5-ampere current connection was used and the 75 volt pressure circuit, although the voltages impressed were from 100 to 125 volts in order to increase the deflection. With 100 volts on the 75 volt circuit, one scale division represents 0.005 ampere in phase with the voltage, which means 0.1 per cent. in ratio or a phase angle of 3.3 minutes at full load current of 5 amperes. Readings were estimated to tenths of one division.

In the use of this wattmeter certain precautions must be observed in order to avoid errors. Although shielded, the instrument should be placed at a considerable distance from sources of stray fields such as current transformers, the phase-shifting transformer, and conductors carrying heavy currents. The over excitation of the pressure coil makes this particularly necessary. Also, connections should be so made that the pressure terminal marked ± is at the potential of the current circuit in order to avoid deflections due to electrostatic effects within the meter. This may be done by seeing that the ± pressure terminal is connected through switch V_p to the side of the lamp bank next the wattmeter. A connection across the double-throw switch, as shown in Fig. 1, is necessary for the same purpose in regard to V_Q. If the wattmeter reads negative the current leads should be reversed at the wattmeter terminals.

Silsbee points out the possibility of errors due to voltages induced in the current coil from the flux of the moving coil and suggests setting the moving coil in the position of zero mutual inductance. This was not done in the present instance because it was found that in curves which cross the zero line there is no appreciable break in continuity of the curve in spite of the fact that the current connections are reversed. This is possibly because the voltage induced in the current coil would be in quadrature with the current in the moving coil.

The impedance of the current coil of the wattmeter should be as low as possible in order to avoid any appreciable voltage drop between the points to which it is connected. Such impedance tends to make the secondary current more nearly equal to the primary current both in magnitude and phase, and therefore changes the ratio and phase angle of the transformer. In other words it changes the voltage which must be produced by the secondary, and therefore has the effect of a change in the secondary burden. The disturbing effect of the impedance of the wattmeter current coil in these measurements may be calculated from the following formula. The effective *increase* in the secondary burden due to the effect of the wattmeter stated as a complex quantity is:—

$$-\frac{i}{I_s}\left\{R_w \sin\,(\delta+\beta)+X_w \cos\,(\delta+\beta)\right\}+j\frac{i}{I_s}\left\{R_w \cos\,(\delta+\beta)-X_w \sin\,(\delta+\beta)\right\}$$

where I_s is the secondary current;

i is the vector difference between primary and secondary currents, $I_p - I_s$;

R_w is the resistance of the wattmeter current coil and leads;

X_w is the reactance of the wattmeter current coil and leads;

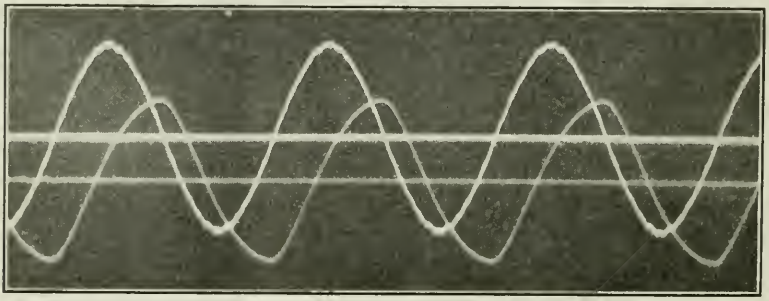
δ is the angle of lead of i with regard to ϕ, the working flux;

β is the angle of lead of E_s with regard to I_s.

If the above value is small with regard to the impedance of the secondary circuit, the effect of the wattmeter on the ratio and phase angle will be correspondingly small. In the wattmeter used, the resistance of the 5 ampere current circuit was 0.13 ohm and its inductance 0.00021 henry, and the value of the impedance calculated from the above expression was rarely more than 0.5 per cent. of the impedance of the secondary burden. By way of practical confirmation the effect of increasing the resistance of the wattmeter circuit was observed. The wattmeter reading showed no perceptible change when the impedance in the wattmeter circuit was doubled for readings up to ten scale divisions.

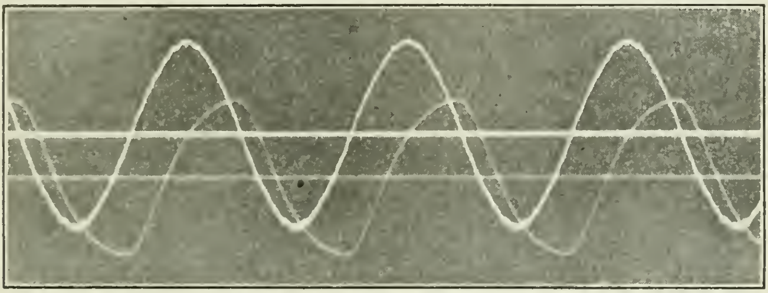
It is assumed throughout this method that all quantities are sine waves, but that is not strictly true. The primary ammeter will read the mean effective value of the primary current regardless of its wave form, but the indications of the wattmeter carrying the difference-current will depend upon the wave forms of both the current and the voltage. This difference-current in the case of an uncompensated current transformer is the exciting current of the transformer, which may therefore be expected to contain harmonics.

In this research a 60-cycle alternator was used which gave a fair approximation to a sine wave, a small tooth ripple being the only noticeable harmonic. With this alternator as supply a series of oscillograms was taken of the exciting current of a sample current transformer at various densities. The core length was 15.7 inches, net cross-section 1.94 square inches, weight 8.56 pounds, being in the form of a ring without joints. In Osc. Nos. 11 to 14 the upper wave represents the



Oscillogram No. 11.

voltage applied to the exciting winding and the lower wave the exciting current, connections being as shown in Fig. 3. Notice that at the low densities the exciting current is very little distorted while with increasing density the effects of hysteresis and saturation introduce harmonics. The greatest crest density in well designed current transformers under normal operating conditions does not usually exceed 25,000 lines per square inch.



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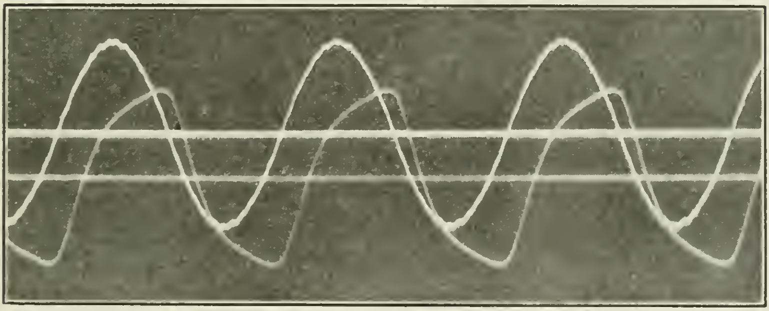

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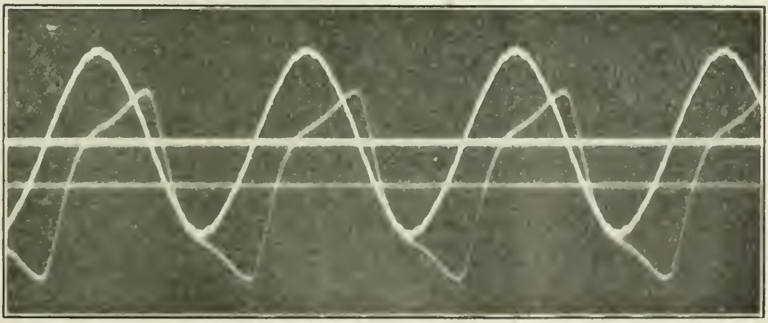


Oscillogram No. 14.

Oscillograms were also taken with connections as in Fig. 1, the elements being connected to show the wave forms of I_p, i and I_s respectively. In Osc. Nos. 17 to 20 the impedance of the secondary was $R=0.54$, $X=0.94$, which produces a power factor in the secondary circuit equal to that of the core and causes the primary and secondary currents to be in phase with the exciting current. The density is 8800 lines per square inch, which is somewhat greater than that in Osc. No. 12. In Osc. No. 17 there is no compensation, hence the difference-current is the exciting

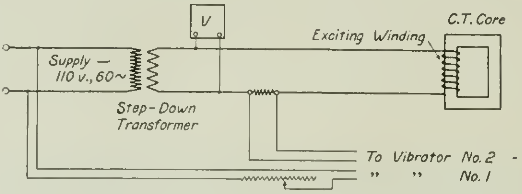
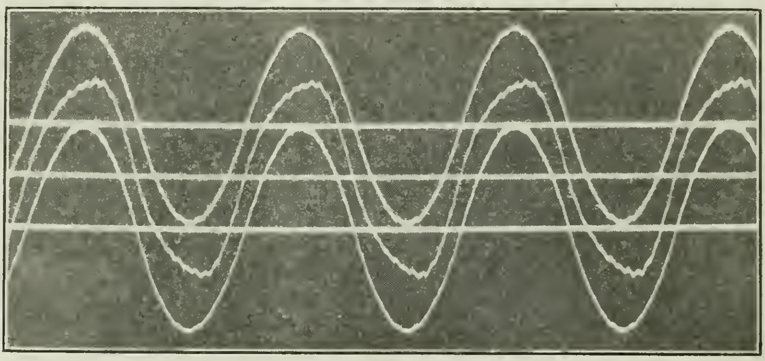
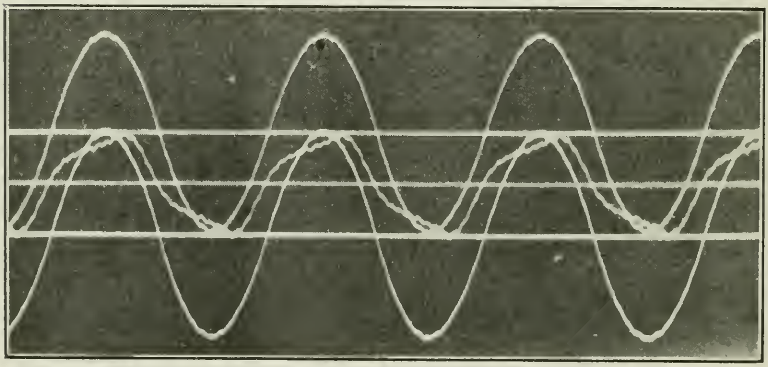


Fig. 3.—Oscillograph connections to obtain wave form of exciting current.

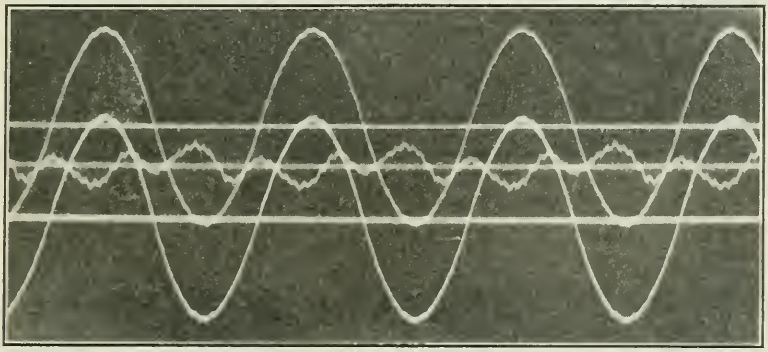


Oscillogram No. 17.

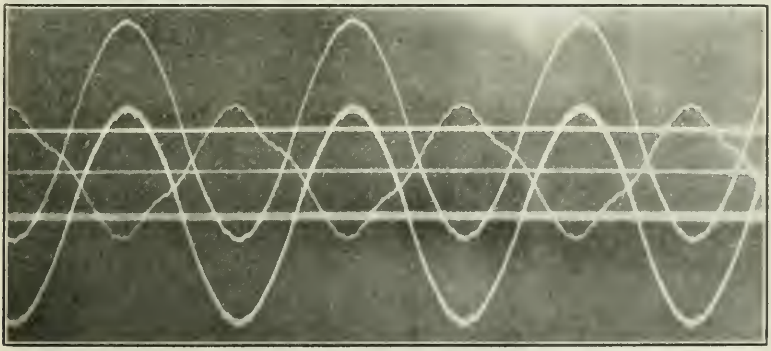


Oscillogram No. 18.

current of the transformer. In Osc. No. 18, the number of secondary turns has been reduced by one. There is thus subtracted from the difference-current a component in phase with and of the same wave form as the secondary current. Since the secondary current is approximately a sine wave the difference-current has the same harmonics in it as before, but these harmonics form a greater proportion of the total current and are therefore more noticeable. In Osc. No. 19 the compensation is two turns and it so happens that this almost exactly neutralizes the fundamental in the difference-current, leaving only the harmonics of the exciting current. In Osc. No. 20 the compensation has been carried to three turns, resulting in a reversal of phase of the difference-current and a dilution of the harmonics.

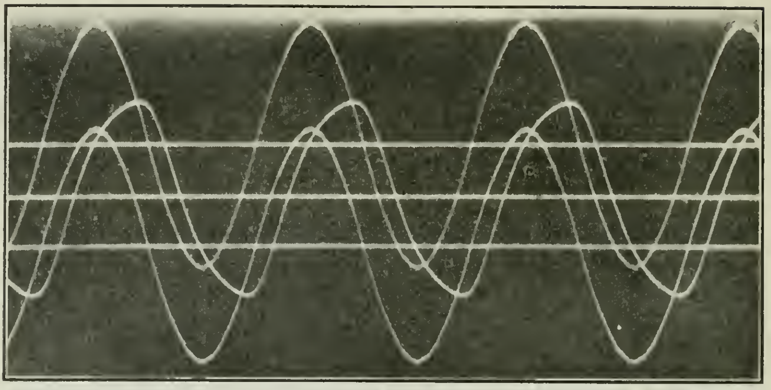


Oscillogram No. 19.



Oscillogram No. 20.

In Osc. Nos. 21 to 24 the total impedance in the secondary circuit was 1.18 ohms non-inductive resistance, and the density was 9600 lines per square inch. The high power factor of the secondary circuit causes the exciting current to lag with regard to the primary and secondary



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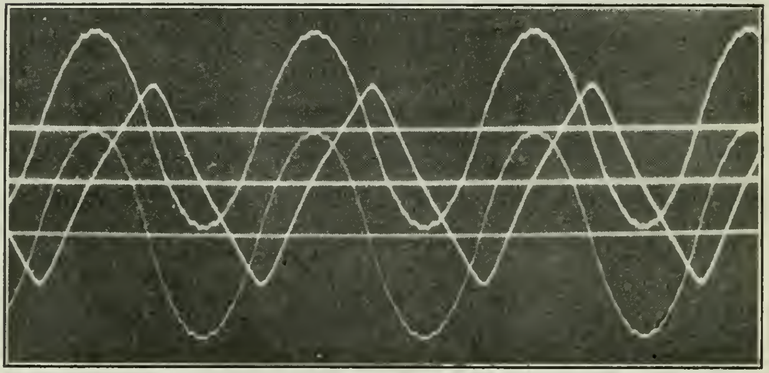
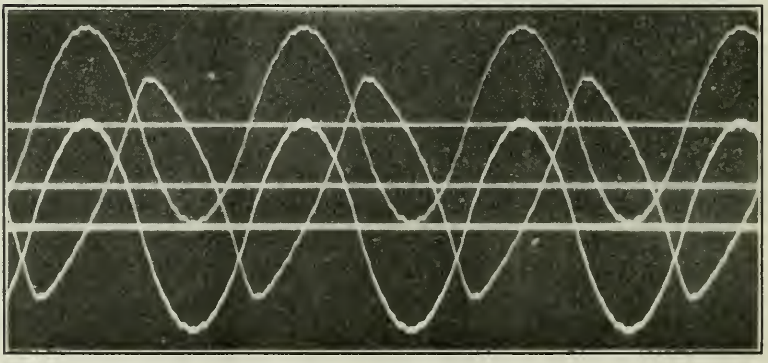


Oscillogram No. 22.



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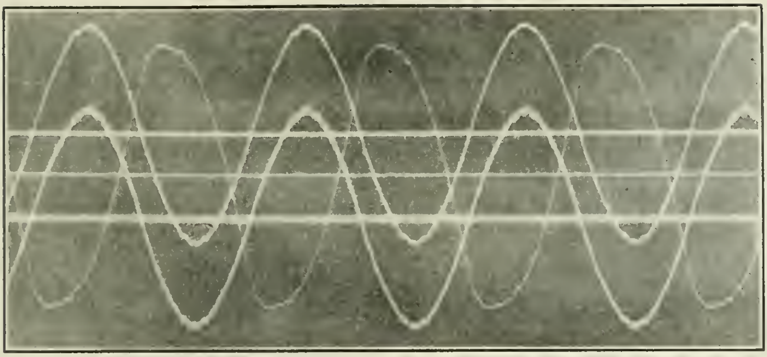

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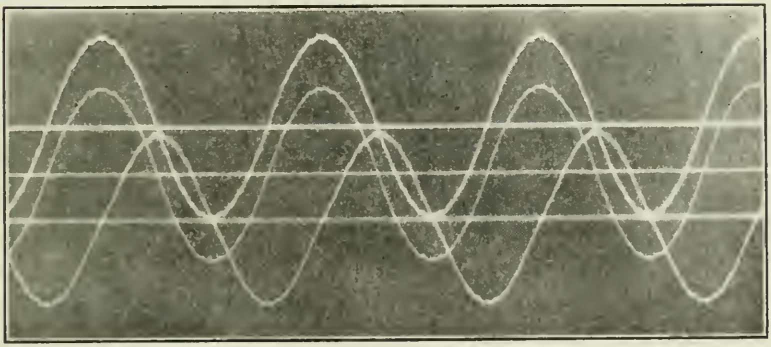
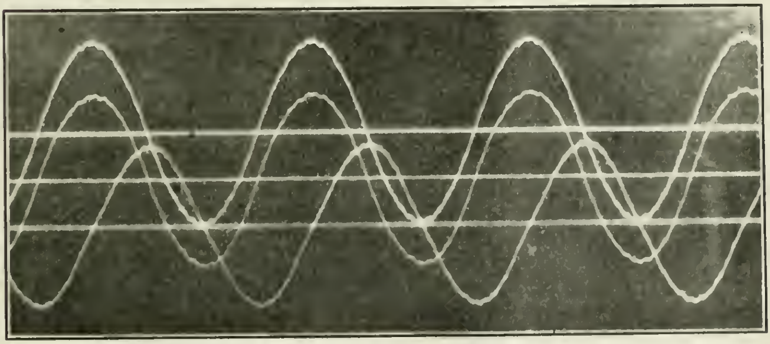


Oscillogram No. 28.



Oscillogram No. 29.

currents. In Osc. No. 21 there is no compensation, while in Osc. Nos. 22, 23 and 24 the number of secondary turns is reduced by one, two and three turns respectively with effects on the difference-current as shown in the oscillograms.

As before stated the readings of the wattmeter carrying this difference-current will depend upon the wave form of the voltage applied to it. The voltage V_p will be of the same wave form as I_p, which will be the same as that of the alternator voltage. Voltage V_Q is obtained from the secondary of a phase-shifting transformer, the wave form of which is shown in Osc. Nos. 28 and 29, the first wave in each case being that of primary voltage, and the other two those of two secondary voltages which are nominally in quadrature. In Osc. No. 28 the primary windings of the phase shifter were connected in Y while in Osc. No. 29 they were in delta. There is no noticeable difference between the wave forms of the primary and secondary voltages.

Since the wattmeter is excited by a voltage of approximately sine form the readings will be proportional to the fundamental components of the difference-current, the harmonics being neglected. Thus in the case of Osc. No. 19 the wattmeter reading would be practically zero in both phases, indicating correct ratio and phase angle although there really exists a difference of about 0.01 ampere between the primary and secondary currents as shown by the second wave on the oscillogram. Therefore the results obtained by this method are apparently in error as to ratio, but not as to phase angle, which is taken to mean the phase angle between the fundamental harmonics of primary and secondary currents. The error in the ratio measurement depends upon the value of the higher harmonics in the exciting current but is usually very small. Assuming in a certain case that the effective value of the harmonics is 25 per cent. of the effective value of the exciting current, and that the exciting current is 1.8 per cent. of the load current, the harmonics will constitute 0.45 per cent. of the load current. If conditions are such that with 5 amperes secondary current the difference-current is composed only of harmonics as in Osc. No. 19, the wattmeter will read zero and indicate a ratio of unity. Actually there is a difference-current of $\frac{0.45}{100} \times 5 = 0.0225$ ampere, and thus the true primary current is $\sqrt{5^2 + 0.0225^2} = 5.00005$ and the true ratio is 1.00001. The error in this case, due to neglect of harmonics by the wattmeter, is only 0.001 per cent., which is quite negligible. Also, as has been shown in the oscillograms, the percentage of harmonics in the exciting current of current transformers at their normal densities is not large.

Most of the chief precision methods of calibrating current transformers are subject to the same errors from differences between primary and secondary current wave form, including null methods involving balancing of shunt drops or mutual inductances whether the indicator of balance is a vibration galvanometer, separately excited dynamometer, or permanent-magnet galvanometer with mechanical rectifier. The same applies to deflection methods with separately excited instruments. An exceptional case is the method involving the use of two dynamometers for the separate measurement of primary and secondary currents. Therefore the errors due to the exciting current wave form in the method of measuring ratio and phase angle in this research are no greater than those due to the same cause in most of the best known precision methods, and are usually negligibly small.

THROUGH-TYPE PORTABLE CURRENT TRANSFORMERS

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C. Kent Duff,
Research Assistant

It is generally known that the ratio and phase-angle errors of through-type portable current transformers are affected by changes in the position of the primary, but little definite information is available on this subject. The following results obtained from ratio and phase-angle measurements on several different transformers of well-known manufacture may be of interest.

Through-type portable current transformers are extensively used in making field tests on alternating-current apparatus where the current reaches values in excess of 200 amperes. The nominal ratio can be varied over a considerable range by using different numbers of turns in the primary which consists of the conductor of the circuit being measured, passed usually 1, 2, or 4 times through the hole in the case to give ratios in proportion 4:2:1.

The effect of various changes in the position of the primary depends very much upon the construction of the current transformer, especially as regards the arrangement of the secondary winding on the core. The three most usual types of construction are:—

Type 1.—Circular core with uniformly distributed secondary.

Type 2.—Rectangular core with secondary in the form of two coils on opposite sides of the core.

Type 3.—Rectangular core with a single secondary coil on one side of the core.

These types have been investigated experimentally.

Type 1.—In this type of current transformer the phase-angle error is practically independent of primary position, while the ratio error is only slightly affected by changes in the position of the primary, less than 0.1 of one per cent. variation having been observed in the measurements made.

As to ratio, the secondary current is least, for a given primary current and secondary burden, when the primary conductor passes in a straight line axially through the transformer; or an equivalent arrangement when the primary consists of four or more turns uniformly distributed, that is, with loops equally spaced around the outside of the case. All other arrangements of the primary give a greater secondary current.

The maximum secondary current for given primary current is obtained when all primary turns are looped as closely as possible around one side of the transformer.

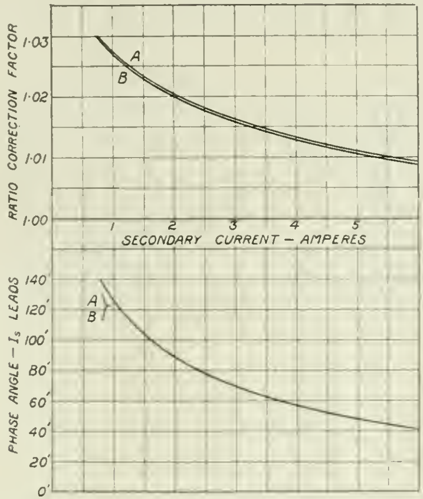

Fig. 1.—Ratio and phase-angle curves of type 1 current transformer; 400 to 5 amperes. *A*, straight axial primary; *B*, primary looped around one side.

Fig. 1 shows the ratio and phase-angle curves of a 400 to 5 ampere transformer with rated secondary burden of 10 volt-amperes, 80 per cent. power factor, at 60 cycles.

Ratio correction factor is the factor by which the marked ratio must be multiplied to give the true ratio. If it be greater than 1.0, the secondary current is too small, and *vice-versa*.

Curve *A* is for a straight axial primary; or its equivalent, a uniformly distributed primary of four or more turns. Curve *B* is the ratio curve when the primary is a coil one foot in diameter placed around one side of the transformer as in Fig. 2. The maximum difference is about 0.05 of one per cent. in the ratio. A primary coil of smaller diameter will give a greater secondary current, but one foot is about the minimum diameter practicable for stiff cables used in practice. All other positions of the primary give results between these two curves.

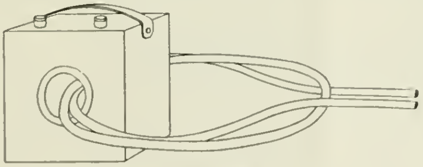

Fig. 2.—Type 1 current transformer; circular core.

Another transformer of the same type, of ratio 600 to 5 amperes, gives the curves shown in Fig. 3 with the same frequency and burden as above. Curve A is for a straight axial primary passing through the centre of the hole, or equivalent distributed primary. Curve B shows the result of moving the straight primary conductor close to one side of the hole. Curve C is obtained with the primary in the form of a coil one foot in diameter around one side of the core.

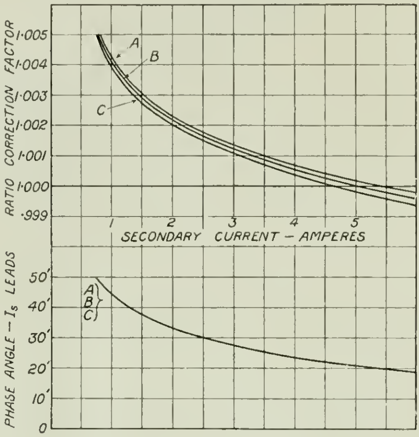

Fig. 3.—Curves for type 1 current transformer; 600 to 5 amperes. A, straight axial primary; B, straight primary close to one side of hole; C, primary looped around one side of core.

The phase-angle curves A and B of Fig. 1 are shown as identical, which means that the observed difference was definitely less than half a minute. The same is true for the phase-angle curves A, B, C of Fig. 3.

In endeavouring to obtain minimum errors of transformation in a transformer of Type 1 by suitable arrangement of the primary, ratio error only need be considered since phase angle is practically unaffected by changes in primary position. For the transformers investigated the minimum errors for the particular burden and frequency used are obtained with the primary as in Fig. 2. This may not always be the case, however, because most transformers are compensated, which means that the ratio of secondary to primary turns is less than the marked ratio of primary to secondary currents. It is, therefore, possible to have the true ratio curve lie partially or even totally below the line of marked ratio. The object of compensation is to make the ratio curve cross the line at about 4 amperes secondary current under average conditions of frequency and secondary burden so as to minimize the average ratio error. However, it is not always possible to attain this object perfectly, and thus it may be that the ratio curve lies mostly below the line. If compensation be the best for average conditions the ratio curve will be made too low by lower secondary burden, or higher power factor of secondary burden, or higher frequency. In such cases it may be best to use a straight axial or uniformly distributed primary.

It is evident, therefore, that in the general case of a compensated current transformer of Type 1, unless the approximate position of the ratio curve is known for the particular frequency and burden used, it is impossible to say how the primary should be placed for minimum average ratio error.

Type 2.—In this type of current transformer both ratio and phase angle are affected by variations in the position of the primary, observed variations in ratio in the investigations here discussed being about 0.1 of one per cent., and in phase angle about 2 minutes.

Regarding phase angle, any change in the position of the primary which increases the secondary leakage reactance will reduce the angle of lead of the secondary current with regard to the primary current. The angle of lead decreases as the primary is successively placed in the following positions:—

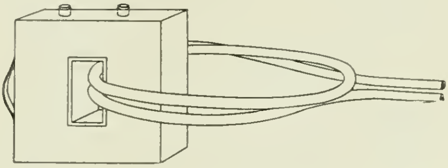

Fig. 4*a*.—Type 2 current transformer; primary in position 1.

1. Looped around one secondary coil, Fig. 4*a*.
2. Equally divided around the two secondary coils.
3. A straight axial conductor, or its equivalent of four or more turns uniformly distributed.
4. Looped around one end of the core between secondary coils.
5. Equally divided around the two ends of the core, Fig. 4*b*.

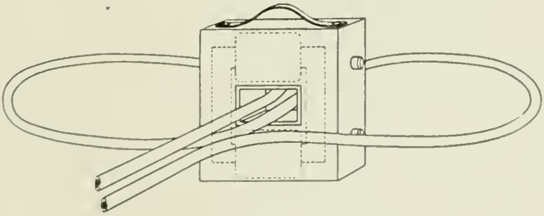

Fig. 4*b*.—Type 2 current transformer; primary in position 5.

The first and last of these positions produce the two extremes of phase-angle error, and these are shown in Fig. 5, curves *A* and *B* respectively, for a transformer of ratio 1,600 to 5 amperes. The frequency was 60 cycles and the secondary burden 25 volt-amperes at 80 per cent. power factor. The diameter of the primary loop was 14 inches, about the minimum for actual conductors in practice. A larger primary loop produces smaller variations. The maximum variation here is less than 2 minutes.

The ratio error is also affected by primary position, the secondary current decreasing as the primary is successively placed in the positions listed above. The ratio curve *A* is obtained with the primary as in Fig. 4*a*. Curve *B* shows the results with primary as in 4*b*. The maximum difference in the ratio error is 0.09 of one per cent.

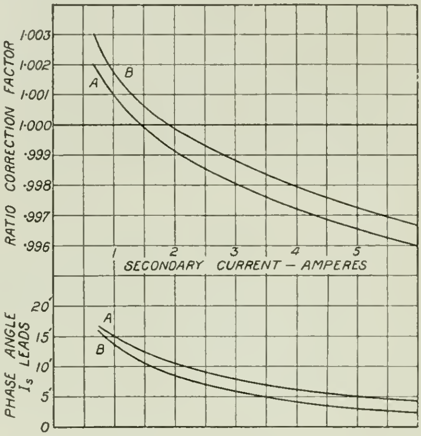

Fig. 5.—Curves for type 2 current transformer. *A*, primary as in Fig. 4*a*; *B*, primary as in Fig. 4*b*.

As to the best position of the primary to produce minimum errors, both ratio and phase-angle errors must be considered. In the above transformer, owing to over-compensation for the particular frequency and burden used, curve *B* is best for both ratio and phase angle, that is, the primary should be placed as in Fig. 4*b*. However, if the ratio curves had been above the line of marked ratio the position for least phase-angle error would have involved also the greatest ratio error, and *vice-versa*. It would then have been necessary to decide whether the primary coil position should be arranged to favour approximately 0.09 of one per cent. reduction in ratio error at the expense of 2 minutes' increase in phase angle, or *vice-versa*. It is worth noticing also that in power measurement 2 minutes in phase angle may, depending on load power factor, be a much more important change if the current transformer has a large phase angle than if the phase angle is small.

The decision depends upon the kind of measurements being made. Ammeter indications are affected only by ratio. Power-factor meters are affected only by phase angle. Wattmeter indications are dependent on both quantities, but phase-angle errors are more important at low power factors and ratio errors at high power factors.

If the phase-angle curves had been below the zero line, that is if the secondary current had been lagging instead of leading with regard to the primary current, the condition for least errors would again be changed. Conditions tending to make the secondary current lead less (or lag more) are lower power factor of secondary burden, or higher frequency, or any other thing which will reduce the power factor of the total secondary circuit. For the same reason a transformer of high ratio is more likely to have a secondary current lagging with regard to primary current than one of low ratio because it has more secondary turns and hence higher internal reactance and lower power factor.

If the power factor of the total secondary circuit is equal to that of the core the phase error will be zero, and if the secondary power factor is less than this the secondary current will lag behind the exciting current and therefore behind the primary current. See Fig. 6.

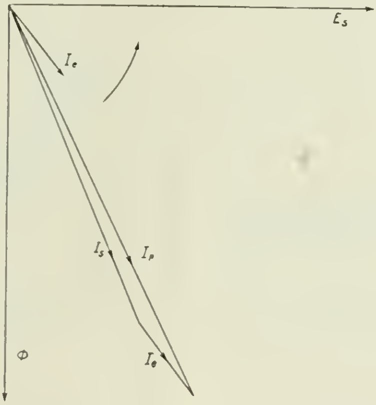

Fig. 6.—Vector diagram showing how secondary current may lag behind primary current when power factor of secondary circuit is low.

It is evident, therefore, that to know how to place the primary to minimize the errors, it is necessary to know whether the ratio and phase angle curves lie above or below the zero line.

Type 3.—Both ratio and phase-angle errors in this type of current transformer are greatly affected by changes in the position of the primary, observed variations in the transformer tested being 40 minutes in phase angle and 0.5 of one per cent. in ratio at rated secondary current and approximately rated burden, and greater variations at smaller currents.

As to phase angle, the secondary current is made to lead less (or lag more) with regard to the primary current by any change in primary position which tends to increase the secondary leakage reactance. The latter is minimum when the primary is wound as closely as possible over the secondary coil, and maximum when wound as closely as possible on the opposite side of the transformer core. The angle of lead of the secondary current with regard to the primary current decreases as the primary is successively placed in the following positions:—

1. Wound closely over the case on the same side as the secondary coil.
2. A coil 11 inches in diameter around the secondary coil, Fig. 9.
3. A straight axial conductor or its equivalent of four or more turns uniformly distributed around the core.
4. Two 11-inch coils around the two ends of the core.
5. A coil 11 inches in diameter around the side opposite the secondary coil.
6. Wound closely over the case on the side opposite the secondary coil.

Regarding ratio, increase of leakage reactance increases the total secondary impedance and hence ratio also. Therefore the ratio increases as the primary is successively placed in the six positions listed above.

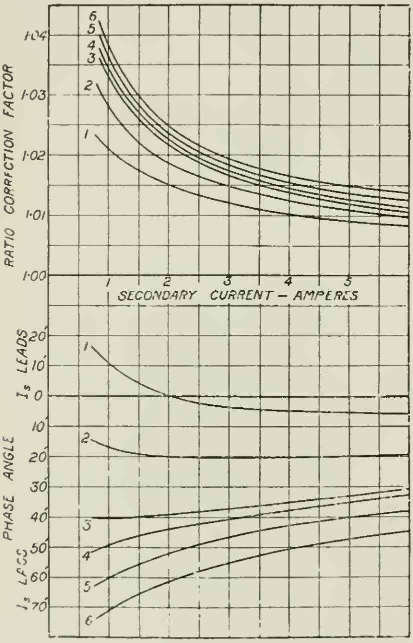

Fig. 7.—Curves for type 3 current transformer at 60 cycles; primary in positions 1 to 6.

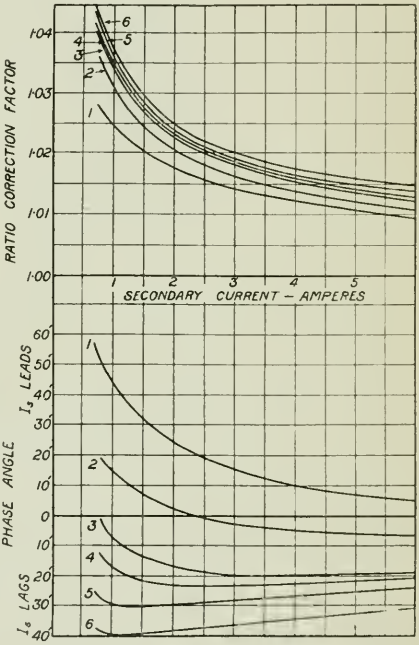

Fig. 8.—Curves for type 3 current transformer at 25 cycles; primary in position 1 to 6.

The ratio and phase-angle curves for a 1,600 to 5 ampere transformer with secondary burden of 11.2 volt-amperes at 90 per cent. power factor, 60 cycles, are given in Fig. 7 for the primary in the six positions listed above, curves 1 to 6 corresponding to positions 1 to 6. The corresponding curves for the same transformer and the same burden at 25 cycles are given in Fig. 8 for the same six positions of the primary.

The secondary leakage reactance in this transformer with primary in position 2 was about 3 ohms at 60 cycles, and with primary in position 5 about 7 ohms. These values are much larger than the total resistance in the secondary circuit, and the result is very low power factor of the total secondary circuit, which accounts for the secondary current lagging behind the primary current. For the same reason it is evident that the position of the primary has a greater influence on the characteristics of the transformer than a considerable change in the secondary burden because it produces a greater effect on the total impedance of the secondary circuit. These effects would be still more marked in a transformer of higher ratio, but less pronounced in one of lower ratio.

A transformer of this type in which the secondary winding is highly inductive is quite sensitive to frequency variations even with a non-inductive burden, but the effect of changes in primary position quite buries the effect of possible variation in frequency on a commercial system. Even a change in frequency from 60 to 25 cycles with constant primary position produces a smaller change in both ratio and phase-angle errors than does a change in primary from position 5 to position 2 with constant frequency. By comparing, for example, the phase-angle curve 5 in Fig. 7 with the phase-angle curve 5 in Fig. 8 the effect of a change in frequency will be seen. In contrast to this, by comparing phase-angle curves 5 and 2 in Fig. 7 the effect of a change in primary position will be seen. Similar relations exist in the corresponding ratio curves. It is interesting to observe that the variations in phase angle are approximately the same at the two frequencies, which is not true of the ratio errors.

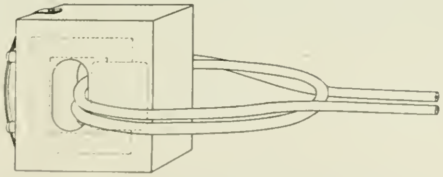

Fig. 9.—Type 3 current transformer; primary in position 2.

No general statement can be made as to the best arrangement of the primary for minimum errors in transformers of Type 3. In regard to the transformer tested, the best arrangement for average conditions would be position 2, that is, primary looped around the secondary coil as in Fig. 9. Stiff cable used in practice would probably not be bent smaller than 11 inches diameter which would give the ratio and phase angle curves marked 2 in Figs. 7 and 8. The transformer tested was not compensated, probably due to an oversight in construction. With proper compensation and properly disposed primary winding this type of transformer can be made to have reasonably small errors, but an indiscriminate arrangement of the primary may cause serious differences between actual and supposed transformation.

Summary—1. Ratio and phase-angle errors in through-type portable current transformers depend upon the arrangement of the primary winding. The nature and extent of the effect of changes in primary position depend upon the arrangement of the secondary winding on the core.

2. Transformers with circular cores and uniformly distributed secondaries are very little affected by changes in primary position, phase angle being practically independent and ratio varying less than 0.1 of one per cent. in the two transformers tested.

3. Transformers with rectangular cores and secondaries in two coils on opposite sides of the core are affected in both ratio and phase-angle errors by change in primary position. These changes were found to produce variations in phase angle of 2 minutes and in ratio about 0.1 of one per cent. in the transformer tested.

4. Transformers with rectangular cores and single secondary coils on one side of the core are very much dependent on primary position; ratio varying about 1 per cent., and phase angle about 1 degree in the transformer tested.

5. The position of the primary for minimum errors in measurements cannot be stated in general terms. It depends upon the type of transformer used, its compensation, the frequency, and the properties of the secondary burden. In addition, the kind of measurement must be considered since current measurement depends only on ratio, power factor by power-factor meters only on phase angle, while power measurement is affected by both quantities; ratio being more important at high power factors and phase angle at low power factors of load.

DETERMINATION OF TURN-RATIO OF CURRENT TRANSFORMERS

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Research Assistant

A simple method of finding the turn-ratio of current transformers without the use of auxiliary exploring-coils has been used in connection with investigations in the School of Engineering Research, University of Toronto. Nothing on this subject has hitherto been published as far as the writer is aware. Such a method and the resulting data might have several useful applications, two of which are here suggested.

(1) Manufacturers of current transformers would be able to check the turn-ratio of their product and detect any errors in compensation due to a mistake in the number of turns in either winding. This is mentioned especially because among a small number of transformers investigated two were found in which it was quite clear that errors had been made in the number of secondary turns, one being uncompensated and the other about 1 per cent. over-compensated.

(2) In certain methods of testing current transformers a knowledge of turn-ratio is required. See for example *Transmission Line Relay Protection*, by Woodrow, Roper, Traver, and McGahan, Proc. A.I.E.E., June 1919, p. 637, and *A Study of the Current Transformer With Particular Reference to Iron Loss*, by P. J. Agnew, Bulletin, Bureau of Standards, Vol. 7, No. 3, p. 431. The manufacturer's data on turn-ratio, even if obtainable, is not always correct as already noted.

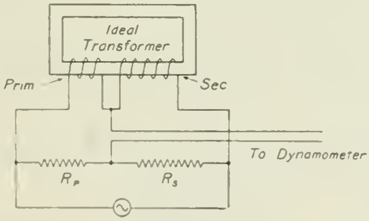

Fig. 1.—Simple bridge method of determining turn ratio of ideal transformer.

The general principle of the method is that in an ideal transformer without resistance or core reluctance an alternating voltage applied to the two windings connected in series in the same sense will be divided between them in the ratio of the number of turns in each. The turn-ratio might then be determined by a potentiometer or bridge balance method as suggested in Fig. 1. In actual transformers there are two conditions which modify the application of the above principle. These are copper resistance of the windings and leakage flux, by which is meant flux that does not completely interlink both windings and will, therefore, cause a greater average voltage drop per turn in one winding than in the other The resistance drops in the two windings, if not proportional to the number of turns in each, will cause a greater average voltage per turn in one winding than in the other and will, therefore, give slightly erroneous results. These conditions necessitate certain precautions the details of which are explained below.

The resistances of the two windings are made approximately proportional to turns by adding the necessary compensating resistance externally to one winding, usually the primary. If r_p and r_s are the primary and secondary resistances respectively and n the marked ratio of primary to secondary currents, the necessary extra primary resistance is $\frac{r_s}{n} - r_p$. If this gives a negative result it means that extra resistance is required in the secondary of value $(r_p \times n) - r_s$. Strictly n should be the turn-ratio and not the marked ratio, but the former is supposed to be unknown and in most cases will not differ from the latter more than 2 or 3 per cent. Also, the resistance drop in the windings will usually be quite small compared to the reactance drop since both current and resistance are small. The leakage flux is in most cases a more serious source of error than the resistance of the windings. Air forms part or all of the path of such flux, and the greater part of the reluctance of the leakage paths will, therefore, be in air which will cause the leakage flux to be proportional to and in phase with the current in the windings, while the core flux is approximately proportional to, and in quadrature with, the impressed voltage. These facts suggest that error may be avoided by using as an indicator of balance a sensitive dynamometer or A.C. galvanometer with its field separately excited in phase with the current in the transformer. The dynamometer will then be insensible

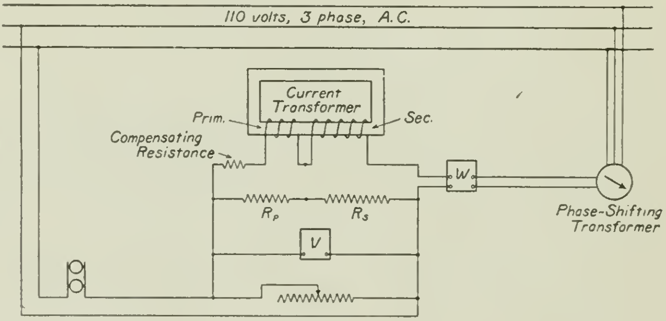

Fig. 2.—Diagram of connections to obtain voltage in quadrature to current through transformer.

to all voltages in quadrature with the current, that is, to voltages induced by the leakage flux.

In order to excite the dynamometer in phase with the transformer current a preliminary test is made in which a phase-shifting transformer as shown in Fig. 2, is adjusted to give a secondary voltage in quadrature with the current-transformer current. The dynamometer field current, adjusted in quadrature with the phase-shifter voltage, will then be in phase with the current in the current transformer. Observations are, therefore, made of the phase-shifter settings required to make the wattmeter *W* indicate zero for each of a number of voltages applied to the current transformer. If a phase-shifting transformer is not available the potentiometer scheme suggested in Fig. 3*a* may be used on three-phase circuits. The phase across which the potentiometer should be connected depends upon the phase rotation of the system and on the power factor of the current-transformer core, but is easily determined by trial. A similar scheme may be used on two-phase circuits. On single-phase circuits a condenser and resistor in series are connected across the line and the wattmeter is connected across the resistor as shown in Fig. 3*b*. A 100 ohm rheostat and 25 telephone condensers of 2 microfarads each give the required range of phase displacement at 60 cycles. The ratio of R_s to R_p should be made approximately equal to the expected turn-ratio so that subsequent alterations will be small.

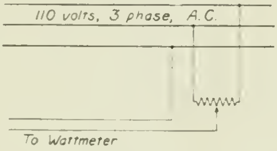

Fig. 3*a*.—Alternative method of obtaining quadrature voltage.

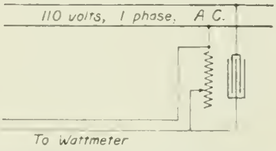

Fig. 3*b*.—Method of obtaining quadrature voltage from single phase circuit.

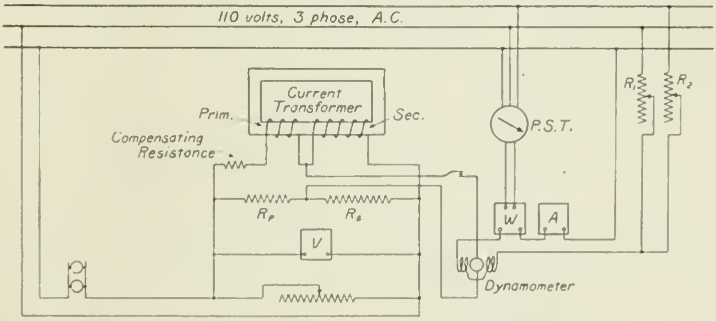

Fig. 4.—Connections for determining turn-ratio of current transformer.

The wattmeter is then removed from the current-transformer circuit and placed in the dynamometer field circuit and other connections are made as indicated in Fig. 4. With the same combinations of voltage and phase-shifter settings as found in the preliminary test, the dynamometer field current is adjusted to make $W=0$ by suitably proportioning the two resistances R_1 and R_2. On a single-phase circuit a combination of a variable reactor and resistor in series with the dynamometer field will effect the same result. The dynamometer field current is thus placed in quadrature with the phase-shifter voltage, which is itself in quadrature with the transformer current. Hence the dynamometer is excited in phase with the transformer current as required by the conditions. The non-inductive resistances R_s and R_p are then adjusted to make the dynamometer indicate zero. The ratio of R_s to R_p is then the turn-ratio of the transformer. The true values of the resistances R_s and R_p should be measured as accurately as possible for close results. Results should be the same for any applied voltage provided always that the dynamometer is excited in phase with the transformer current. A voltage which gives the maximum core power factor as indicated by the phase-shifter settings is a very good voltage to use, though somewhat higher voltages may also be used to advantage providing saturation is not too closely approached. The highest available commercial frequency is most suitable because it permits of higher transformer voltages with which closer balance can be obtained. A frequency of 60 cycles was used in obtaining the following results on two different types of current transformer. The dynamometer used gave a perceptible deflection (½ mm. at 1 metre radius) with 0.0003 volt on the moving coil and full field excitation of 2 amperes in phase with the voltage applied to the moving coil.

A 15,000 volt switchboard current transformer of marked ratio 150 and 75 to 5 amperes was tested. It was known to have 296 secondary turns and a total of 20 primary turns, or a true turn-ratio of 14.8. The primary resistance was 0.0085 ohm with sections connected in series and the secondary resistance was 0.455 ohm. Therefore extra primary resistance required to make the ratio of IR drops equal to the marked ratio of 75 to 5 was $\frac{0.455}{15}-0.0085=0.0219$ ohm. Phase-shifter settings (given as read on an arbitrary scale on which 90 degrees represents electrical quadrature) are included in the following results in which $\frac{R_s}{R_p}$ is the measured turn-ratio.

Impressed volts on C.T.	Phase-shifter settings	R_s	R_p	$\frac{R_s}{R_p}$	Per cent. error in turn-ratio measurement
75	68°	3005.9	203.08	14.8016	0.01 of 1 per cent.
60	$69\frac{1}{2}$°	3005.9	203.08	14.8016	0.01 of 1 per cent.
45	$72\frac{1}{2}$°	3005.9	203.08	14.8016	0.01 of 1 per cent.

It is not intended to claim the high accuracy in the measurement of R_s and R_p, which the number of significant figures used might seem to imply. However, extreme care was taken in arriving at the above values in order to determine whether there are any errors inherent in the method itself or whether the accuracy is limited only by the errors involved in the practical details of execution. The resistances given are the average of a number of measurements made on an unusually high grade certified bridge. In the determination of the ratio of two resistances, relative accuracy is the only essential and the relative accuracy of the bridge used is even better than its absolute accuracy. The order of magnitude of the possible error in measurement of the relative values of R_s and R_p would be about the same as the actual difference between the measured value and the true value of the turn-ratio, namely, 0.01 of 1 per cent.

A through-type portable current transformer of 1,600 to 5 amperes marked ratio was also tested. It had 320 secondary turns, which means that it was uncompensated though probably not so intended by the maker. In order to give a turn-ratio of 10 to 1, 32 turns were wound on as primary and the following results were obtained.

Impressed volts on C.T.	Phase-shifter settings	R_s	R_p	$\frac{R_s}{R_p}$	Per cent. error in turn-ratio measurement
60	54°	4999.6	500.16	9.996	0.04 of 1 per cent.
40	$56\frac{1}{2}$°	4999.6	500.16	9.996	0.04 of 1 per cent.
25	$61\frac{1}{2}$°	4999.6	500.16	9.996	0.04 of 1 per cent.

In both these cases the error in the determined turn-ratio is no greater than that which is to be expected due to the measurement of the resistances and hence we may reasonably conclude that any other errors inherent in the method are negligible. In most practical cases the limit of accuracy will be determined by the accuracy in resistance measurement, but in high ratio transformers, especially those with single-turn primary, the limiting factor may prove to be the sensibility of the dynamometer.

MINIMIZING THE ERRORS OF CURRENT TRANSFORMERS BY MEANS OF SHUNTS

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In the measurement of large quantities of energy for which correspondingly large charges are made a small percentage error in the current transformers operating the meters may involve quite a considerable amount of money. The maximum possible accuracy is, therefore, worth while in current transformers used for this class of service.

Current transformers are usually "compensated" for ratio by adjusting the number of secondary turns. This type of compensation will be termed "turns-compensation" to distinguish it from the "shunt-compensation" herein discussed.

Turns-compensation is accomplished by making the ratio of secondary to primary turns slightly less than the nominal ratio of primary to secondary currents. Since the secondary ampere-turns are equal to the vector difference between the primary ampere-turns and the exciting ampere-turns, the true ratio of primary to secondary currents is always greater than the ratio of secondary to primary turns. Turns-compensation, therefore, reduces the ratio error, but the correction is only an approximate one for average conditions of use. When it is considered that one current transformer is usually intended to serve for all frequencies from 25 to 133 cycles, and for a rated secondary burden of unstated power factor, it is apparent that under the particular conditions of frequency and burden in a given case, there will generally be room for further improvement in the accuracy of the ratio. The phase-angle error cannot be reduced by turns-compensation.

In any particular case it is usually possible to reduce both the ratio and phase-angle errors of a current transformer by suitable turns-compensation combined with the use of a non-inductive or a condensive shunt connected across either the primary or secondary terminals.

Primary shunts have advantages from another point of view, in that they tend to protect the current transformer and secondary circuit from the effect of high-frequency disturbances. Secondary shunts tend to protect only the secondary instruments in case high frequency currents are transmitted to the secondary. Condensive shunts are more effective for these purposes than non-inductive shunts since the impedance of the former to high-frequency currents is much less than their impedance at commercial frequencies, while the impedance of non-inductive shunts is independent of frequency. The protective action of the latter, therefore, depends upon the existence of a certain amount of inductance in the circuits across which they are connected.

TURNS-COMPENSATION

It has been stated that the secondary ampere-turns are equal to the vector difference between the primary ampere-turns and the exciting ampere-turns, or in symbols,

$$I_s n_s = I_p n_p - I_e n_p \text{ vectorially } = n_p (I_p - I_e)$$

Rearranging,

$$\frac{I_p - I_e}{I_s} = \frac{n_s}{n_p}$$

If there is no turns-compensation $\frac{n_s}{n_p}$ is the nominal ratio. The true ratio, $\frac{I_p}{I_s}$, is greater than $\frac{I_p - I_e}{I_s}$, therefore greater than $\frac{n_s}{n_p}$, therefore also greater than the nominal ratio. In other words, the secondary current is too low, the vector difference $I_p - I_e$ being usually numerically less than I_p. By reducing n_s to n_{s1}, that is by turns-compensation, the secondary current increases from I_s to I_{s1}. Assuming I_e to remain unchanged the secondary ampere-turns will be the same as before or $n_s I_s = n_{s1} I_{s1}$.

The new value of the true ratio would therefore be

$$\frac{I_p}{I_{s1}} = \frac{I_p}{I_s \times \frac{n_s}{n_{s1}}} = \frac{I_p}{I_s} \times \frac{n_{s1}}{n_s},$$

or, in other words, the true ratio would vary directly with the number of secondary turns, primary turns remaining constant. Such is not exactly true, however, because the increased secondary current requires proportionately increased secondary induced voltage, and the number of turns in which that voltage must be induced is smaller than before, hence the flux must increase as the square of the increase in current. This requires a greater exciting current than before, and therefore the percentage reduction in the ratio is somewhat less than the percentage reduction in secondary turns.

The ratio and phase-angle curves of a certain current transformer* with fixed burden and frequency and various compensations are shown in Fig. 1. The burden was 25 volt-amperes at $87\frac{1}{2}$ per cent. power factor, the frequency was 60 cycles, and the nominal ratio was one to one, the number of primary turns being 224. The numbers of secondary turns were 224, 223, 222 and 221 in curves *A*, *B*, *C*, and *D* respectively.

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*The current transformer used in this and the following measurements was of shell type. The frequency throughout was 60 cycles. The burden (except for Fig. 5) was 25 v.a., at 5 amperes and 87½ per cent. power factor; that is, with $R_s = 0.875$ ohms and $X_s = 0.487$ ohms.

Table I shows the percentage reduction in ratio, which is seen to be always less than the percentage reduction in turns.

TABLE I

Change in secondary turns	Per cent. change in turns	Per cent. change in observed ratio at 1 ampere secondary current	Per cent. change in observed ratio at 5 amperes secondary current
224-223	0.448	0.400	0.430
223-222	0.450	0.402	0.432
222-221	0.452	0.404	0.434

The number of figures shown in the values of per cent. change in observed ratio is not intended to imply such extreme accuracy in ratio observations. The last figures are estimated for the purpose of comparison with the per cent. change in turns.

The effect of turns-compensation is, therefore, to lower the whole ratio curve by an amount which is slightly less than the percentage reduction in secondary turns, and at the same time to slightly increase the slope of the ratio curve. The phase angle is not noticeably affected by a small percentage change in secondary turns,

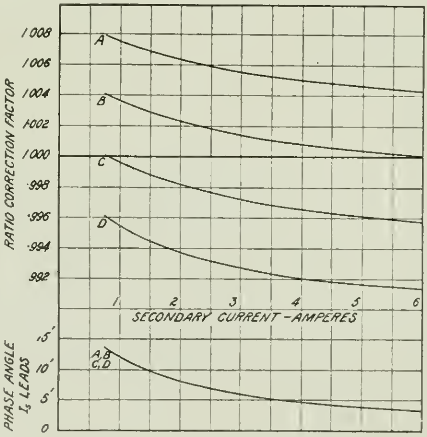

Fig. 1.—Ratio and phase-angle curves of a current transformer with four different turns-compensations. Primary turns, 224; Secondary turns: *A*, 224: *B*, 223; *C*, 222; *D*, 221.

In Fig. 1 the most favourable ratio curve would be either *B* or *C*. In most cases *B* would probably be preferred since its ratio error is less than that of *C* between 2 and 6 amperes secondary current. It would be still better if the ratio curve could be made to cross the line at about 4 amperes. However, that is not possible in this case by turns-compensation alone because the least change that can be made is one turn, which causes nearly 0.45 per cent. change in ratio.

SECONDARY NON-INDUCTIVE SHUNTS

The effect of a non-inductive shunt connected across the secondary terminals of a current transformer depends upon the characteristics of the secondary burden.

When the secondary burden is non-inductive the effect of a secondary non-inductive shunt will be to increase the ratio* without affecting the phase angle. If the resistance of the shunt is high compared to that of the secondary burden, the total impedance of the secondary circuit will not be appreciably affected by it, and the current in the secondary burden will be approximately $\frac{r_c}{r_c+R_s}$ times its former value, where R_s is the resistance of the secondary burden and r_c is the resistance of the non-inductive shunt. The new ratio will, therefore, be $\frac{r_c+R_s}{r_c}$ times its former value, or the percentage increase is $\frac{R_s}{r_c}\times 100$. The ratio curve can thus be raised by any desired amount according to the value of the resistance of the shunt, instead of by steps as in turns-compensation.

If the secondary burden is inductive the effect of a secondary non-inductive shunt will depend upon the magnitude and power factor of the burden. Consider first the effect in the case of an inductive burden of zero power factor if such were possible. The non-inductive shunt in this case would take a current in quadrature with the current in the burden, and if the resistance of the shunt were large enough to make its effect on the total secondary impedance negligible, the ratio would be unchanged, while the phase angle, considered positive when the secondary current leads with regard to the primary current, would be reduced by an angle $\tan^{-1}\frac{X_s}{r_c}$ where X_s is the reactance of the secondary burden. Since in most current transformers under operating conditions the secondary current leads, the secondary non-inductive shunt would thus reduce the phase-angle error.

In practice the secondary burden usually has a power factor between

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*In referring to current transformers provided with primary or secondary shunts, ratio is considered to mean the ratio of the primary line current to the current in the secondary burden, not the ratio of the currents in the primary and secondary windings.

50 and 100 per cent. depending on the frequency of the circuit and the type of instruments used. In such cases a secondary non-inductive shunt affects both ratio and phase angle, its effect on ratio being proportional to the resistance of the secondary burden and its effect on phase angle proportional to the reactance.

The ratio is increased approximately $\frac{R_s}{r_c} \times 100$ per cent. and the phase angle is reduced approximately by an angle $\tan^{-1} \frac{X_s}{r_c}$.

The effect of the shunt can be understood more readily by reference to the vector diagram, Fig. 2, in which I_s is the secondary current and I'_s its counterpart in the primary assuming a turn-ratio of one to one. V_s is the secondary terminal voltage leading I_s by an angle $\tan^{-1} \frac{X_s}{R_s}$, and I_c is the current in the non-inductive shunt which is in phase with V_s, and is represented in the primary by I'_c. E_s is the secondary induced voltage, and ϕ the flux producing it, while the exciting current I_e is required to maintain the flux. I_e and I_c are both exaggerated in this diagram for the sake of clearness. Without any secondary shunt the primary current would be represented by the broken line I'_p. With a shunt on the secondary the primary current must be equal to I_p to hold the same secondary current. The ratio $\frac{I_p}{I'_s}$ is greater than $\frac{I'_p}{I'_s}$, but the phase angle between I_p and I'_s is less than that between I'_p and I'_s. The current in the secondary shunt, or rather its counterpart in the primary, is seen to have the same effect as so much addition to the exciting current.

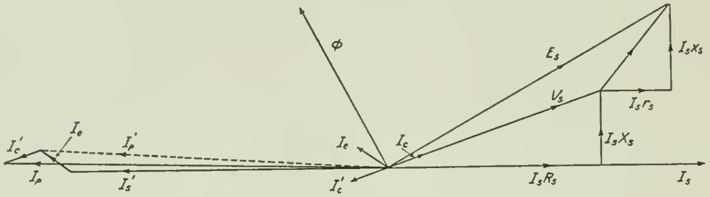

Fig. 2.—Vector diagram of current transformer with a non-inductive shunt in the secondary circuit.

In the case of the transformer before referred to, with 222 secondary turns and a burden of 25 volt-amperes at 87½ per cent. power factor, 60 cycles, using a secondary non-inductive shunt of 220 ohms, curves A, Fig. 3, were obtained. The improvement over curves B and C, which are the same as B and C in Fig. 1, is obvious. In this case R_s is 0.875.

X_s is 0.487, and r_c is 220 ohms. The calculated change in ratio is $\frac{R_s}{r_c}\times 100=\frac{0.875}{220}\times 100=0.4$ per cent.

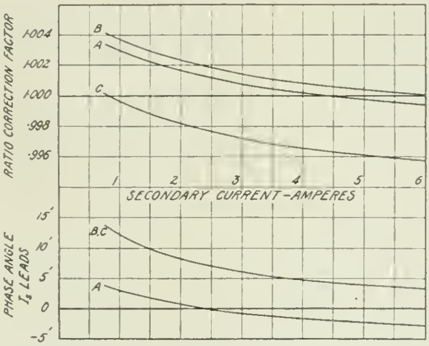

Fig. 3.—Curves showing effect of a non-inductive shunt in the secondary circuit. *A*, secondary turns 222, secondary non-inductive shunt of 220 ohms. *B* and *C*, same as in Fig. 1, (no sec. shunt).

Similarly the calculated change in phase angle is $\tan^{-1}\frac{X_s}{r_c}=\tan^{-1}\frac{0.487}{220}=\tan^{-1}0.0022=7\frac{2}{3}$ minutes. The observed changes at different secondary currents are shown in Table II. The differences between the calculated and observed changes is due to the effect of the shunt on the total secondary impedance, and perhaps also to small inaccuracies in the observations.

TABLE II

Secondary amperes	1	2	3	4	5	6	Average observed change	Calculated change
Per cent. ratio change	0.37	0.38	0.39	0.4	0.4	0.4	0.39	0.40
Phase-angle change in minutes	9'	7'	$6\frac{1}{3}$'	6'	6'	6'	$6\frac{3}{4}$'	$7\frac{2}{3}$'

Predetermination of the effect of a secondary non-inductive shunt requires only a knowledge of the resistance of the shunt and of the resistance and reactance of the secondary burden. Conversely the resistance of the secondary shunt required to effect desired changes in ratio or phase angle may be calculated if the constants of the secondary burden be known. For example, in Fig. 1, curve *C*, at 4 amperes secondary current the ratio is 0.375 per cent. low, and the phase angle 4 minutes high. The approximate shunt resistance required to make the ratio correct at 4 amperes is $\frac{0.875}{0.375}\times 100=234$ ohms. The approximate shunt resistance required to make the phase angle zero at 4 amperes is $\frac{X_s}{\tan 4'}=\frac{0.487}{0.0012}=410$ ohms. Since both ratio and phase angle cannot be made simultaneously zero at 4 amperes secondary current, it must be decided which error it is more important to eliminate or else a compromise has to be made

A secondary non-inductive shunt will not reduce the ratio error unless the turns-compensation is such that the ratio is too low. If ratio is too high it becomes necessary to reduce secondary turns if improvement is to be made by this type of shunt-compensation. The number of turns on the secondary of a transformer already built cannot readily be changed. However, it is sometimes possible to pass an extra turn or two through the core and insulate them sufficiently from the primary. If connected in series with the secondary and in the same sense on the core, the total number of secondary turns is increased; while, if opposing the secondary, the effect is practically the same as if an equal number of secondary turns were removed.

Non-inductive resistors suitable for use as secondary shunts may be easily and cheaply obtained in compact form from various manufacturers. The normal voltage that secondary shunts are required to stand would be from 10 to 15 volts, but the voltage might rise to several times this value for short periods as, for instance, when an increase in current is occasioned by a heavy overload or a short circuit on the system. A high voltage would also result from a break in the secondary circuit of the current transformer, and the shunt would quickly be burnt out.

SECONDARY CONDENSIVE SHUNTS

A condenser connected across the secondary terminals of a current transformer in parallel with the burden will take a current which is proportional to the secondary terminal voltage and approximately in quadrature with it. Replacing I_c and I'_c in Fig. 2 with a leading current such as would be taken by a condenser, and following a similar course of reasoning to that given in regard to the secondary non-inductive shunt, the effects of a secondary condensive shunt will readily be seen. Its effect on ratio will be proportional to the reactance of the secondary burden, and its effect on phase angle proportional to the resistance, which are converse effects to those of a non-inductive shunt. Assuming that the condenser does not appreciably affect the total secondary impedance, that the voltage applied to it is a sine wave, and that its power factor is zero, it will *reduce* the ratio by an amount $X_s\,C\omega\times 100$ per cent., and lower the phase angle by an angle $\tan^{-1} R_s\,C\omega$, where

X_s and R_s are the constants of the secondary burden, C the capacity of the secondary condensive shunt in farads and ω is $2\pi f$ where f is the frequency of the circuit.

Actually there are several factors which make the above assumptions untrue so that the formulae are only approximate, though in many cases the results will hold with sufficient accuracy for practical purposes.

The power factor of a condenser is to all intents and purposes zero, its phase angle being very nearly 90 degrees. The impedance of the shunt may be of the order of magnitude of one per cent. of the impedance of the secondary burden which will cause a very small change in the total impedance of the secondary circuit. On the other hand the secondary voltage will scarcely ever have the assumed sine form and in some cases it may be something quite different. The wave form of the secondary voltage is influenced by at least three factors. If the primary current is not a sine wave it is practically certain that neither the secondary current nor secondary terminal voltage will be a sine wave. If the primary current is a sine wave the secondary current will not be quite free from harmonics because of the harmonics in the exciting current. If the secondary current is a sine wave the secondary terminal voltage will not be so unless the secondary circuit contains no iron core devices such as relays or induction meters which have cyclically varying impedances. The matter of harmonics and frequency variation is of more importance in condensive shunts than in non-inductive shunts which do not change their impedance with frequency. Primary current wave form may vary greatly, being much distorted under certain conditions, such as in a synchronous motor running light at unity power factor, while the same machine under load would take a current of much smoother form. Shunt-compensation, by means of a condenser, might be correct for the latter case, but it might cause serious errors in the former. However, the measurements made at light load would probably not be so important.

Paper telephone condensers in units of ½ to 2 microfarads are inexpensive, compact and otherwise well suited for use as condensive shunts. They are usually tested to 500 volts d.c., which is ample for any but very abnormal conditions, such as a severe short circuit on the system or an open secondary circuit which would cause voltages endangering the condenser insulation.

Fig. 4 gives the ratio and phase-angle curves of the current transformer already mentioned, with the same conditions of burden and frequency as in Fig. 1. Curves *B* and *C* are obtained with 223 and 222 secondary turns respectively, and no secondary shunt. Curve *A* results when a secondary condensive shunt of about 7 microfarads is used with 223 turns in the secondary. The calculated effect of a 7 microfarad

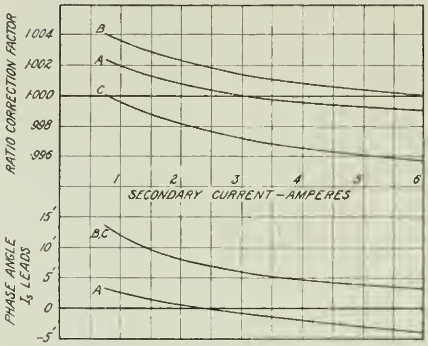

Fig. 4.—Curves showing effect of a condensive shunt in the secondary circuit. *A*, secondary turns 223, secondary condensive shunt of 7 m.f.d. *B* and *C*, same as in Fig. 1 (no sec. shunt).

shunt is to reduce the ratio $0.487 \times 7 \times 10^{-6} \times 377 \times 100$ per cent. or 0.13 per cent., and to lower the phase angle by $\tan^{-1} 0.875 \times 7 \times 10^{-6} \times 377 = \tan^{-1} 0.0023$ or 8 minutes. The observed changes, that is the differences between curves *A* and *B*, Fig. 4, are given in Table III.

TABLE III

Secondary amperes	1	2	3	4	5	6	Average observed change	Caclculated change
Per cent ratio change	0.10	0.11	0.12	0.12	0.12	0.12	0.115	0.13
Phase-angle change in minutes	$9\frac{1}{2}'$	$7'$	$6\frac{1}{2}'$	$6\frac{1}{2}'$	$7'$	$7'$	$7.2'$	$8'$

The secondary condensive shunt is useful when the original ratio is too high and the turns-compensation cannot be changed, since it reduces the ratio. The required capacity of a secondary condensive shunt to effect desired changes in ratio or phase angle may be calculated approximately from the formulae already given. It will generally be found that different values of capacity are required for the desired ratio and phase angle corrections. In such cases a compromise has to be made.

The above discussion has been on the application of condensive shunts to current transformers operating on a fixed burden. A secondary condensive shunt is advantageous also in current transformers not operating on a fixed burden, as, for instance, portable current transformers in which the secondary burden may be frequently changed. If the burden is small, in the limit zero, a secondary shunt has little or no effect since it is practically short-circuited. The ratio and phase angle would, therefore, be unchanged by the removal of the secondary shunt. Assuming the shunt to be removed, an increase of the secondary burden would tend to raise the ratio and to increase the phase angle, if the power factor of the secondary burden is greater than that of the secondary winding, as is usually the case with portable current transformers supplying meters. With a secondary condensive shunt the shunt tends to neutralize the above effects of increasing burden and hence to keep the ratio and phase angle curves nearly constant over a considerable range of secondary burdens and power factors. If these sets of nearly identical curves can be located near the line of zero errors, a very desirable condition is realized. In order to do this, the transformer must be so designed that the errors are a minimum with the secondary short-circuited. To keep the phase angle small under this condition the transformer must have considerable leakage reactance so that the power factor of the secondary winding will be low and about equal to that of the core. Turns-compensation may be employed to make the ratio as near correct as possible with secondary short-circuited. A ratio and phase angle test should then be made at rated frequency, using a burden which in magnitude and power factor represents the average burden which the transformer will be called upon to supply. From this data the required capacity can be calculated or the capacity can be determined experimentally during the ratio and phase angle test.

Such a current transformer equipped with a secondary condensive shunt, if designed for minimum errors at one given frequency, will not have such good characteristics at a different frequency. It is also subject to wave-form errors as before shown.

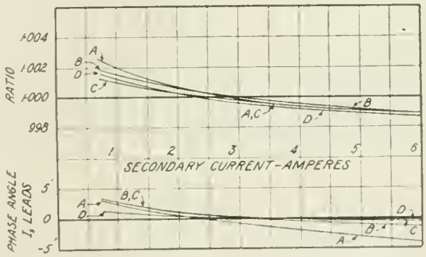

Fig. 5.—Curves showing effect of various burdens. Prim. turns 224; sec. turns 223; sec. shunt 7 m.f.d. Burden: *A*, 25 v.a. @ 87½ % p.f.; *B*, 16 v.a. @ 93½ % p.f.; *C*, 8.55 v.a. @ 93.9 % p.f.; *D*, sec. short-circuited.

An example of the results of this type of design is given in Fig. 5, which shows ratio and phase angle curves of the transformer of Fig. 1 with various burdens made up of random combinations of Weston dynamometer-type portable indicating ammeters and wattmeters. The frequency was 60 cycles and the capacity of the secondary shunt 7 microfarads.

Non-inductive shunts are not applicable to this type of service as they cause the ratio to vary more with change in burden than if no shunt were used.

COMBINATIONS OF SECONDARY NON-INDUCTIVE AND CONDENSIVE SHUNTS

As has been shown, it rarely occurs that a given shunt will make the desired corrections in both ratio and phase angle, though it may reduce both these errors. However, with the proper turns compensation, a combination of a non-inductive shunt with a condensive shunt connected in parallel across the secondary terminals will often make possible the desired changes in both errors. Equating the desired percentage *reduction* in ratio to $100\left(X_s C\omega - \frac{R_s}{r_c}\right)$ and the desired reduction in phase angle to $\tan^{-1}\left(R_s\, C\omega + \frac{X_s}{r_c}\right)$ and solving as simultaneous equations, values of C and r_c are obtained which denote approximately the required constants of the condensive and non-inductive shunts respectively. If the results are negative, or not real quantities, it means that it is not possible to completely neutralize both errors with the existing turns-compensation. If secondary turns can be changed, say from n_s to n_{s1}, recalculation should be made on the assumption of this change in secondary turns. Thus the desired percentage *reduction* in ratio is equated to $100\left\{(X_s\, C\omega - \frac{R_s}{r_c}) + \frac{(n_s - n_{s1})}{n_s}\right\}$. It may be necessary to make n_{s1} one or possibly two turns greater or less than n_s according to conditions. Conditions are sometimes such that no change in secondary turns will make a real solution possible. Only partial compensation can then be obtained.

SECONDARY INDUCTIVE SHUNTS

By reference to Fig. 2 it will be seen that a secondary inductive shunt, taking a current lagging with regard to V_s, will have little or no beneficial effect on either ratio or phase angle.

Some experiments were made in an attempt to use the saturation properties of an iron core inductive shunt in order to flatten out the ratio curve while employing turns-compensation to correct the ratio and non-inductive or condensive shunts to compensate for phase angle. The conclusion was reached that this is not practically feasible because, though part of the ratio curve may be flattened, its slope is increased in other parts, and ratio rises rapidly as the current increases above its rated value, due to saturation in the iron core impedance.

PRIMARY NON-INDUCTIVE SHUNTS

A non-inductive resistance shunted across the primary windings of a current transformer has the same general effect as a secondary non-inductive shunt, viz., it increases the ratio and reduces the phase angle. The effect on ratio is proportional to the total equivalent resistance of the transformer and burden in terms of the primary circuit, and the effect on phase angle is proportional to the total equivalent reactance. If the power factor of the primary and secondary windings is less than that of the burden, as is frequently the case, the effect on phase angle as compared to the effect on ratio will be greater than with a secondary non-inductive shunt. This is generally to be desired.

A primary non-inductive shunt increases the ratio by approximately $\frac{R_t}{r_c} \times 100$ per cent., and reduces the phase angle by an angle $\tan^{-1} \frac{X_t}{r_c}$, where R_t and X_t are respectively the total equivalent resistance and reactance of the transformer and burden in terms of the primary, and r_c is the resistance of the primary non-inductive shunt. These formulae are approximate for reasons similar to those previously mentioned in connection with secondary non-inductive shunts.

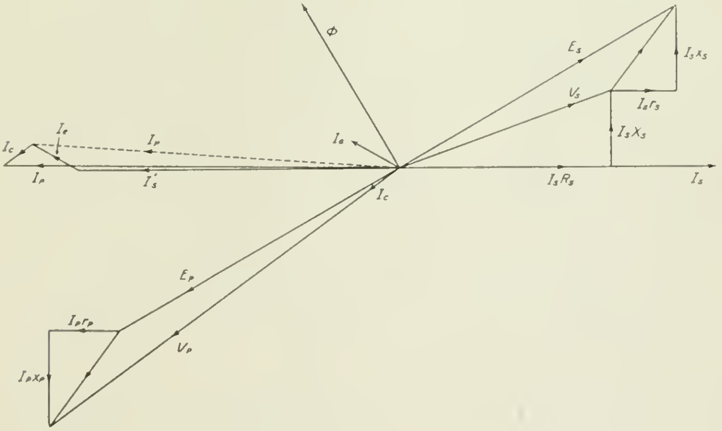

Fig. 6.—Vector diagram of a current transformer with non-inductive shunt in the primary.

Fig. 6 is a vector diagram illustrating the effect of a primary non-inductive shunt. By comparison with Fig. 2, the notation of which is similar, the difference between the effects of primary and secondary non-inductive shunts will be observed.

For illustration actual ratio and phase angle curves of a current transformer with and without a primary shunt are given in Fig. 7. Curves *B* and *C* are the same as curves *B* and *C* in Fig. 1, the transformer being the same and conditions the same. Curves *A* are obtained with a 4,000 ohm primary shunt with 223 turns in the secondary. As regards ratio, curve *A* is not as good as curve *B*, but the phase angle is reduced to the minimum. If 222 secondary turns are used with a 4,000 ohm primary shunt, curve *D* results, but *D* is not as good as *B*, particularly near rated current. Cases such as this will occur in which the reduction of phase angle error is accomplished at the expense of ratio no matter how the turns-compensation is adjusted. Some other kind of shunt would be necessary in such cases, for instance, a secondary non-inductive or primary or secondary condensive shunt or some combination of them. As an alternative some resistance may be deliberately added in series with the secondary burden and 222 secondary turns used with a primary non-inductive shunt of resistance which might need to be somewhat less than 4,000 ohms. The increased burden would tend to bring ratio curve *D* up to the desired position and the simultaneous tendency to increase the phase angle would be counteracted by a change in the value of resistance of the shunt.

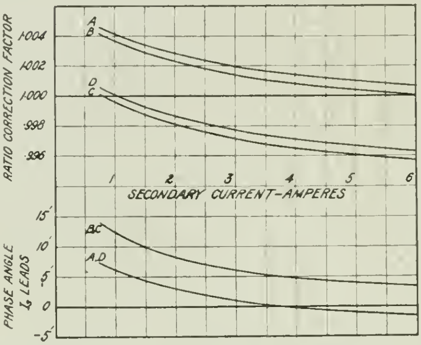

Fig. 7.—Curves showing effect of a non-inductive shunt in the primary. *A*, sec. turns 223, prim. non-inductive shunt of 4000 ohms. *B* and *C*, same as in Fig. 1 (no primary shunt). *D*, sec. turns 222, prim. non-inductive shunt of 4000 ohms

The calculated change in ratio due to the shunt in Fig. 7 is $\frac{R_t}{r_c} \times 100 = \frac{2.48}{4000} \times 100 = 0.06$ per cent. The calculated change in phase angle is $\tan^{-1} \frac{6.87}{4000} = \tan^{-1} 0.0017 = 6$ minutes. Observed changes are given in Table IV.

A primary non-inductive shunt has another kind of advantage, namely, that it serves as a by-pass for high frequency current surges in

TABLE IV

Secondary amperes	1	2	3	4	5	6	Average observed change	Calculated change
Per cent. ratio change	0.04	0.04	0.04	0.05	0.06	0.06	0.05	0.06
Phase-angle change in minutes	6′	5′	5′	5′	5′	4½′	5′	6′

the main line which might otherwise injure the insulation of the current transformer.*

PRIMARY CONDENSIVE SHUNTS

A condenser connected across the primary terminals of a current transformer has qualitatively the same effect on the ratio and phase angle as one connected across the secondary. The ratio is decreased $X_t C\omega \times 100$ per cent., and the phase angle is reduced by an angle $\tan^{-1} R_t C\omega$, approximately. As an example Fig. 8 again reproduces curves A, B and C of Fig. 1, while curves D represent the ratio and phase angle with a 2 microfarad primary shunt, with 224 secondary turns which is also the number of primary turns. The calculated effect on ratio is

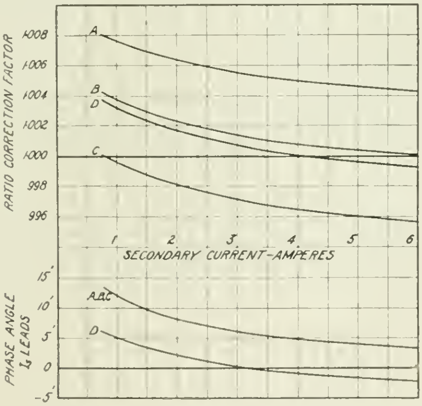

Fig. 8.—Curves showing effect of a condensive shunt in the primary. A, B and C, same as in Fig. 1. D, sec. turns 224, primary condensive shunt of 2 m.f.d.

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*Voltage Rises in Current Transformers, E. Wirz and Schweiz. Elektrotek-Verein, Bull. 7, p. 121, July, 1915.

Protection of Current Transformers against Voltage Rises, E. Wirz, Elekt-Zeits., 36, p. 450 and 467, Sept., 1915.

Protection of Current Transformers against Voltage Rises, E. Wirz and Geweche, Elekt.-Zeits., 37, p. 69, Feb., 1916.

Insuring Protection of Current Transformers, V. Karapetoff, Electrical World, 73, p. 473, 1910.

$6.87 \times 2 \times 10^{-6} \times 377 \times 100$ or 0.51 per cent., and on phase angle $\tan^{-1} 2.48 \times 2 \times 10^{-6} \times 377$ or $\tan^{-1} 0.00187$, *i.e.* $6\frac{1}{2}$ minutes. The observed changes, *i.e.* the differences between curves *A* and *D* are tabulated in Table V.

TABLE V

Secondary amperes	1	2	3	4	5	6	Average observed change	Calculated change
Per cent. ratio change	0.42	0.46	0.48	0.48	0.48	0.48	0.47	0.51
Phase-angle change in minutes	$7\frac{1}{2}'$	$6'$	$6'$	$6'$	$5\frac{1}{2}'$	$5\frac{1}{2}'$	$6'$	$6\frac{1}{2}'$

In curve *D* we have the peculiar condition that, although the transformer has equal primary and secondary turns, the true ratio falls below the turn ratio, or the secondary current is greater than the line current above 4 amperes. This is due, of course, to a partial resonance effect between the condenser connected across the primary and the reactance of the transformer and burden.

In current transformers of high ratio the primary impedance is low and the capacity required for a primary condensive shunt might be quite high. It would then be more economical to use a primary non-inductive shunt.

The primary condensive shunt as a corrective device is subject to disturbing effects of harmonics and frequency variation. It will pass high frequency current surges even more readily than the non-inductive shunt.

COMBINATIONS OF SHUNTS

Any or all of the shunts herein mentioned may be used simultaneously, and the effects of each will be added to those of the others, and may be calculated from the given formulae. The accuracy of the calculations depends upon the extent to which the assumptions on which the formulae are based, are realized.

SUMMARY

1. Primary and secondary non-inductive shunts tend to increase the ratio of a current transformer and reduce the angle of lead of the secondary current with regard to the primary current.

2. Primary and secondary condensive shunts tend to decrease the ratio of a current transformer and reduce the angle of lead of the secondary current with regard to the primary current.

3. With suitable turns-compensation one or more of the above mentioned shunts may be used to reduce the ratio and phase angle errors of a current transformer operating with fixed burden and frequency. The constants of the shunts may be calculated approximately from the formulae given if the required changes in the ratio and phase angle and the constants of the transformer and burden are known.

4. Condensive shunts may, under certain conditions, be useful in current transformers without fixed secondary burden.

5. Inductive shunts are not generally useful for reducing current transformer errors.

6. Primary shunts have the advantage of protecting the transformer and burden against high frequency current surges. Secondary shunts tend to protect only the secondary burden.

TRUE NET SECTIONS OF RIVETED TENSION MEMBERS

By C. R. Young

Associate Professor of Structural Engineering

Comparison of the net areas of square and zigzag sections across a riveted tension member and the selection of the least, does not form the correct basis for arriving at the capacity of the member. Nevertheless, this method is commonly followed. In a number of structural handbooks there are tables for ready use giving the stagger required to maintain net section, and the rule for their application is that if the stagger is ever so little less than this critical one, the entire area of the second hole must be deducted. Thus referring to Fig. 1(a), if s is less than $(2gh+h^2)^{\frac{1}{2}}$, two full holes must be deducted, whereas if s is greater than $(2gh+h^2)^{\frac{1}{2}}$, only one hole is to be deducted. Similarly, for the case shown in Fig. 1(b), three full holes are to be deducted if s is less than $(2gh+h^2)^{\frac{1}{2}}$.

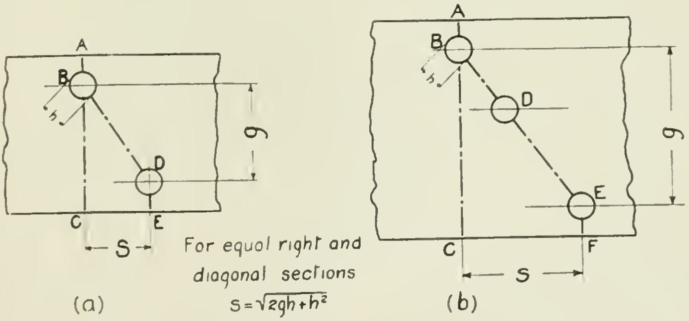


Fig. 1—Stagger Ratio to Maintain Net Section

Apart from the illogical procedure involved in deducting a full hole when the stagger is but slightly under the critical one for maintaining equal sections, the existence of a component of shearing stress on each diagonal section renders the familiar handbook method incorrect. This shearing stress, when combined with the normal stress on the section, produces a resultant stress greater than that on a right section of the same area.

Several writers have already pointed out the theoretically correct method of computing deductions and its agreement with actual experiment. Victor H. Cochrane, in *Engineering News* of April 23, 1908, indicated the method; T. A. Smith, in *Engineering News* of May 6, 1915, submitted a diagram based on true maximum stresses, from which the number of holes to be deducted for 7/8-in rivets could be determined; and D. B. Steinman, in *Engineering News-Record* of June 14, 1917, proposed two alternative approximate rules for calculating the correct deduction. The formula for determining the theoretically correct deduction may be conveniently established in the following manner.

Let a flat tension member of breadth b and thickness of one unit be subjected to a tensile stress, which for a right net section AF or GK through any rivet hole (Fig. 2), has an *average* value of f lb. per sq. in. Let the gauge lines be g in. apart, the rivet stagger be s in., and the diameter of holes for deduction purposes be h (diameter of rivet plus 1/8 in.).

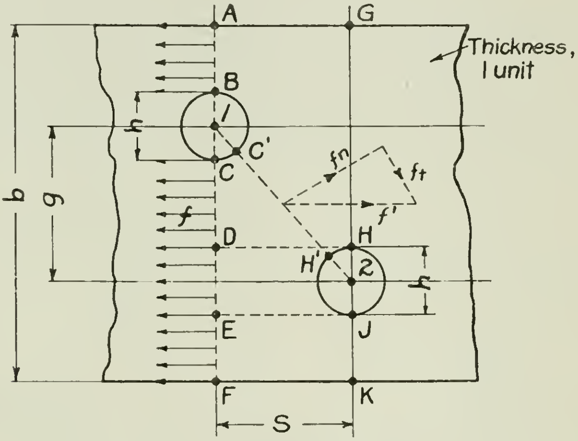


Fig. 2—Relation of Stresses on Right and Inclined Sections

The apparent total stress on the strip CE is then fg, but since the resistance on the portion DE may be lowered by the proximity of rivet 2, the average intensity of stress on CE must be

$$f \times \frac{g}{g - xh}$$

where x is some fraction of one hole. For equalized strength on a right section passing through hole No. 1 and on a zigzag section such as A12K, the value of x should be such that the normal stress on CE should not be exceeded by the maximum diagonal stress on the inclined section $C'H'$.

The intensity of stress on C′H′ parallel to the axis of the member, must be such that this area will resist the total stress on the strip CE, that is fg lb. It, therefore, equals

$$f' = \frac{fg}{\sqrt{g^2+s^2}-h}$$

The normal stress on C′H′ is then

$$f_n = f' \times \frac{g}{\sqrt{g^2+s^2}}$$

$$= \frac{fg^2}{g^2+s^2-h\sqrt{g^2+s^2}}$$

The tangential stress on C′H′ is

$$f_t = f_n \times \frac{s}{g} = \frac{fgs}{g^2+s^2-h\sqrt{g^2+s^2}}$$

But in accordance with the well-known principle of compound stress the maximum stress on the section C′H′ is

$$f_m = \frac{1}{2}(f_n + \sqrt{f_n{}^2+4\,f_t{}^2})$$

$$= \frac{f_n}{2}\,(1+\sqrt{1+4\frac{s^2}{g^2}})$$

Introducing the value of f_n found above,

$$f_m = \frac{fg(g+\sqrt{g^2+4\,s^2})}{2(g^2+s^2-h\sqrt{g^2+s^2})}$$

But this maximum intensity of stress on C′H′ should, if the proper rivet hole deduction has been made, be equal to the intensity on the right section CE, that is

$$f_m = f \times \frac{g}{g-xh}$$

Solving for x, we obtain

$$x = \frac{g}{h} - \frac{2(g^2+s^2-h\sqrt{g^2+s^2}}{h(g+\sqrt{g^2+4\,s^2})} \qquad \text{(1)}$$

If more than two gauge lines are used, the deduction for a rivet on the third gauge line would be calculated in the same manner. The total number of rivet holes to be deducted from the gross right sectional area of a tension member to give true effective section is, therefore,

$$n = 1+x_1+x_2+x_3+ \qquad \text{(2)}$$

The terms x_1, x_2, etc., represent the fractions of successive holes that are to be deducted in addition to the whole of the first hole on the section considered. Their values are found from Eq. (1) by the insertion of the appropriate values of g and s.

Formula (2) is to be applied to alternative sections, the successive terms representing the deductions for successive holes considered in a chain across the member. The particular group of rivets to be considered is that which will give the greatest total deduction, whether the rivets lie on adjacent gauge lines or not.

The theory considered above is fortunately well supported by experimental research. Tests made on riveted tension splices by the Deutscher Eisenbauverband, and reported in 1915, showed that the strength of net sections was in entire accord with the indications of this theory.

APPROXIMATE RULES

The labour involved in applying formula (1) to alternative sections would preclude its use in the form given on all except the most important work. Fortunately, it is possible to devise approximate rules based on the correct theory which will give results sufficiently accurate for most purposes.

Early efforts to frame specifications so as to take account of the observed tendency of joints to fail along zigzag lines unless a certain excess of material were provided along such sections resulted in rules such as the following:

"Deduct from the gross transverse section all holes either in a transverse line or in a diagonal line when the net section of the latter does not exceed the former by more than 30%." (Greiner's *General Specification for Bridges*, 1911).

"All rows of rivet holes shall be deducted unless so staggered that the net section along a zigzag line, taking all distances in a diagonal direction at only three-fourths of their value, exceeds the ccrresponding net section taken square across the shape or plate." (Quoted by D. B. Steinman.)

Such rules are unsatisfactory in character. In the first the percentage increase is required on all portions of the zigzag section, regardless of their inclination to the axis of the member. Obviously this procedure is 30% in error for right sections and only a little less so for those that are nearly normal to the axis. The rule, as a whole, is consequently too severe. The deduction of 25% from the diagonal distances provided in the second rule is also excessive, since application of the correct theory, now amply supported by experimental investigation, shows that for the critical diagonal section which makes an angle of 55°

with a right section, the effect of shear is to increase the stress by only about 16%.

A popular form of specification recognizing, although inadequately, the weakness of diagonal sections is one such as the following:

"The greatest number of rivet holes which can be cut by a transverse plane, or come closer than 2 in. of it, are to be deducted from the gross area of the section."

Whether the exact distance specified be 2 in. or some similar dimension, the rule is dangerous in that it takes no account of the effect of gauge, or distance between rivet lines. While it is safe and proper to neglect entirely a second $\frac{3}{4}$-in. hole staggered 2 in. with respect to a first when the gauge is $1\frac{1}{2}$ in., it is not correct to neglect the second hole when the gauge is over that amount. When it is 3 in., 0.56 of the second hole should be deducted, and when it is 6 in., the portion is 0.87. If the holes be 1 in. diameter, a stagger of 2 in. with a gauge of 2 in. would require the deduction of 0.40 of the second hole, while with a gauge of 6 in. the deduction would be 0.90 hole. To give, therefore, a rule which is independent of gauge is to lead to serious error, particularly where the gauges are large.

GRAPHICAL COMPARISON OF RULES

In order to establish a rational basis for an approximate rule for net sections there have been plotted in Fig. 3 curves giving the correct deductions required in certain critical and in certain typical cases. The most serious spacing of rivets, so far as deductions are concerned, is that pitch which will cause the rivets to be the minimum allowable distance apart, say 3 diameters, centre to centre. Accordingly, there have been plotted in Fig. 3 curves giving the correct deductions for $\frac{5}{8}$-in. and $\frac{7}{8}$-in. rivets for minimum diagonal pitches giving stagger ratios varying from 0 to 1.60 and from 0 to 1.57 respectively. The gauges vary from 1.88 in. to 1.0 in. and from 2.63 in. to 1.4 in. respectively. The curves thus represent the correct deductions for the full range of applicable stagger ratios with the diagonal spacing always at the permissible minimum. Curves have also been plotted, giving the correct deductions for $\frac{5}{8}$-in. rivets for the appropriate range of stagger ratios with gauges of 3 and 6 in. Corresponding curves for $\frac{7}{8}$-in. rivets with gauges of 3 and 6 in. are also shown.

From an examination of these curves it is evident that

(1) For a given stagger ratio and a given gauge the fraction of a hole to be deducted is greater for large than for small rivets.

(2) For a given size of rivets and stagger ratio, the fractional deduction is greater for small gauges than for large ones.

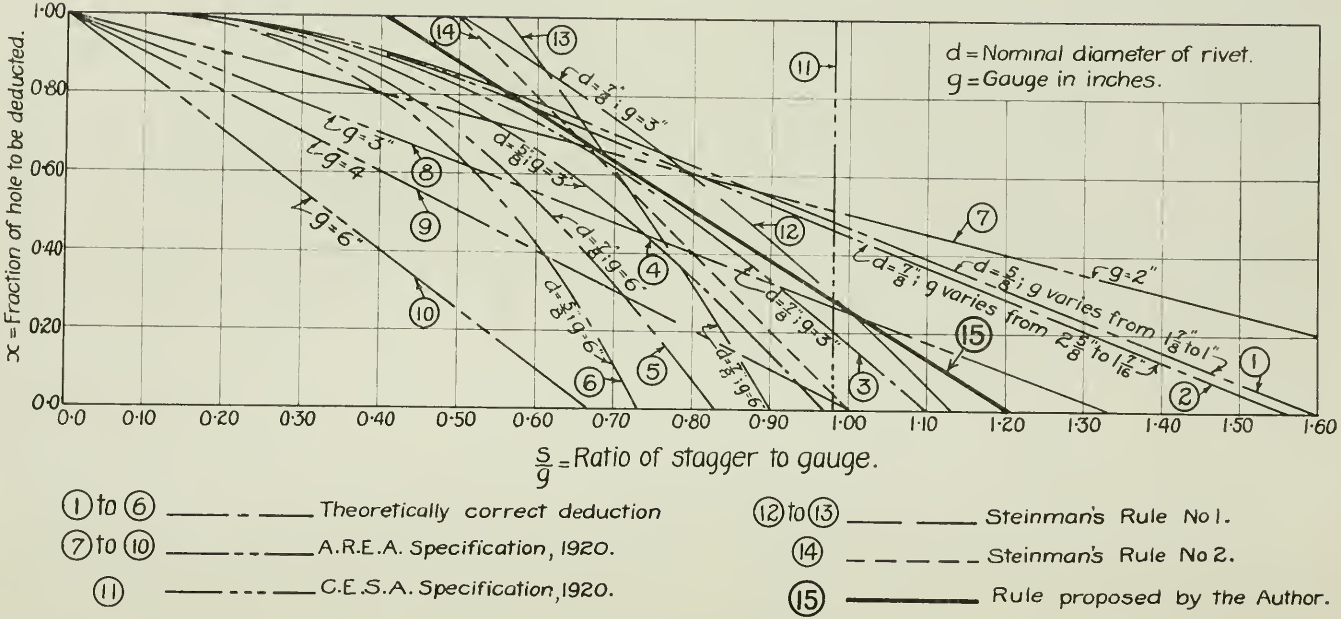


FIG. 3—COMPARISON OF APPROXIMATE DEDUCTION RULES

With Fig. 3 as a basis it will be useful to examine some of the approximate deduction rules that have been proposed in order to ascertain whether they are sufficiently accurate to be satisfactory.

Some recognition of the influence of the stagger ratio on net area is contained in the deduction rule adopted in the 1920 specification of the American Railway Engineering Association. This rule is as follows:

"The net section of riveted members shall be the least area which can be obtained by deducting from the gross sectional area the areas of holes cut by any perpendicular to the axis of the member and parts of the areas of other holes on one side of the plane within a distance of 4 inches, which are on gauge lines one inch or more from those of the holes cut by the plane, the parts being determined by the formula:

$$A=(1-\frac{p}{4})$$

where A = area of one hole,

p = distance in inches of the centre of the hole from the plane."

In Fig. 3 there have been plotted for various stagger ratios the deductions required under this rule for gauges of 2, 3, 4, and 6 in. Comparing the lines so drawn with the curves for correct deductions it is seen that for all gauges with moderate stagger ratios the deductions are too small. For the smaller gauges and larger stagger ratios the deductions may be too large. It should be noted that the number of holes to be deducted is independent of the size of the rivet.

An approximate rule established in the specifications for steel railway bridges of the Canadian Engineering Standards Association is as follows:

"There shall be deducted from each member as many rivet holes as there are gauge lines, unless the distance centre to centre of rivets measured in the diagonal direction is at least 40% greater than the distance between the gauge lines."

This rule, which requires the deduction of a full rivet hole in every case, where the stagger with respect to the last rivet would give a diagonal distance less than 140% of the gauge, is needlessly severe. It means, as will be seen from Fig. 3, that whenever the ratio of stagger to gauge is less than 0.98 a full second hole must be deducted, and when this ratio is greater than 0.98 no part of a second hold need be taken away.

A rule proposed by D. B. Steinman in *Engineering News-Record* for June 14, 1917, also based on calculating the section along diagonal lines is as follows:

"The net section of a plate or shape shall be defined as the least section obtainable across the rivet holes, square or zigzag, taking every net distance in a diagonal direction at 85% of its value, except where

85% of the distance is less than the square projection, in which case the latter shall be used instead."

This rule, which will be referred to as Steinman's rule No. 1, may be plotted with respect to various stagger ratios so as to give necessary deductions from the right section. Such has been done for $\frac{7}{8}$-in. rivets and gauges of 3 and 6 in. in Fig. 3. It is thus seen that the rule gives a close approximation to the deductions where the gauge is small, but deductions that are too wide of the mark for the larger gauges. It is rather more difficult to apply than rules which depend wholly on the relation of stagger to gauge, by reason of the necessity of calculating the net section along diagonal lines.

Dr. Steinman proposed a second, and more readily applied, although a more approximate, rule which was stated as follows:

"After deducting the first hole, each successive hole is to be deducted if its stagger s, with respect to the proceeding hole, does not exceed one-half the gauge g, and no deduction is to be made if the stagger equals or exceeds the gauge. For intermediate values of the stagger ratio, a fraction of the hole is to be deducted as may be obtained by interpolation, that is, deduct $2(g-s)/g$."

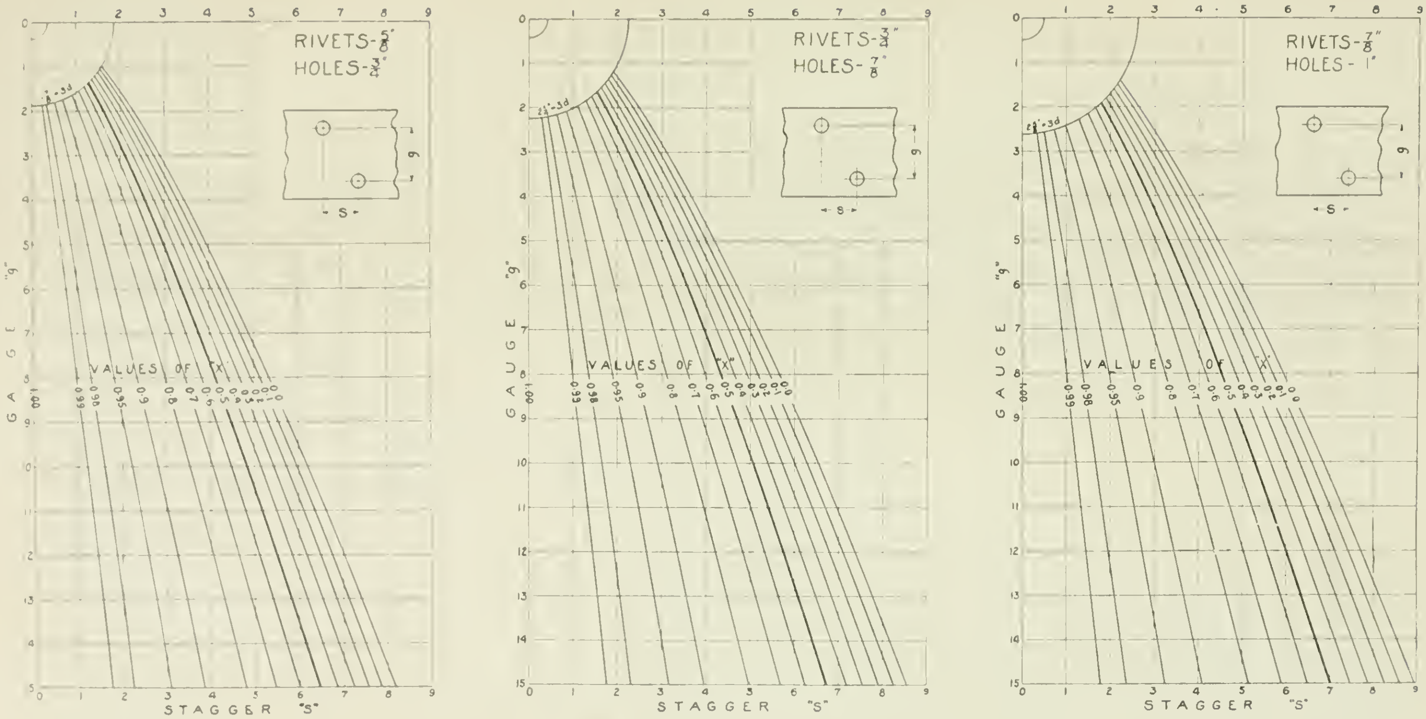
Comparing the graph of this rule, Fig. 3, with the curves giving theoretically-correct deductions, it is seen that for close spacing of rivets it gives deductions that are too great for the smaller values of the stagger ratio and deductions that are too small for large values of this ratio. It gives deductions that are too great for the larger gauges with all applicable values of the stagger ratio. The results given by it, however, represent a fair average between those obtained for the usual range of gauges by rule No. 1. For a simple, easily-applied working rule, to be applied in cases where great accuracy is not required, Steinman's rule No. 2 is very satisfactory. It is, for such cases, much better than any of the others mentioned.

Since any single deduction rule intended for general use with various sizes of rivets and gauges should be based rather more on the worst conditions than on the average ones, it is believed that such a rule should conform more closely to the conditions existing with large rivets and small gauges than does Steinman's rule No. 2. With this in view the writer would propose the following rule:

"The fraction of the area of a rivet hole that must be deducted for the second of any two longitudinal rivet lines shall be calculated by the formula:

$$x = 1.50 - 1.25\frac{s}{g}$$

where x = fraction of the rivet hole in question to be deducted,



(a) (b) (c)

Fig. 4—Theoretically Correct Deductions of Rivet Holes for Various Gauges and Staggers

s = stagger of rivet with respect to rivet on last gauge line,
g = distance between gauge lines.
The deduction for any one rivet is in no case to exceed unity."

This formula, when plotted in Fig. 3, shows close conformity to the conditions likely to obtain in riveted work with close spacings. The curve lies reasonably near those for small gauges for the higher stagger ratios and approaches them for the lower stagger ratios more closely than the curves for the other approximate rules.

DEDUCTIONS BY DIAGRAM

To obviate the large amount of work required in solving formula (1), the diagrams of Fig. 4 have been prepared. These give the theoretically correct deductions for any usual ratio of stagger to gauge. The curves have been drawn for $\frac{5}{8}$, $\frac{3}{4}$ and $\frac{7}{8}$-in. rivets, for staggers up to 9 in. and for gauges up to 15 in.

The use of the diagrams is simple. Thus, suppose it is desired to find the required deduction for a $\frac{3}{4}$-in. rivet on a second gauge line 9 in. from the first gauge line and having a stagger of 5 in. with respect to the preceding rivet. From Fig. 4(b) it is seen that the deduction for the second rivet should be 0.4 hole. Adding this to the first hole the total deduction would be 1.40 holes.

The saving in time effected through the use of diagrams for true effective section will be evident from a study of Fig. 5. The weakest section in this case can be precisely determined only by comparing the deductions on sections A B C D, A B F C D, A B F C I J, A B F G, A B F I J, E F C D, E F G and E F I J. Of course it is usually possible to eliminate some of the zigzag sections by inspection. In the present case, for example, it is evident that the deduction along sections A B F G,

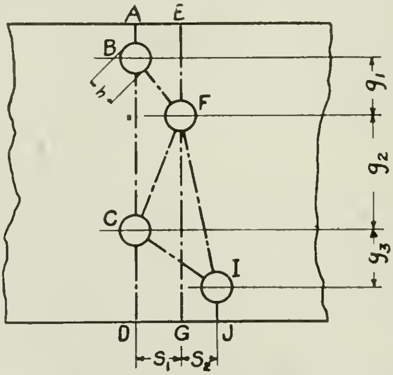


FIG. 5—COMPARISON OF ALTERNATIVE SECTIONS

E F G and E F I J are less than along A B C D. These sections, therefore, need not be considered in making the comparison.

To take a numerical case assume the following values for the indicated dimensions of Fig. 5:

$g_1 = 2\frac{1}{2}$ in. $s_1 = 2$ in.
$g_2 = 5$ in. $s_2 = 1\frac{1}{2}$ in.
$g_3 = 2\frac{1}{2}$ in. $h = 1(\frac{7}{8} + \frac{1}{8})$ in.

By making use of Fig. 4(c) the deductions that apply to such sections as cannot be eliminated by inspection can readily be set down. It is found that the total deductions in the various cases are then as follows:

A B C D $= 1 + 1 =$ 2.00 holes
A B F C D $= 1 + 0.55 + 0.85 =$ 2.40 "
A B F C I J $= 1 + 0.55 + 0.85 + 0 =$ 2.40 "
A B F I J $= 1 + 0.55 + 0.96 =$ 2.51 "
E F C D $= 1 + 0.85 =$ 1.85 "
E F I J $= 1 + 0.96 =$ 1.96 "

Thus the most dangerous section is found to be A B F I J, for which 2.51 holes should be deducted.

It is to be remembered that the formula on which these diagrams is based is so fashioned that the deduction is to be made from the gross right section of the member. If an 8 by $\frac{1}{2}$-in. plate contained two 1-in. holes on a 5-in. gauge and 2-in. stagger the net effective area to be considered in design would be $(8 \times \frac{1}{2}) - (1.85 \times 0.5) = 3.08$ sq. in.

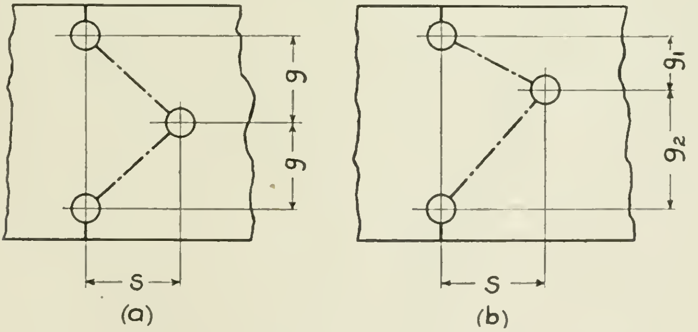


Fig. 6—Stagger to Give Total Deduction of Two Holes

In design it is often desirable to determine the necessary stagger in order that the true deduction may be exactly one hole, or two holes. Thus, for the case shown in the upper right-hand corner of Fig. 4(b), assuming $\frac{3}{4}$-in. rivets, the stagger required to make n, the number of holes to be deducted, $= 1$ is found by noting on Fig. 4(b) the horizontal scale reading of the point where the proper gauge line intersects the curve $x=0$. For the case shown in Fig. 6(a) the value of s required to make $n=2$ is found, if the rivet be $\frac{3}{4}$-in., as before, by reading from Fig. 4(b) the stagger corresponding to the point where the gauge line intersects the curve $x=0.5$. Where the situation is as illustrated in Fig. 6(b) the stagger necessary to make the total deduction equal to exactly two holes, is found by noting on the horizontal scale the reading for the point where a vertical intercept between the gauge lines representing g_1 and g_2 is bisected by the curve $x=0.50$.

In order to apply the methods described, to the calculation of net sections in angles or channels, it is only necessary to consider the angle or channel developed. If, for example, there is one line of rivets in each leg, of an angle, both the gauges are g_1 and the thickness of the angles is t, then the angle would be equivalent to the plate shown in the upper right-hand corner of Fig. 4 (a), g being equal to $2g_1-t$.

INVESTIGATION OF THE PROPORTIONING OF CONCRETE MIXTURES

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(This study is part of a co-ordinated investigation conducted in various laboratories on behalf of the American Society for Testing Materials.)

Although for many years concrete has been recognized as an excellent material of construction, scientific methods in the making of it have not to any extent been heretofore employed. The common practice has been to specify the volumetric ratios of cement, sand and coarse aggregate —such as a 1:2:4 mix—leaving the quantity of water to the judgment of the operator, regardless of the size or grading of the aggregates. It was not generally known that the quantity of water in the mix had any marked effect on the strength of the concrete, and because water made his work easier, the operator used plenty of it. There were so many uncontrolled variables in such methods that the quality of the resulting concrete was a very uncertain thing. In recent years the lack of standard methods in mixing concrete has stimulated investigation, and in 1918 two new theories of proportioning concrete were advocated.

"THE SURFACE AREA THEORY"*

Captain L. N. Edwards, Bridge Engineer, Department of Works, City of Toronto, announced in 1918 the discovery of a fixed relation between the surface area of the mixed aggregates on the one hand and the cement, water and strength of the final mortar or concrete on the other. His work was done mostly on cement mortars involving chiefly surface area of sands. He divided his sands into sizes with the aid of sieves having 4, 8, 10, 20, 30, 40, 50 and 80 openings to the linear inch, and counted the number of particles per gramme of each size. He then determined by experiment the volume displaced by a known number of particles of each size, and from these figures, assuming all particles to be spherical, computed the surface area of a particle of each size. With the number of particles per gramme and the surface area of each particle known, he estimated the surface area per gramme and per 100 lb. From experiment he had previously found that the water necessary for what he called a normal uniform consistency mortar was that required to reduce the cement to a normal consistency paste plus a quantity measured in c.c's. equal to the surface area of the aggregate in square inches divided by 210. He had also found that with consistency constant, the strength depended on the ratio of surface area to weight of cement and he was able to evolve a law establishing that relation. A table giving the surface area per 100 lbs. of all sizes of aggregate ordinarily occurring was easily prepared. When a new aggregate is met, a sieve analysis of it is made and the surface areas are read from the table. These are added and with the fixed relation between strength and the ratio of surface area to cement the quantities of cement and water to be used are readily obtained.

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* Proceedings A. S. T. M., Part 2, 1918.

"THE FINENESS MODULUS THEORY"*

In Bulletin No. 1, Structural Materials Research Laboratory, Lewis Institute, Chicago, Professor D. A. Abrams enunciates his theory as to the correct method of designing concrete mixtures. He states that the old method of proportioning in which fixed quantities of fine and coarse aggregates are mixed without regard to the size and grading of the constituent materials is far from satisfactory, that the surface area method is subject to serious limitations and is based on an erroneous theory, and that all methods previously proposed fail to give sufficient attention to the water content of the mix. He then discusses the effect on strength of changes in mix, consistency and grading of aggregate.

The method Prof. Abrams has developed for measuring the size and grading of an aggregate is called the "Fineness Modulus Method". He uses Tyler standard sieves numbers 100, 48, 28, 14, 8, 4 and the $\frac{3}{8}''$, $\frac{3}{4}''$ and $1\frac{1}{2}''$ sizes, nine sieves in all, the opening in each being just one half that of the one next larger. The fineness modulus of an aggregate is the sum of the percentages coarses than each sieve divided by 100. By making a sieve analysis of any aggregate, using the above series of sieves, one can readily determine its fineness modulus. Having determined the fineness modulus of fine and of coarse aggregates one can obtain the proportions of each in a mixture having a desired fineness modulus by applying the equation:

$$p = 100\frac{A-B}{A-C}$$

where p = percentage of fine aggregate in the mixture
A = fineness modulus of coarse aggregate
B = " " of mixed "
C = " " of fine "

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*Bulletin No. 1, Structural Materials Research Lab., Lewis Institute.

He discovered that when the cement, water and fineness modulus were kept constant while the grading of the material that gave the fineness modulus was varied the strength remained constant.

The volume of water divided by the volume of cement is called the *water-cement ratio*. Professor Abrams states that the water is the most important ingredient in concrete mixtures because small changes in the water content cause more important changes in the quality of concrete than corresponding changes in the other ingredients. The quantity of water required to produce a desired consistency is obtained from his water formula given herewith:

$$x=R\left(\frac{3}{2}p+(0.22-\frac{m}{42}+a-c)n\right)$$

where x = ratio of volume of water to volume of cement in batch.

R = relative consistency of concrete or "Workability factor". Normal consistency (relative consistency = 1.00) requires the use of such a quantity of mixing water as will cause a "slump" of $\frac{1}{2}$ to 1 inch in a freshly molded 6 by 12-inch cylinder of about 1:4 mix upon withdrawing the form by a steady, upward pull. A relative consistency of 1.10 requires the use of 10% more water and under the above conditions will give a slump of about 5 to 6 in.

p = ratio of water to cement by weight for normal consistency.

m = fineness modulus of aggregate.

n = volumes of mixed aggregate to one of cement.

a = absorption of aggregate or ratio of volume of water absorbed to volume of aggregate.

c = moisture contained in aggregate or ratio of volume of water contained to volume of aggregate. (Assumed as zero for room-dry aggregate.)

He shows how strength of concrete increases as the water ratio decreases so long as one still has a workable mix, and also shows that with the consistency and mix constant the strength varies directly as the fineness modulus. In *The Canadian Engineer*, June 26, 1919, he states that the strength and other desirable properties of concrete depend on the quality of the cement paste and are little affected by the quantity so long as we have sufficient to produce a workable mix. The term "workable mix" is taken to mean the driest that can be handled.

The severest criticism of the above two theories for proportioning concrete mixtures came from Messrs. G. M. Williams and W. Davis of the U.S. Bureau of Standards. They assert in several published articles that the figures of each proponent contradict those of the other, that the results obtained by the Bureau of Standards do not substantiate either theory and that neither method is practical for the varying conditions to be met in the field. Mr. R. B. Young, of the Ontario Hydro-Electric Power Commission's Testing Laboratories, has contributed much to the discussion, criticizing each theory but finding much of value in both. He has developed a modified surface area water-cement ratio method which is used for proportioning all the concrete work being carried on by the Commission.

There are so many varying factors in the proportioning of concrete—quality of aggregate, size and grading of aggregate, voids, cement, water, the proportion of each in the mix and the consistency necessary for the job on hand, that it becomes a very difficult undertaking to devise a formula that will fit all the varying conditions of work in the field.

Engineering News-Record, June 12, 1919, in an editorial says: "Until a program of experiments, laid out by some central body, and conducted in many responsible laboratories, develops some concordant results, the proponents of the various theories or methods can hardly wonder that the man in the field still sticks to his old 1:2:4 mix".

Early in 1920 committee C-9 on Concrete and Concrete Aggregates of the A.S.T.M. prepared a programme of experiments (Series 201) intended to constitute a study of the effect of size and grading of aggregates, and of variations in the quantities of cement and water on the strength and other properties of concrete, and secured the co-operation of the following laboratories to carry out the work:

Hydro-Electric Power Commission of Ontario, Toronto.
Department of Public Works, Philadelphia, Pa.
University of Wisconsin, Madison, Wis.
" " Texas, Austin, Texas.
" " Toronto, Toronto, Ontario.

The following summary concluded the specifications of Committee C-9:

CO-OPERATIVE TESTS OF CONCRETE

Outline of Tests

(7th revision, February 3, 1920)

"This series of tests constitutes a study of the effect of size and grading of aggregates, quantities of mixing water and cement on the strength of concrete. These tests will furnish a basis for comparing different proposed methods of designing concrete mixtures.

"Compression tests to be on 6 x 12-in. cylinders.

"Mix to be done by volume.

"Concrete for each specimen to be proportioned and mixed separately by hand.

"The five cylinders in a set to be made on different days.

"Aggregates: local materials to be mixed in a room-dry condition. A commercial sand is to be used. The coarse aggregate to be graded from No. 4 to $1\frac{1}{4}$ or 2 in.

"Portland cement to be a mixture of equal parts of three or more local brands.

"Normal consistency of concrete (relative consistency = 1.00) is based on the use of such a quantity of mixing water as will cause a slump of $\frac{1}{2}$ to 1 inch in a freshly molded 4 by 8 by 12-in. frustum of a right circular cone upon withdrawing the form immediately after molding. If desired, duplicate slump tests may be made by other methods.

"Cylinders to be capped with neat cement 2 or 4 hours after molding.

"Specimens to be stored in damp sand or saturated atmosphere and tested damp.

"Age at test to be 28 days".

PROPORTIONING AND MIXING

All concrete was proportioned by volume, that is, one volume of cement to a given number of volumes of mixed aggregate. The exact quantities of materials to be used in the batch were determined by weight, assuming cement to weigh 94 lb. per cu. ft. The required quantities of each size of aggregate were determined on the basis of the unit weights of the mixed aggregates.

The variables in the series of cylinders to be made were:

Mix	*Consistency*	*Fineness Modulus*
1:3.5	0.8	x
1:5	0.9	4.5
1:7	1.0 normal	5.5
....	1.1	5.75
....	1.25	6.0
....	1.5	6.25
....	....	6.5

The different values of "fineness modulus" of the mixed aggregate were obtained by mixing the fine and coarse aggregates in proportions which produced the values shown. The fineness modulus marked (x) refers to the value of the fine aggregate only. Over 32 bags of three varieties of Portland cement were emptied into a pile, thoroughly mixed and re-bagged. The sand secured was clean, free from organic material, had a fineness modulus of 4.0 and weighed 109 lbs. per cu. ft. The coarse aggregate was clean, tough limestone with a fineness modulus of 7.5 and weighed 99 lbs. per cu. ft. There are 126 possible combinations and it was intended to make 5 cylinders of each combination. Only 4, however, were made. This gave 504 batches in all. The four duplicates were made on separate days.

THE DETERMINATION OF NORMAL CONSISTENCY

In order to eliminate the personal factor where one judges the consistency to be normal or otherwise, several tests have been suggested:

(1) *The Chute Test.*—Here the mixed concrete is placed in one end of a metal chute and the other end is lowered until the concrete starts flowing. The angle at which the material begins to flow is a measure of the workability or consistency.

(2) *The Flow-table Test.*—Here a molded mass of freshly mixed concrete is placed on a table and the table is jarred in a prescribed manner. The relation between the diameters of the base of the mass after and before jarring is a measure of the consistency.

(3) *The Slump Test.*—This is referred to above. A truncated cone 4 in.×8 in.×12 in. high was used instead of a cylinder.

Of the three methods Committee C-9 chose the "Slump Test". The instructions were followed very carefully, but such variable results were obtained that the slump test was abandoned. The intention of the Committee was to have the consistencies range from very dry through normal to very wet, using some method of determining consistency more dependable than the operator's judgment. After a study of Professor Abram's water formula referred to above, and after several trials in which it was used, the conclusion was reached that it gave consistencies that appeared to be normal. In order not to delay the work this formula was adopted and Table No. 1 prepared which indicates the quantity of cement, sand, stone and water for each of the 126 mixes.

The most difficult part of the work was to avoid loss of water during and after tamping the batch into the form. The procedure finally adopted was to have the cylindrical form very clean and the base of it ground perfectly flat, to paint the seam of the form with hot paraffin, to oil a square of plate glass and place on it a sheet of waxed paper, to paint the paper with hot paraffin, to place the cylinder on the prepared base and to paint hot paraffin around the bottom of it to seal it to the base. Great care was necessary to avoid breaking this seal while tamping the concrete into the form. The success of these efforts to avoid loss of water is indicated by the closeness of the strength results for duplicate

TABLE No. 1.—WORKING CHART

CEMENT TO AGGREGATE 1:7 – 1/4 cu. ft. batches

Fineness Modulus	Cement		Sand		Stone		Water in c.c's.					
							0.8 N	0.9 N	N	1.1 N	1.25 N	1.5 N
	lbs.	oz.	lbs.	oz.	lbs.	oz.	c.c.	c.c.	c.c.	c.c.	c.c.	c.c.
4.0	3	5.6	27	4	..	....	1024	1152	1281	1410	1600	1920
4.5	....		23	3.2	3	11.5	956	1076	1196	1315	1495	1794
5.5	....		15	8	10	9.6	820	921	1024	1126	1280	1536
5.75	....		13	9.7	12	8	788	887	986	1084	1230	1480
6.0	....		11	11.2	14	3.2	752	846	940	1034	1175	1410
6.25	....		9	13.7	15	14.4	717	807	897	986	1120	1346
6.5	....		7	5.6	18	3	683	768	854	939	1070	1280
Mix=1:5 – 1/4 cu. ft. batches												
	lb.	oz.										
4.0	4	11.2	Same as		above		1140	1282	1425	1567	1780	2140
4.5	....						1076	1210	1345	1480	1680	2017
5.5	....						937	1054	1172	1290	1465	1760
5.75	....						904	1020	1131	1244	1414	1700
6.0	....						865	975	1082	1190	1352	1620
6.25	....						830	934	1038	1140	1295	1560
6.5	....						800	900	998	1100	1250	1500
Mix=1:3.5 – 1/4 cu. ft. batches												
	lb.	oz.										
4.0	6	11.2					1316	1480	1645	1810	2056	2470
4.5	....		Same as		above		1248	1400	1560	1720	1950	2340
5.5	....						1112	1250	1390	1530	1740	2085
5.75	....						1080	1215	1350	1485	1690	2025
6.0	....						1048	1179	1310	1441	1640	1965
6.25	....						1012	1138	1265	1391	1580	1900
6.5	....						976	1098	1220	1342	1525	1830

TABLE No. 2

Average Strength in lb. per sq. in.

Mix	Consistency	Fineness Modulus							
		4.0	4.5	5.5	5.75	6.0	6.25	6.5	Average
1:7	0.8	650	1040	1637	1804	2034	2064	1810	1577
	0.9	846	1035	1823	2165	2297	2446	2498	1873
	1.0	764	872	1727	1965	2270	2075	2060	1676
	1.1	693	817	1470	1549	1781	1806	1898	1430
	1.25	506	618	970	1062	1214	1200	1212	969
	1.5	358	377	586	635	705	728	781	596
Average		636	793	1352	1530	1717	1719	1709	
1:5	0.8	1155	1460	2573	2662	2998	2570	2427	2263
	0.9	1395	1696	3243	3382	3541	3580	3330	2881
	1.0	1448	1692	2873	2918	3174	3208	2692	2572
	1.1	1368	1534	2283	2485	2737	2616	2262	2183
	1.25	1105	1119	1662	1770	1966	1955	1603	1597
	1.5	589	740	1040	1004	1058	1069	1055	935
Average		1177	1373	2279	2370	2579	2499	2228	
1:3.5	0.8	2012	2370	2873	3517	3452	3782	3603	3087
	0.9	2325	2997	3675	3770	3915	4309	4172	3595
	1.0	2665	2899	3457	3522	3740	3869	3703	3408
	1.1	2325	2450	2613	3360	3389	3415	2947	2928
	1.25	1775	1614	2303	2405	2617	2732	2038	2212
	1.5	1120	1060	1425	1372	1436	1548	1294	1322
Average		2037	2231	2724	2991	3091	3276	2960	

TABLE No. 3

Water-cement ratio for all mixes

Mix	Consistency	Fineness Modulus						
		4.0	4.5	5.5	5.75	6.0	6.25	6.5
1:7	0.8	1.01	0.94	0.81	0.77	0.74	0.70	0.67
	0.9	1.14	1.06	0.91	0.87	0.83	0.79	0.75
	1.0	1.27	1.18	1.01	0.97	0.93	0.88	0.84
	1.1	1.39	1.30	1.11	1.07	1.02	0.97	0.93
	1.25	1.58	1.47	1.27	1.22	1.16	1.11	1.06
	1.5	1.90	1.78	1.52	1.47	1.40	1.33	1.27
1:5	0.8	0.80	0.76	0.66	0.64	0.61	0.58	0.56
	0.9	0.90	0.85	0.74	0.72	0.69	0.66	0.63
	1.0	1.0	0.95	0.82	0.80	0.76	0.73	0.70
	1.1	1.10	1.05	0.91	0.88	0.84	0.80	0.78
	1.25	1.26	1.18	1.03	1.0	0.95	0.91	0.88
	1.5	1.51	1.42	1.24	1.20	1.14	1.10	1.06
1:3.5	0.8	0.65	0.61	0.56	0.53	0.51	0.50	0.48
	0.9	0.73	0.69	0.62	0.60	0.58	0.56	0.54
	1.0	0.81	0.77	0.69	0.67	0.65	0.62	0.60
	1.1	0.89	0.85	0.76	0.73	0.71	0.69	0.66
	1.25	1.01	0.96	0.86	0.83	0.81	0.78	0.75
	1.5	1.22	1.15	1.03	1.0	0.97	0.94	0.90

batches. Of course, water was lost in a few cases, and for this and other reasons not apparent, a very few eccentric results were obtained from the several hundred cylinders tested. Table No. 2 gives the average strength of these four cylinders for each of the 126 different mixes.

Referring to Table No. 1 it is very apparent that as the fineness modulus and the proportion of stone to sand increases, the quantity of water necessary for a given consistency decreases. In the 1:7 mix the quantity of water that makes a batch, whose fineness modulus is 4.0, of normal consistency, makes a batch, whose fineness modulus is 6.5, of 1.5 consistency. The mix is the same, the water the same, but the size of the aggregate changes the consistency. In every case the consistencies given as 0.8 were all very dry; the normal consistencies were all apparently of equal moistness and the 1.5 consistencies were semi-fluid. After completing the work it is considered that Professor Abram's water formula is a satisfactory means of determining the water necessary for normal consistency.

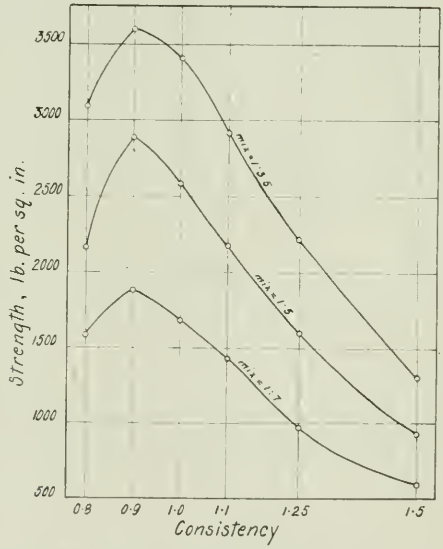

Fig. No. 1. Showing the Relation between Strength and Water Content.

From Table No. 2 and the curve in Fig. No. 1 it may be noted that the proportion of water in the mix has a decided effect on the strength of the concrete. With few exceptions the maximum strength is obtained when the consistency is 0.9. From that point the strength rapidly falls until with a consistency of 1.5 (one commonly seen in practice) the

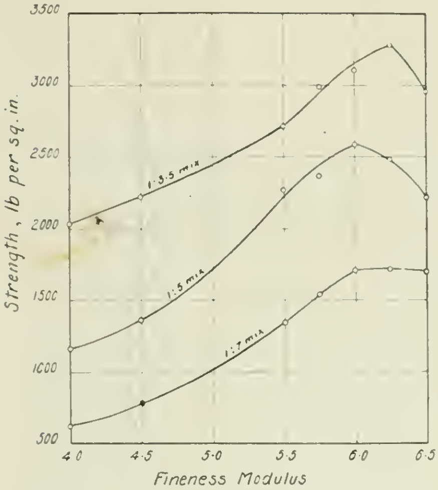

Fig. No. 2. Showing the Relation between Strength and Fineness Modulus.

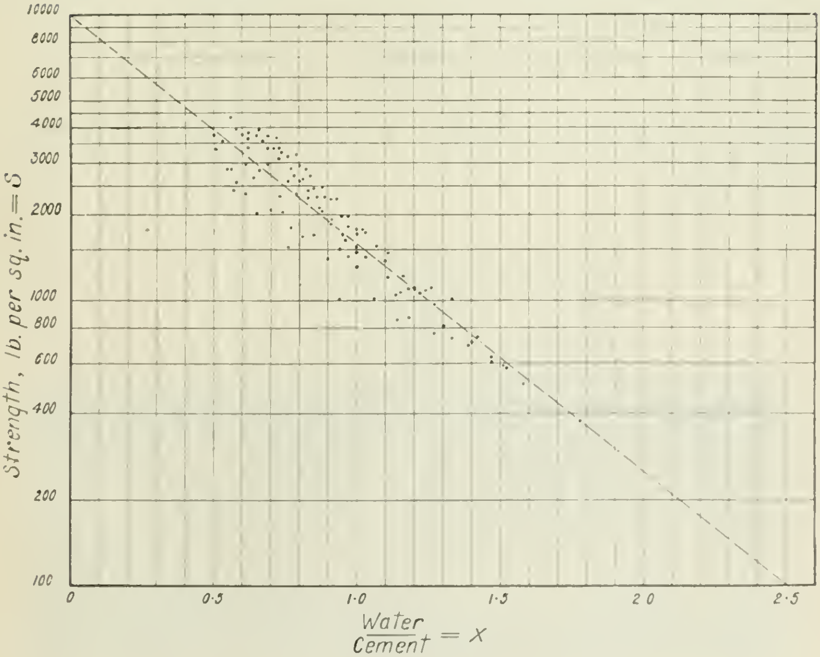

Fig. No. 3. Strength is here plotted against Water-Cement Ratio. The Points roughly follow the law $S = \frac{10{,}000}{6.3^x}$.

concrete has only about one quarter the strength that might have been developed with less water. The same table of figures and the curve in Fig. 2 show the effect of fineness modulus on strength. The strength increases directly as the fineness modulus up to a point where the mixture is difficult to handle on account of its containing too large a proportion of stone and too little sand. From the tests it is obvious that greater strength results from the driest workable consistency and the largest workable fineness modulus.

As stated previously, Professor Abrams asserts that so long as one has a workable mix the strength of concrete is more dependent on the water-cement ratio than on any other factor, the strength varying inversely as this ratio. In Fig. No. 3 the average strength of four clyinders has been plotted against the water-cement ratio for 126 different mixes using semi-logarithmic paper. Apparently there is no precise relation between strength and the water-cement ratio, but rather a general tendency for the strength to increase as the water-cement ratio diminishes. Referring to the three ordinates where the ratio is 0.65, 0.8, and 1.0, it will be seen that the strength may vary 100% or more above a minimum. The law in so far as a law exists, seems to be according to the equation:

$$S = \frac{10{,}000}{6.3^x}$$

where S is compressive strength in lb. per sq. in., and x is the water-cement ratio.

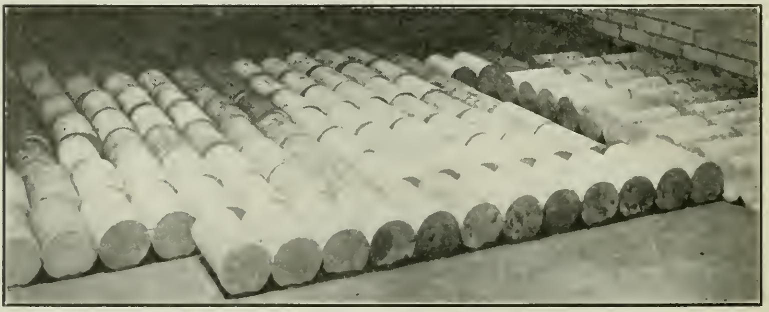

Fig. No. 4. Test Cylinders in Storage.

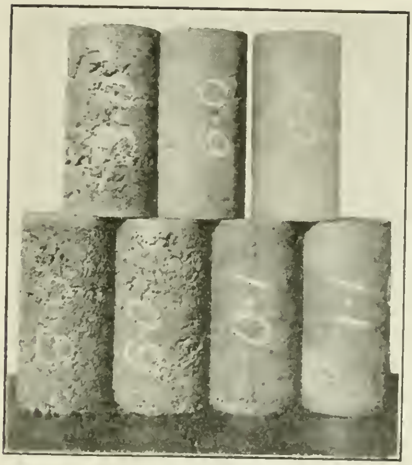

Fig. No. 5. The Consistency numbers are shown on the Cylinders.

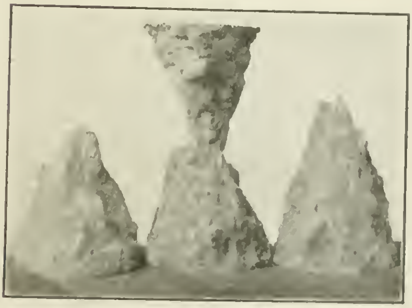

Fig. No. 6. Typical Failures.